

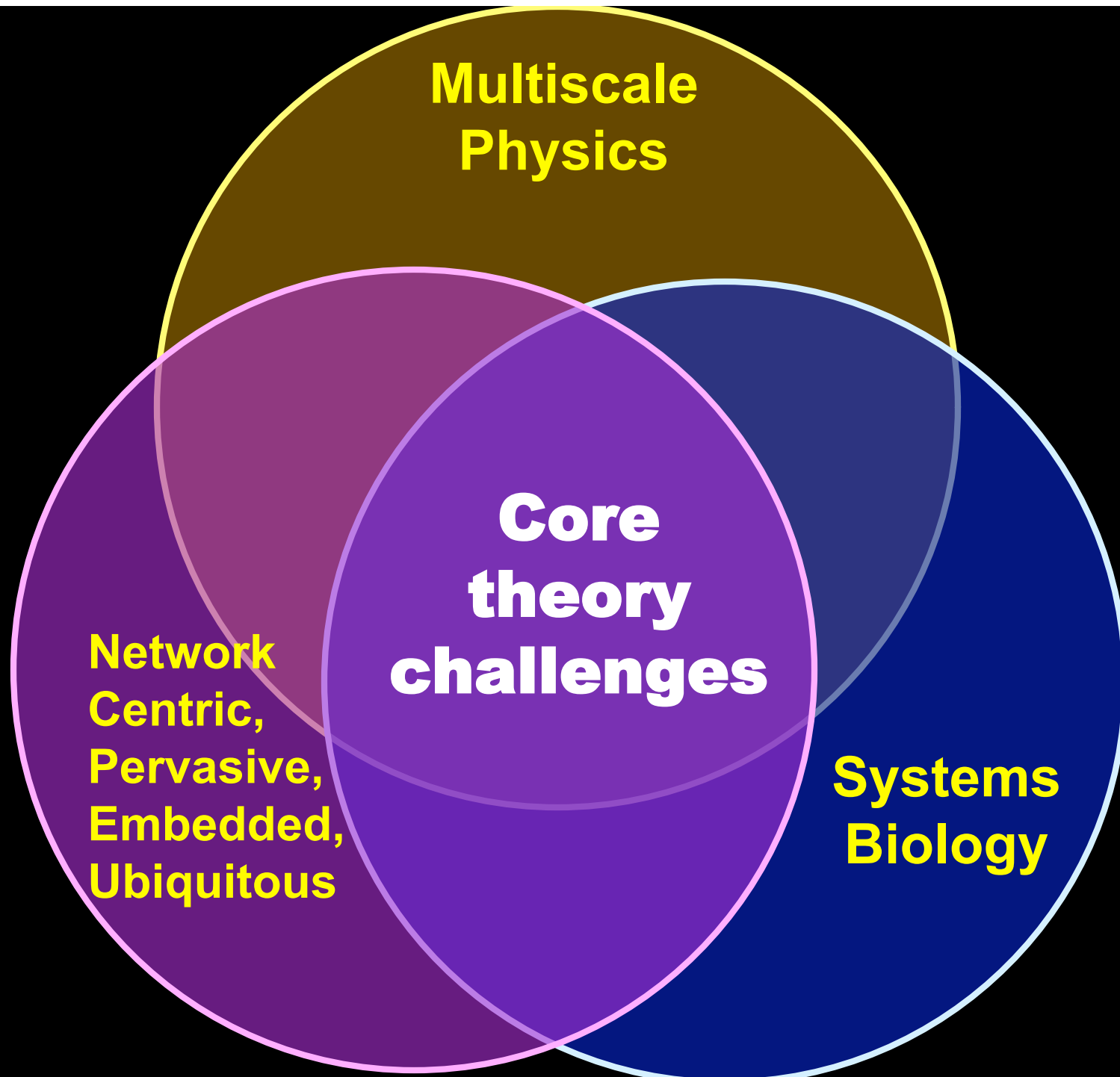
# Connections II

## Fundamentals of network science

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BioEng, and ElecEng  
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[www.cds.caltech.edu/~doyle](http://www.cds.caltech.edu/~doyle)



# Core theory challenges

- Hard limits
- Short proofs
- Small models
- Architecture

# Architecture?

- *“The bacterial cell and the Internet have*
  - architectures*
  - that are robust and evolvable”*
- What does “architecture” mean?
- What does it mean for an “architecture” to be robust and evolvable?
- Robust yet fragile?

