Control of the MVWT II Hovercraft

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1 Model

The equations of motion can be written as

$$\begin{aligned} m\ddot{x} &= -\mu\dot{x} + f_x \\ m\ddot{y} &= -\mu\dot{y} + f_y \\ J\ddot{\theta} &= -\psi\dot{\theta} + T, \end{aligned}$$

where m and J are the mass and moment of inertia of the vehicle, μ and ψ are the linear and rotational friction coefficients, f_x and f_y are the x and y forces exerted by the fans, and T is the torque exerted by the fans. The physical parameters of the vehicles should be read in from a parameter file and should be allowed to vary from vehicle to vehicle. Reasonable parameters to start with are $m = 0.749$kg, $J = 0.0031$kg m^2, $\mu = 0.15$kg/s, and $\psi = 0.005$kg m^2/s.

In matrix form, the equations become:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{\mu}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\mu}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\psi}{J} \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & \frac{1}{m} & 0 \\ 0 & 0 & \frac{1}{J} \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ T \end{bmatrix}$$

2 Control

The controller takes the state $\mathbf{x} = (x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})$ and a reference velocity $\mathbf{v}_{ref} = (u, v)$ and calculates the desired thrust F and torque T. As this happens on board the hovercraft, the simulator will need to perform this step. It first transforms everything into "error coordinates" $(\theta_e, \dot{\theta}, \xi_1, \xi_2)$ that represent how far away the vehicle is from the desired velocity. The equations to do this are:

$$\begin{aligned}
\theta_e &= \theta - \tan^{-1}(v/u) \\
\xi_1 &= \frac{\dot{x}u + \dot{y}v}{\|\mathbf{v}\|} - \|\mathbf{v}\| \\
\xi_2 &= \frac{-\dot{x}v + \dot{y}u}{\|\mathbf{v}\|}
\end{aligned}$$

We then calculate the controls by

$$\begin{bmatrix} F \\ T \end{bmatrix} = -K \begin{bmatrix} \theta_e \\ \dot{\theta} \\ \xi_1 \\ \xi_2 \end{bmatrix}.$$

The 2×4 gain matrix K will be calculated offline and should be readable from a parameter file to allow us to test different controllers.

We now need to transform these control inputs to the forces f_r and f_r to be sent to the fans. Ideally,

$$\begin{aligned}
f_r &= \frac{1}{2}(F + T + F_0) \\
f_l &= \frac{1}{2}(F - T + F_0),
\end{aligned}$$

where F_0 is a "feed-forward" force that should also be readable from a parameter file. In reality, however, we need to remember that the fans have a minimum and maximum possible thrust. The minimum thrust is 0, i.e. the fans can't thrust backwards. The maximum force should be readable from a parameter file as well. The current estimate of this maximum force is $f_{max} = 0.706N$.

The forces in global coordinates to be used by the simulator are calculated from

$$\begin{aligned}
f_x &= (f_r + f_l)\cos\theta \\
f_y &= (f_r + f_l)\sin\theta \\
T &= (f_r - f_l)r_f,
\end{aligned}$$

where f_r and f_l are the right and left fan forces and r_f is the distance from the center of mass of the vehicle to the axis of the fan. The parameter r_f again should be readable from a file and is $r_f = 0.089m$.

3 Simulation

The simulator needs to take the current state and reference information, calculate the controls as above, then use them as the input to the differential equation given in the first section. This will give the derivative of the state which can be integrated forward for a short period of time to calculate the new state for the next step.