FlightControlEx.m

This example computes the robust stability margins and worst-case disturbance rejection performance for a rigid body transport aircraft with an output feedback control law. The uncertainty model has 14 real parametric uncertainties associated with the aerodynamic coefficients of the aircraft. The uncertainty model also includes unmodeled actuator dynamics. This example is taken from the following textbook: "A Practical Approach to Robustness Analysis with Aeronautical Applications" by G. Ferreres, Kluwer, 1999.

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Contents

Create Open-Loop Model

The open-loop model includes the rigid-body aircraft dynamics. These dynamics contain uncertainties in the aerodynamic coefficients. The open-loop model also includes dynamics for the rudder and aileron actuators. The actuator models also include dynamic uncertainty.

The model parameters are (Appendix A.1, p.187):

```
deg2rad = pi/180;
rad2deg = 1/deg2rad;
gV = 0.146418;          % g/V
tan_theta0 = 0.14;      % tan(theta0)
alpha0 = 8*deg2rad;     % (rad)
```

The uncertain aerodynamic coefficients are:

```
Ybeta = ureal('Ybeta',-0.082,'Percentage',10);
Yp = ureal('Yp',0.010827,'Percentage',10);
Yr = ureal('Yr',0.060268,'Percentage',10);
Ydeltap = ureal('Ydeltap',0.002,'Percentage',10);
Ydeltar = ureal('Ydeltar',0.0118,'Percentage',10);
Lbeta = ureal('Lbeta',-0.84,'Percentage',10);
Lp = ureal('Lp',-0.76,'Percentage',10);
Lr = ureal('Lr',0.74,'Percentage',10);
Ldeltap = ureal('Ldeltap',0.095,'Percentage',10);
Ldeltar = ureal('Ldeltar',0.06,'Percentage',10);
Nbeta = ureal('Nbeta',0.092,'Percentage',10);
Np = ureal('Np',-0.23,'Percentage',10);
Nr = ureal('Nr',-0.114,'Percentage',10);
Ndeltar = ureal('Ndeltar',-0.151,'Percentage',10);
```

The states, inputs, and outputs are given by:

- States = [beta; p; r; phi] = [sideslip; roll rate; yaw rate; roll angle]

- Inputs = [deltap; deltar] = [aileron deflection; rudder deflection]
- Outputs = [ny; p; r; phi] = [accel; roll rate; yaw rate; roll angle]

The state equations (See Eq 2.1/2.2 on p.30 and p.188) are:

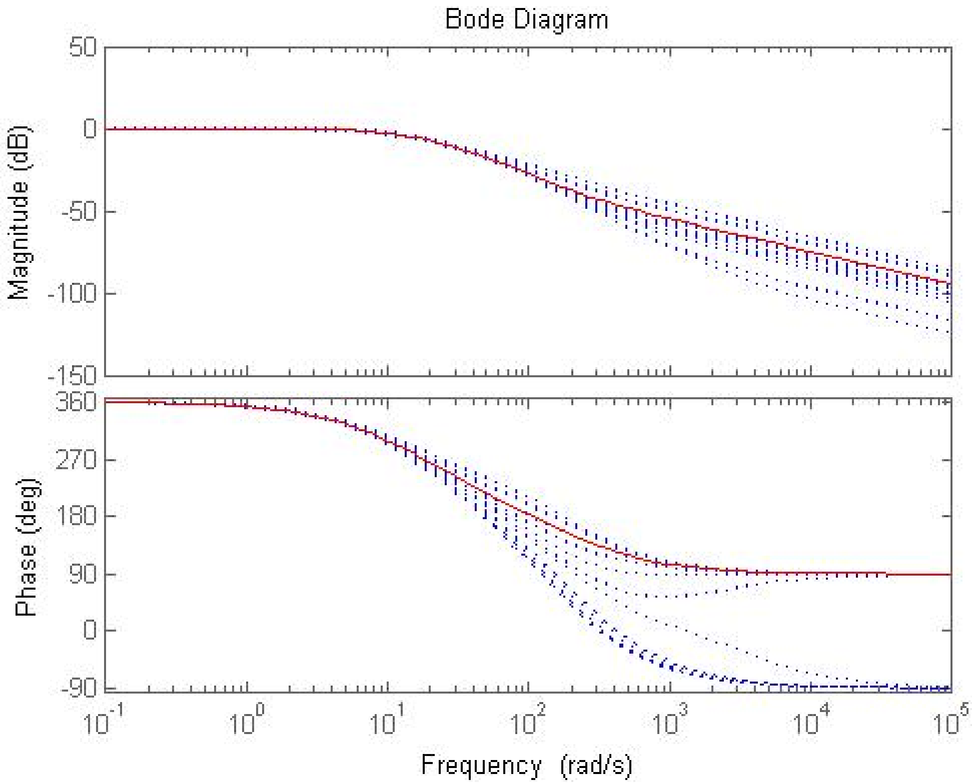
```
A = [Ybeta (Yp+sin(alpha0)) (Yr-cos(alpha0)) gV; ...
    Lbeta  Lp Lr 0; Nbeta Np Nr 0; 0 1 tan_theta0 0];
B = [Ydeltap Ydeltar; Ldeltap Ldeltar; 0 Ndeltar; 0 0];
C = -1/gV*deg2rad*[Ybeta Yp Yr 0];
C = [C; zeros(3,1) eye(3)];
D = -1/gV*deg2rad*[Ydeltap Ydeltar];
D = [D; zeros(3,2)];
AIRCRAFT = ss(A,B,C,D);
```