## Robust

1. Efficient, flexible metabolism
2. Complex development
3. Immune systems
4. Regeneration & renewal
5. Complex societies

## Yet Fragile

1. Obesity and diabetes
2. Rich parasite ecosystem
3. Auto-immune disease
4. Cancer
5. Epidemics, war, genocide, ...

Human robustness and fragility

# Hard limits and tradeoffs

On systems and their components

- Thermodynamics (Carnot)
- Communications (Shannon)
- Control (Bode)
- Computation (Turing/Gödel)

Assume  
*different*  
architectures  
a priori.

- Fragmented and incompatible
- We need a more integrated view  
and have the beginnings

# The nature of simplicity

## Simple questions:

- Simple models
- Elegant theorems
- Elegant experiments

## Simple answers:

- Predictable results
- Short proofs
- Simple outcomes

Reductionist science: Reduce the *apparent complexity* of the world to an underlying simplicity.

Physics has for centuries epitomized the success of this approach.

# 1930s: The end of certainty

## Simple questions:

- Simple models
- Elegant theorems
- Elegant experiments

## Simple answers:

- ~~• Predictable results~~
- ~~• Short proofs~~
- Simple outcomes

- Godel: Incompleteness
- Turing: Undecidability
- Profoundly effected mathematics and computation.
- Little impact on science.



# 1960s-Present: “Emergent complexity”

## Simple questions:

- Simple models
- Elegant theorems
- Elegant experiments



Dominates scientific  
thinking today

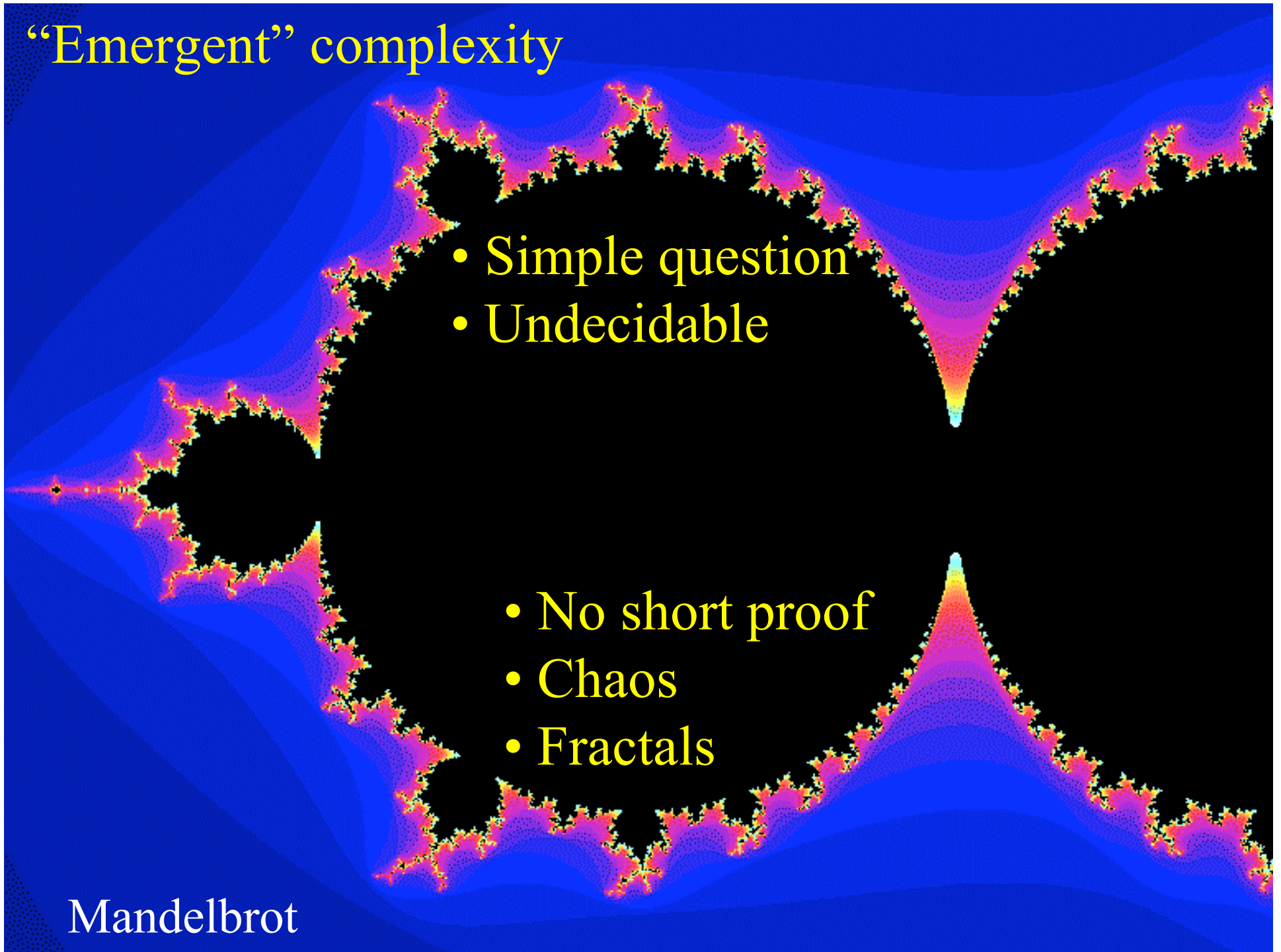
## Complexity:

