CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

ACM 101/AM 125a/CDS 140a

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Problem Set #6

Issued: 8 Feb 11
Due: 17 Feb 11

1. Perko Section 2.14 Problem 1

2. Perko Section 2.14 Problem 7

3. Perko Section 2.14 Problem 11

4. Perko Section 2.14 Problem 12

5. Given the Hamiltonian

$$H(x, y) = \frac{8}{3} - x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{2}y^2$$

a) Write down the first order system in $\mathbb{R}^2$. Find and classify the fixed points. Sketch the phase portrait.
b) Use a Lyapunov function to determine the stabilities of the fixed points not at the origin.
c) Let's add the damping term $-\epsilon y$ ($\epsilon > 0$) to the "$\dot{y}$ equation" in part (a). Use LaSalle's invariant principle to show that as $t \to \infty$ all the solutions approach one of the fixed points in part (a).