The nominal models for rudder and aileron actuators are:

```
N1 = [-1.77, 399];
D1 = [1 48.2 399];
deltap_act_nom = tf(N1,D1);

N2 = [2.6 -1185 27350];
D2 = [1 77.7 3331 27350];
deltar_act_nom = tf(N2,D2);
```

Include multiplicative uncertainty on the actuator dynamics. The uncertainty weight for each actuator specifies 10% uncertainty at low frequencies and 200% uncertainty at high frequencies. The uncertainty weights for each actuator cross 100% uncertainty at roughly 5*wb where wb denotes the -6dB bandwidth of the actuator. The Bode magnitude plot shows the nominal aileron actuator dynamics (red) and samples drawn from the actuator uncertainty set. The actuator gain and phase variation is evident at high frequencies. The rudder actuator has similar uncertainty.

```
Wup = makeweight(0.1,80,2);
deltap_act = deltap_act_nom*(1+Wup*ultidyn('Delp_act',[1 1]));

Wur = makeweight(0.1,170,2);
deltar_act = deltar_act_nom*(1+Wur*ultidyn('Delr_act',[1 1]));

figure(1)
bode(deltap_act,'b:',deltap_act_nom,'r')
```



The uncertain aircraft model is

```
P = AIRCRAFT*blkdiag(deltap_act,deltar_act);
```

Control Law

A constant gain output feedback law is used. The gain below is taken from documentation in the SMT toolbox.

```
K = [-629.8858 11.5254 3.3110 9.4278; ...
  285.9496 0.3693 -2.6301 -0.5489];
```

Closed-Loop

Use the FEEDBACK command to form the closed-loop system from an input disturbance to plant output. The closed-loop is nominally stable. The nominal disturbance rejection performance is assessed by computing the H-infinity norm of the nominal closed-loop transfer function.

```
CLOOP = feedback(P,K);
isstable(CLOOP.Nominal)
ng=norm(CLOOP.Nominal,inf)
```

```
ans =

     1
```

```
ng =

    0.2258
```

Robust Stabilty

The closed-loop is uncertain due to the real parametric and dynamic uncertainty in the plant model. An uncertain frequency response for the closed-loop system is computed with the UFRD command. Robust stability of the closed loop is assessed using the ROBUSTSTAB command. The closed loop is robustly stable and can tolerate up to 276% of the modeled uncertainty.

```
w = logspace(-1,3,100);
CLOOPfr = ufrd(CLOOP,w);
[stabmarg,destabunc,report,info] = robuststab(CLOOP);
stabmarg
```

```
stabmarg =

                LowerBound: 2.7601
                UpperBound: 3.1003
    DestabilizingFrequency: 0.8416
```

ROBUSTSTAB returns destabilizing uncertainty values that are within 310% of the modeled uncertainty. The destabilizing perturbation causes an instability at 0.842 rad/seconds. This destabilizing perturbation could be further investigated, e.g. it can be used in a high fidelity simulation.