- Unpredictability
- Chaos, fractals
- Critical phase transitions
- Self-similarity
- Universality
- Pattern formation
- Edge-of-chaos
- Order for free
- Self-organized criticality
- Scale-free networks

# “Emergent” complexity

- Simple question
- Undecidable
- No short proof
- Chaos
- Fractals

Mandelbrot



# The “New Science of Complexity”

	Simple question
Predictable	Simplicity
Unpredictable	“Emergence”

Even simple systems with little uncertainty  
can yield completely unpredictable behavior.

# 1900s: The triumph (and horror) of organization

## ~~Simple questions.~~

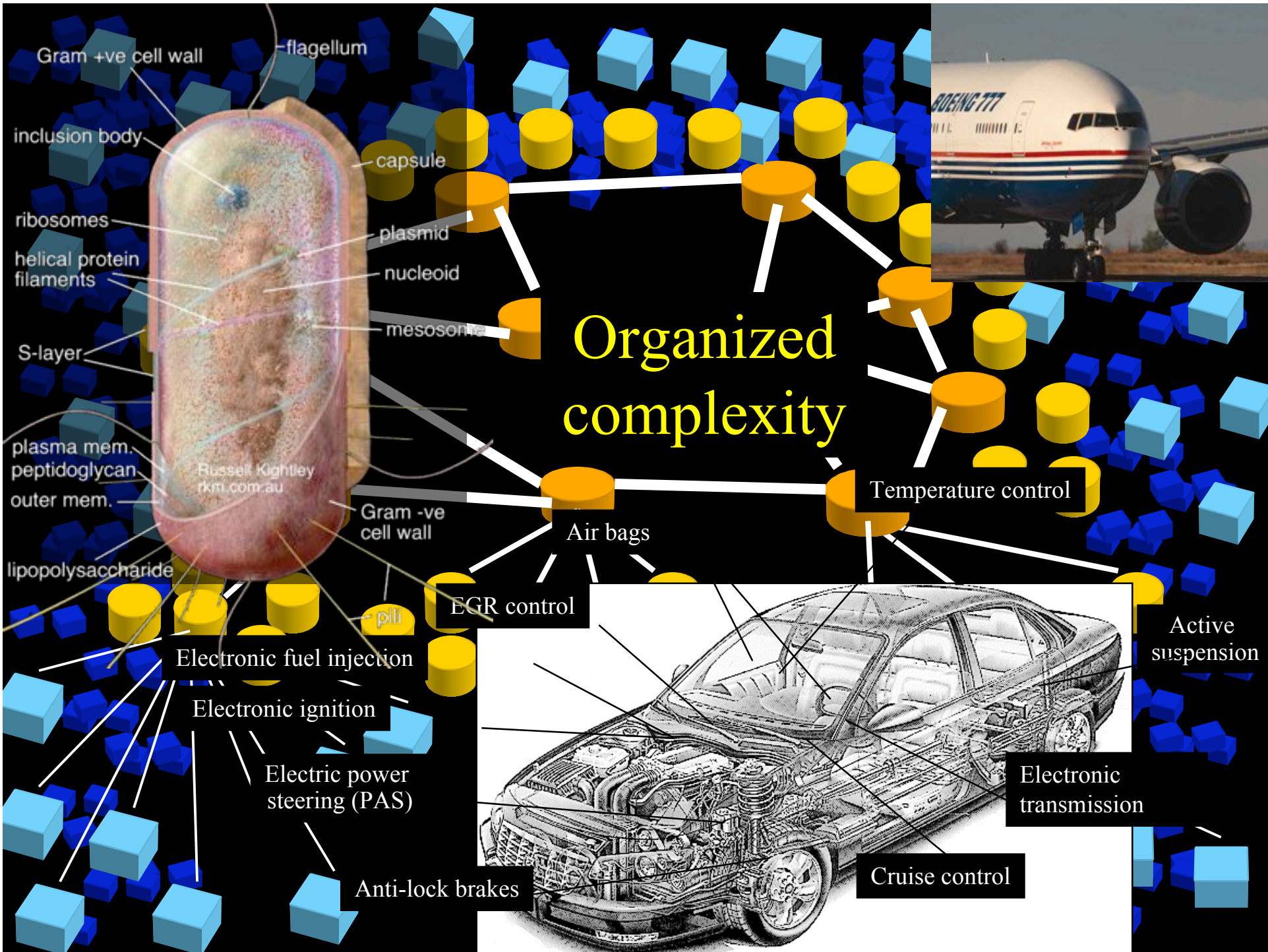
- ~~• Simple models~~
- ~~• Elegant theorems~~
- ~~• Elegant experiments~~

