Constructing the smallest destabilizing perturbation (at Plant input).

A nominal plant and controller are given. There are several tasks to complete, culminating in construction of the smallest input-multiplicative plant perturbation that leads to instability.

MUSYNID

Contents

Plant, controller specification

2-input, 2-output plant and controller are specified in state-space form.

```
Ap = [ -0.2  10; -10  -0.2];
Bp = eye(2);
Cp = [1 8;-10 1];
P = ss(Ap,Bp,Cp,0);

Ac = blkdiag(zeros(2,2),-62.83,[-15.68 11;-11 -15.68],-14.33);
Bc = [0.5343   0.2178;...
     -0.2174   0.5255;...
      3.019   -3.903;...
      0.1649    2.265;...
     -1.308    -2.36;...
     -2.334  0.06362];
Cc = [ -4   0   -1.369  -0.02616  1.771  -3.681;...
        0 -4   -1.467  -3.616   -1.955  -0.6843];
C = ss(Ac,sqrt(3)*Bc,sqrt(3)*Cc,0);
```

Verify closed-loop system is stable

Form all closed-loop maps with `LOOPSENS`

```
help loopsens
```

```
 --- help for DynamicSystem/loopsens ---
```

```
LOOPSENS  Sensitivity functions of plant-controller feedback loop.

   SF = LOOPSENS(P,C) analyzes the multivariable negative feedback loop
   with plant P and controller C:

                  u
          du --->O----->[ P ]-----+---> yP
               - |                |
                 |             y  |
          uC <---+<-----[ C ]-----O<--- dy

   For "2-dof" architectures, C should only include the portion of the
   controller in the feedback path.

   SF is a structure containing the following sensitivity functions:
       Si   Sensitivity at the plant input
       Ti   Complementary sensitivity at the plant input (I-Si)
       Li   Open-loop transfer at the plant inputs (CP)
       So   Sensitivity at the plant output
       To   Complementary sensitivity at the plant output (I-So)
       Lo   Open-loop transfer at the plant outputs (PC)
      PSi   Closed-loop transfer from plant input to plant output
      CSo   Closed-loop transfer from controller input to controller
            output.

   See also LOOPMARGIN, ROBUSTSTAB, ROBUSTPERF, WCSENS, WCMARGIN.
```

Compute Hinf norm of Ti

Ti is complementary sensitivity at plant input

Compute Frequency Response Matrix of Ti at frequency where peak occurs

Find the smallest plant input-multiplicative uncertainty (a constant, complex matrix of dimension 2-by-2) which causes instability

Perturbed closed-loop system

Form perturbed plant, using input-multiplicative form and the perturbation matrix.

Verify Instability of perturbed closed-loop system

Compute perturbed closed-loop system using `loopsens`, and check poles

Connection between unstable pole, and frequency where peak of Ti occurs

Show connection between the two frequencies

(Extra Credit!) Use `cnum2sys` to create real-dynamic uncertainty

Rather than accepting a complex matrix as a destabilizing element, use the command `cnum2sys` to convert the complex matrix uncertainty into a real-valued, linear, dynamic system that leads to instability. The Hinf norm of the dynamic system should match the maximum singular value of the original destabilizing complex matrix.

Frequency-dependent uncertainty

Adopt a frequency-dependent uncertainty model, using a scalar weighting function, wu. Take uncertainty model to be `P*(eye(2) + wu` `(s) Delta)`, using a first order `wu` with:

- DC gain of 0.35 (35% uncertainty at low frequency)
- gain of 1 at 40 rads/TimeUnit (100% uncertainty at 40)
- High-Frequency gain of 10 (1000% uncertainty at high frequency)

Use `makeweight` for the construction.

Mimic/repeat procedure to find smallest destabilizing `Delta`, naming the perturbation `wDelta` in this case. Both **constant, complex** and **real, linear dynamic** perturbations can be constructed, following the same procedure, incorporating the weighting function `wu`. Start by computing the norm of the weighted complementary sensitivity function.

Constant, complex perturbation is constructed with the same steps, based on SVD

Real, linear, dynamic perturbation is constructed in the same manner, using `cnum2sys` on the elements from the SVD construction.

The constructed perturbations are of the same magnitude.

Both result in closed-loop poles on the imaginary axis at the frequency associated with the peak value of the `wu*Ti`.

Conclusions

Using the `svd` command, the smallest destabilizing uncertainty for an input-multiplicative uncertainty model is constructed. It is initially constructed as a complex matrix, but can also be constructed as a real, linear, dynamic system, using `cnum2sys`. If the input-multiplicative uncertainty model is frequency-dependent, the same process can be used, simply incorporating the weighting function

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