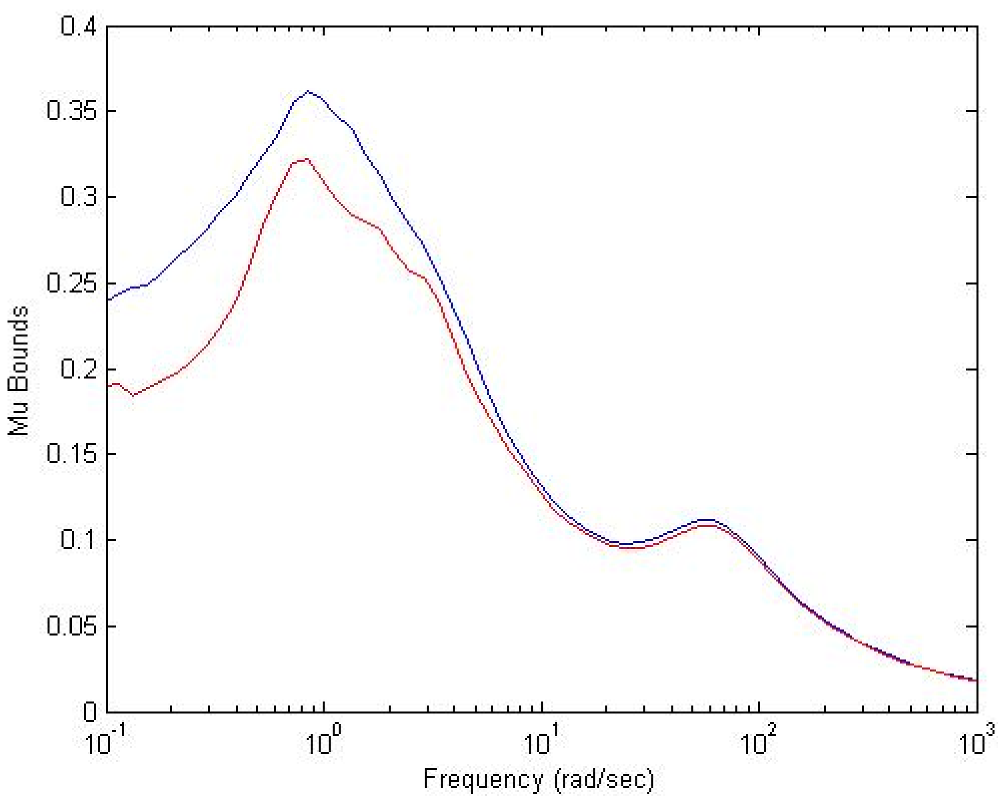
```
damp(pole(usubs(CLOOP,destabunc)))
```

```
       Eigenvalue            Damping     Frequency

 -2.96e+02                   1.00e+00     2.96e+02
 -1.39e+02                   1.00e+00     1.39e+02
 -3.64e+01 + 4.88e+01i       5.98e-01     6.09e+01
 -3.64e+01 - 4.88e+01i       5.98e-01     6.09e+01
 -3.68e+01                   1.00e+00     3.68e+01
 -6.71e+00                   1.00e+00     6.71e+00
 -4.83e+00                   1.00e+00     4.83e+00
 -2.79e+00                   1.00e+00     2.79e+00
 -1.09e+00 + 1.04e+00i       7.23e-01     1.50e+00
 -1.09e+00 - 1.04e+00i       7.23e-01     1.50e+00
 -1.76e-14 + 8.42e-01i       2.09e-14     8.42e-01
 -1.76e-14 - 8.42e-01i       2.09e-14     8.42e-01
 -7.28e-01                   1.00e+00     7.28e-01
 -4.37e-01 + 2.91e-01i       8.32e-01     5.24e-01
 -4.37e-01 - 2.91e-01i       8.32e-01     5.24e-01
 -3.27e-01                   1.00e+00     3.27e-01
 -2.08e-01                   1.00e+00     2.08e-01

(Frequencies expressed in rad/TimeUnit)
```

The ROBUSTSTAB performs a mu analysis in order to compute the stability margins. The plot below shows the mu upper and lower bounds which are stored in the info structure. The stability margins returned by ROBUSTSTAB are inversely related to the peak of the mu lower/upper bound plots.

```
figure
semilogx(info.MussvBnds(1),'b',info.MussvBnds(2),'r')
xlim([w(1) w(end)])
xlabel('Frequency (rad/sec)');
ylabel('Mu Bounds');

stabmarglb = 1/norm(info.MussvBnds(1),inf)
stabmargub = 1/norm(info.MussvBnds(2),inf)
```

```
stabmarglb =

    2.7601


stabmargub =

    3.1003
```



Worst-case performance

The worst-case performance is the largest H-infinity norm achieved over the set of closed-loop transfer functions described by the modeled uncertainty. This is computed with the WCGAIN command. The worst-case performance is approximately 50 percent larger than the nominal performance. The worst-case gain occurs at a low frequency.

```
[wcg,wcu,wcinfo] = wcgain(CLOOPfr);
wcg
```

```
wcg = 

          LowerBound: 0.3321
          UpperBound: 0.3322
    CriticalFrequency: 0.1000
```

WCGAIN returns uncertainty values that achieves the computed lower bound. This can be verified using the USUBS command to evaluate the uncertain closed-loop system at the worst-case uncertainty values

```
norm( usubs(CLOOPfr,wcu), inf)
```

```
ans =

    0.3321
```

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