## Simple answers:

- Predictable results
- Short proofs
- Simple outcomes

- Complex, uncertain, hostile environments
- Unreliable, uncertain, changing components
- Limited testing and experimentation
- Yet predictable, robust, reliable, adaptable, evolvable systems





# Organized complexity

- Requires highly organized interactions, by design or evolution
- Completely different theory and technology from emergence

## Simple answers:

- Predictable results
- Short proofs
- Simple outcomes

- Complex, uncertain, hostile environments
- Unreliable, uncertain, changing components
- Limited testing and experimentation
- Yet predictable, robust, reliable, adaptable, evolvable systems

# Mathematics and technology

Question Answer	Simple	Complex
	Simplicity	<b>Organization</b>
Predictable		
Unpredictable	<b>Emergence</b>	

Emergence and organization are opposites,  
but can be viewed in a unified framework.

# Irreducible complexity?

Question Answer	Simple	Complex
	Predictable	Unpredictable
Predictable	Simplicity	<b>Organization</b>
Unpredictable	Emergence	?

Complexity and unpredictability are  
the key to successful cryptography  
and other security technologies.



# The complete picture



Question \ Answer	Simple	Complex
	Simple	Complex
Simple	Simplicity	<b>Organization</b>
Complex	<b>Emergence</b>	<b>Irreducibility</b>

Simple, predictable, reliable, robust  
versus  
Complex, unpredictable, fragile

# The complete picture



<div> <div>Question</div> <div>Answer</div> </div>	Simple	Complex
Simple	Simple	Organization
Complex	Emergency	Irreducibility

Simple, predictable, reliable, robust  
versus  
Complex, unpredictable, fragile

# The complete picture

Question \ Answer	Simple	Complex
	Simple	Complex
Simple	Simplicity	Organization
Complex	Emergence	Irreducibility

# The challenge

<b>Models</b> <b>Proofs</b>	<b>Small</b>	<b>Large</b>
<b>Short</b>		
<b>Long</b>		

How can we treat complex networks and systems with small models and short proofs?

# The complete picture

Models Proofs	Small	Large
	Simplicity	Organization
Short		
Long	Emergence	Irreducibility

# Breaking hard problems

- SOSTOOLS proof theory and software (Parrilo, Prajna, Papachristodoulou, ...)
- Nested family of (dual) proof algorithms
- Each family is polynomial time
- Recovers many “gold standard” algorithms as special cases, and immediately improves
- Nonlinear, hybrid, stochastic, ...
- No a priori polynomial bound on depth (otherwise  $P=NP=coNP$ )
- **Conjecture: Complexity implies fragility**

# Architecture?

- “*The bacterial cell and the Internet have*
  - *architectures*
  - *that are robust and evolvable (yet fragile) ”*
- What does “architecture” mean?
- What does it mean for an “architecture” to be robust and evolvable?
- Robust yet fragile?
- Rigorous and coherent theory?

