



Lecture 5: Distributed Control Systems



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Goals:

- Define centralized versus decentralized versus distributed control
- Describe analysis results in stability of distributed control systems, with a focus on the role of topology of the information flow
- Summarize synthesis techniques for distributed control systems

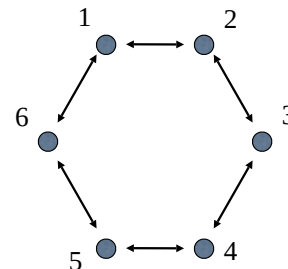
Reading:

- J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations", *IEEE T. Automatic Control*, 2004.
- V. Gupta, B. Hassibi and R. M. Murray, "A sub-optimal algorithm to synthesize control laws for a network of dynamic agents", *Int'l J. Control*, 2005
- R. D'Andrea and G. E. Dullerud, "Distributed Control Design for Spatially Interconnected Systems", *IEEE T. on Automatic Control*, 2003.
- M. Rotkowitz and S. Lall, "A Characterization of Convex Problems in Decentralized Control", *IEEE T. on Automatic Control*, 2006.

Sample Problem: Formation Stabilization

Goal: maintain position relative to neighbors

- "Neighbors" defined by graph
- Assume only sensed data for now
- Assume identical vehicle dynamics, identical controllers?

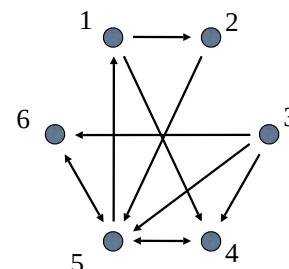


Example: hexagon formation

- Maintain fixed relative spacing between left and right neighbors

$$e_i = \sum_{j \in N_i} w_j (y_i - y_j - h_{ij})$$

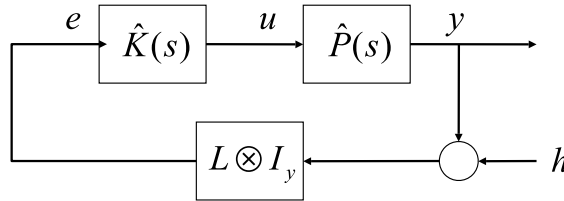
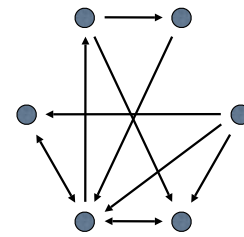
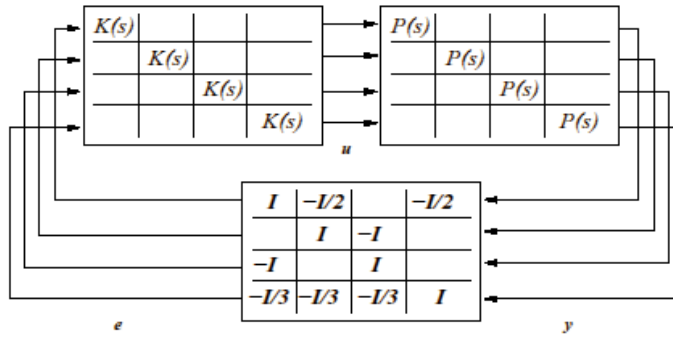
weighting factor
relative position
offset



Can extend to more sophisticated "formations"

- Include more complex spatio-temporal constraints

Mathematical Framework



Analyze stability of closed loop

- Interconnection matrix, L , is the *weighted Laplacian* of the graph
- Stability of closed loop related to eigenstructure of the Laplacian

Stability Condition

Fax and M
IEEE TAC 2004

Agent dynamics

$$\begin{aligned}\dot{x}^i &= Ax^i + Bu^i \\ y^i &= Cx^i\end{aligned}$$

Control law

$$\begin{aligned}\dot{\xi}^i &= F\xi^i + Gz^i \\ u^i &= H\xi^i + Kz^i\end{aligned}$$

