CALIFORNIA INSTITUTE OF TECHNOLOGY
BioEngineering

BE 250C

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Winter 2011

Problem Set #6

Issued: 12 Feb 11
Due: 17 Feb 11

1. Consider a simplified model of the cytotoxic T cell (CTL) immune response to retroviral infection:

$$\frac{dx}{dt} = \lambda - dx - \beta xv$$

$$\frac{dy}{dt} = \beta xv - ay - pyz$$

$$\frac{dv}{dt} = ky - uv$$

$$\frac{dz}{dt} = cyz - bz$$

   x, y, v and z represent the number of uninfected cells, infected cells, free virus and CTLs, respectively. Parameter values are $\lambda = 10^5, d = 1/7, \beta = 2\times10^{-7}, a = p = 0.5, k = 100, u = 6$.

   a) Calculate the equilibrium point $(x*, y*, v*, z*)$ in the absence of a CTL response and determine its stability.

   b) Consider two cases: 1) $cy* < b$ and 2) $cy* > b$. In both cases, plot the dynamics using a semilog scale given initial conditions $x_0 = 10^8$, $y_0 = 100$, $v_0 = 1000$, $z_0 = 100$. Describe what is happening with the CTL response in both cases.

2. In this problem, you will simulate Kimura's random genetic drift model for haploid populations, given a two allele model (A and B). Note that in class, we discussed diploid populations. Both alleles are selectively neutral.

   a) Run a stochastic simulation of the random genetic drift model until T=40, given that the initial frequency of the allele A is p=0.4. Simulate 10 paths and plot the gene frequency for this allele A over time for populations N=20, N=200 and N=2000.

   b) For each particular simulation, calculate the variance for each time step and plot it as a function of time.