Weighted error

$$z^i = \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (y^i - y^j)$$

- Agents have identical, linear dynamics
- Control law is dynamic compensator based on sum of relative errors on neighbors
- Can also allow feedback on internal state (fold into A)

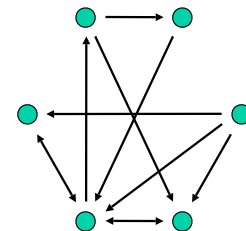
Theorem Let L be the weighted Laplacian of the communications graph $\mathcal{G}$. The closed loop system is (neutrally) stable iff the systems

$$\begin{aligned}\dot{x}^i &= Ax^i + Bu^i & \dot{\xi}^i &= F\xi^i + Gz^i \\ z^i &= \lambda_j Cx^i & u^i &= H\xi^i + Kz^i\end{aligned}$$

are stable for each eigenvalue λ_i of L .

Remarks

- Stability is based on check of n decoupled systems
- λ_i plays the role of a "loop gain": describes how your output affects your input



$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 0 & 1 & -1 & 0 \\ -\frac{1}{3} & 0 & 0 & -\frac{1}{3} & 1 & -\frac{1}{3} \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Sketch of Stability Proof

Notation

- $\hat{A} = I_N \otimes A$: block diagonal matrix with A as elements
- $A_{(n)} = A \otimes I_n$: replace elements of A with $a_{ij}I_n$
- For $X \in \mathbb{R}^{r \times s}$ and $Y \in \mathbb{R}^{N \times N}$, $\hat{X}Y_{(s)} = \hat{Y}X_{(r)}$

$$\dot{x}^i = Ax^i + Bu^i$$

$$z = L\hat{C}x$$

$$\dot{\xi}^i = F\xi^i + Gz^i$$

$$u^i = H\xi^i + Kz^i$$

Let T be a Schur transformation for L , so that $U = T^{-1}LT$ is upper triangular. Transform the (stacked) process states as $\tilde{x} = T_{(n)}x$ and the (stacked) controller states as $\tilde{\xi} = T_{(n)}\xi$. The resulting dynamics become

$$\frac{d}{dt} \begin{bmatrix} \tilde{x} \\ \tilde{\xi} \end{bmatrix} = \begin{bmatrix} \hat{A} + \hat{B}\hat{K}\hat{C}U_{(n)} & \hat{B}\hat{H} \\ \hat{G}\hat{C}U_{(n)} & F \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{\xi} \end{bmatrix}.$$

Slightly simpler version: just prove for output feedback case with no ξ variables. This gets rid of the non-triangular block structure.

block

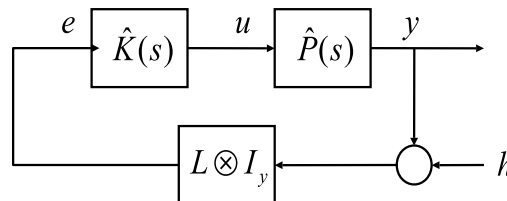
This system is upper triangular, and so stability is determined by the elements on the (block) diagonal:

$$\frac{d}{dt} \begin{bmatrix} \tilde{x}_j \\ \tilde{\xi}_j \end{bmatrix} = \begin{bmatrix} A + BKC\lambda_j & BH \\ GC\lambda_j & F \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{\xi} \end{bmatrix}.$$

Alternatively, rearrange states as (x, ξ) for each block and then you can recover the block structure.

This is equivalent to coupling the process and controller with a gain λ_i .

Frequency Domain Interpretation



Theorem The closed loop system is (neutrally) stable iff the Nyquist plot of the open loop system does not encircle $-1/\lambda_1(L)$, where $\lambda_1(L)$ are the nonzero eigenvalues of L .

Example

$$P(s) = \frac{e^{-s\tau}}{s^2} \quad K(s) = K_d s + K_p$$