# A look back and forward

- The Internet architecture was designed without a “theory.”
- Many academic theorists told the engineers it would never work.
- We now have a nascent theory that confirms that the engineers were right (Kelly, Low, Vinnicombe, Paganini, Papachristodoulou, ...)
- Parallel stories exist in “theoretical biology.”
- For future networks, “systems of systems,” and other new technologies, as well as *systems biology of the cell, organism and brain*, ...
- ...let’s hope we can avoid a repeat of this history. (Looks like we have a good start...)



# Architecture?

- “*The bacterial cell and the Internet have*
  - *architectures*
  - *that are robust and evolvable*”
- For the Internet, we know how all the parts work, and we can ask the architects

# The Internet hourglass

## Applications

Web

FTP

Mail

News

Video

Audio

ping

napster

Ethernet

802.11

Power lines

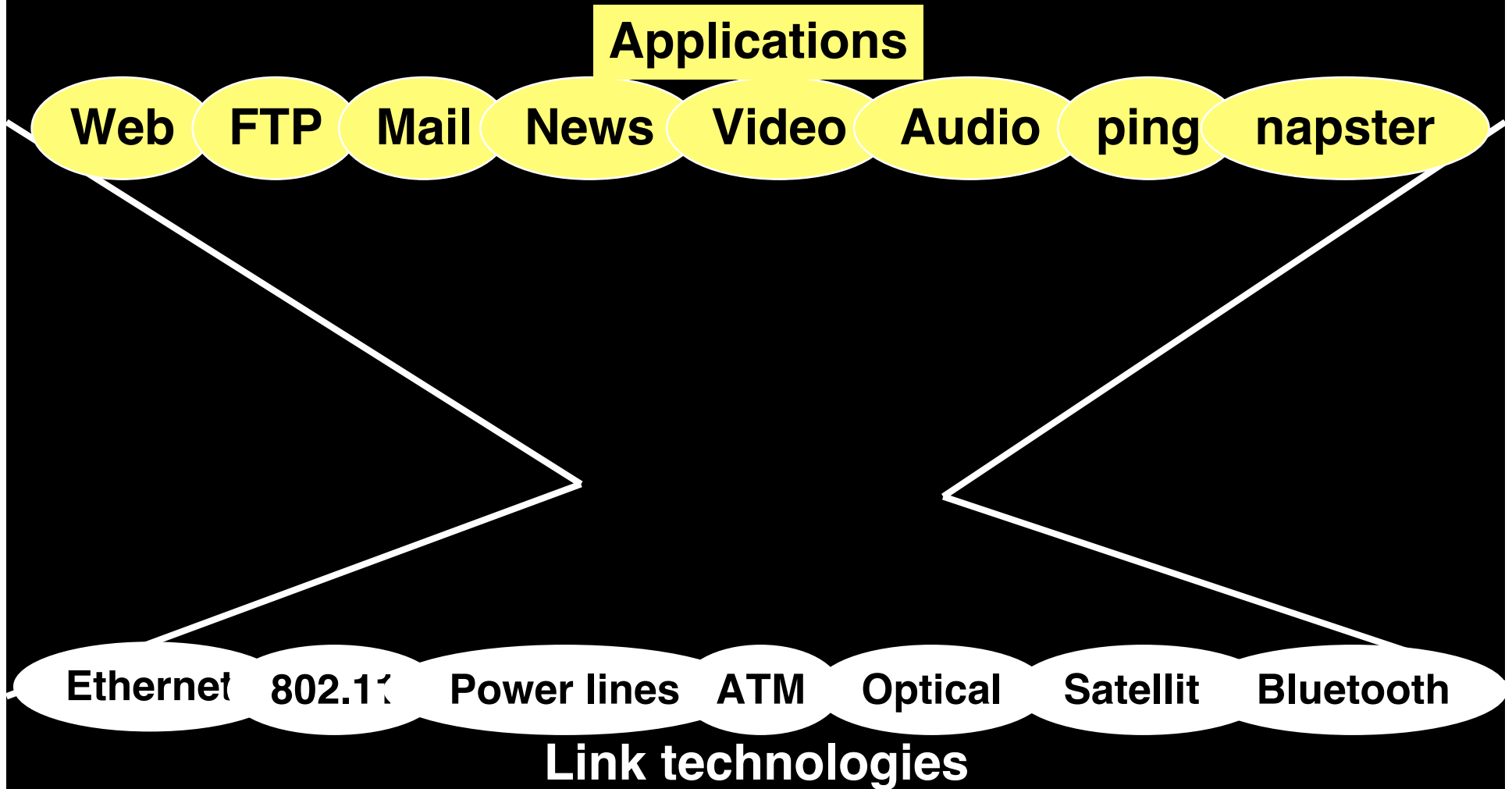
ATM

Optical

Satellit

Bluetooth

## Link technologies



# The Internet hourglass

## Applications

Web

FTP

Mail

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TCP

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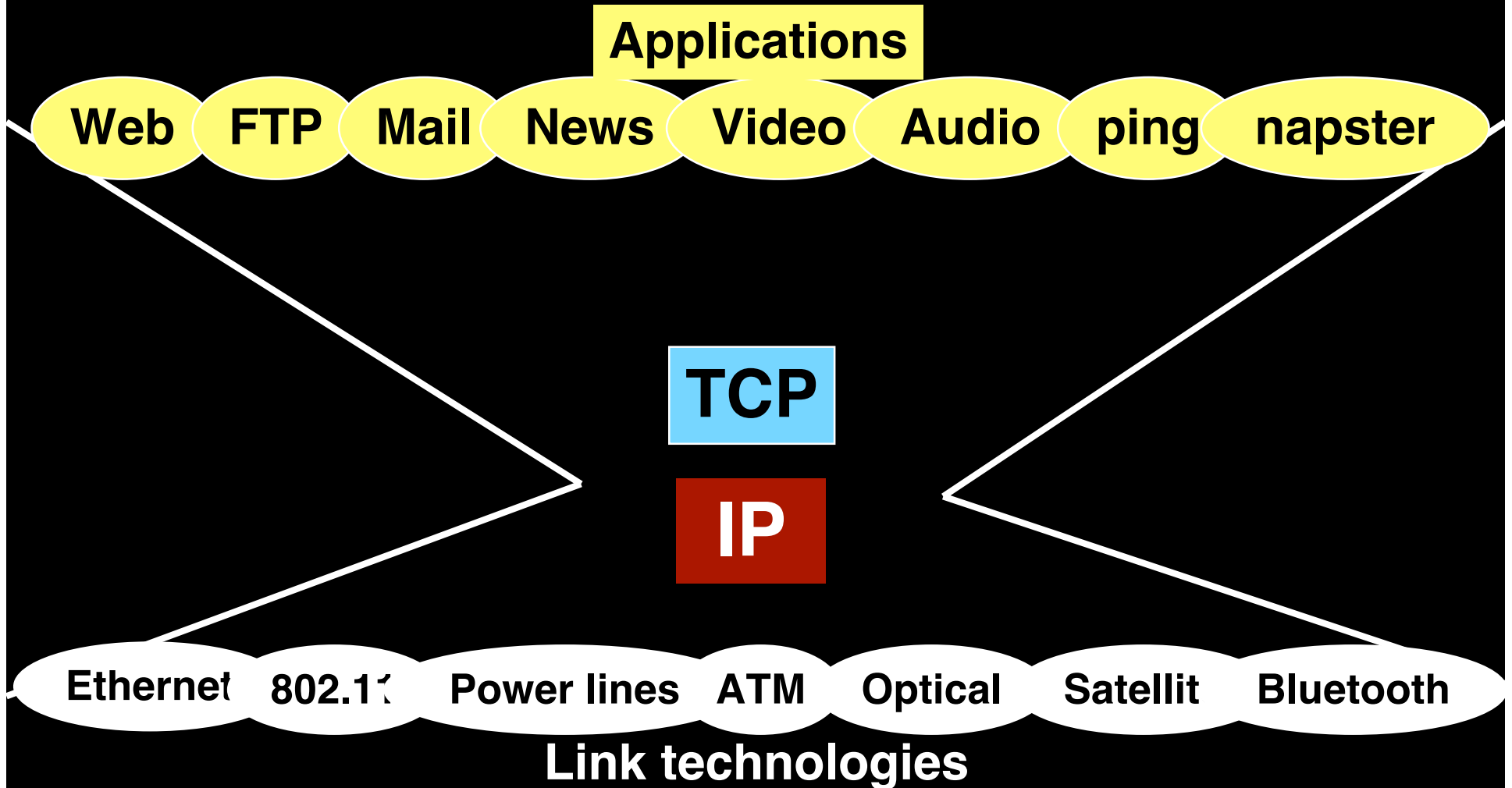
ATM

Optical

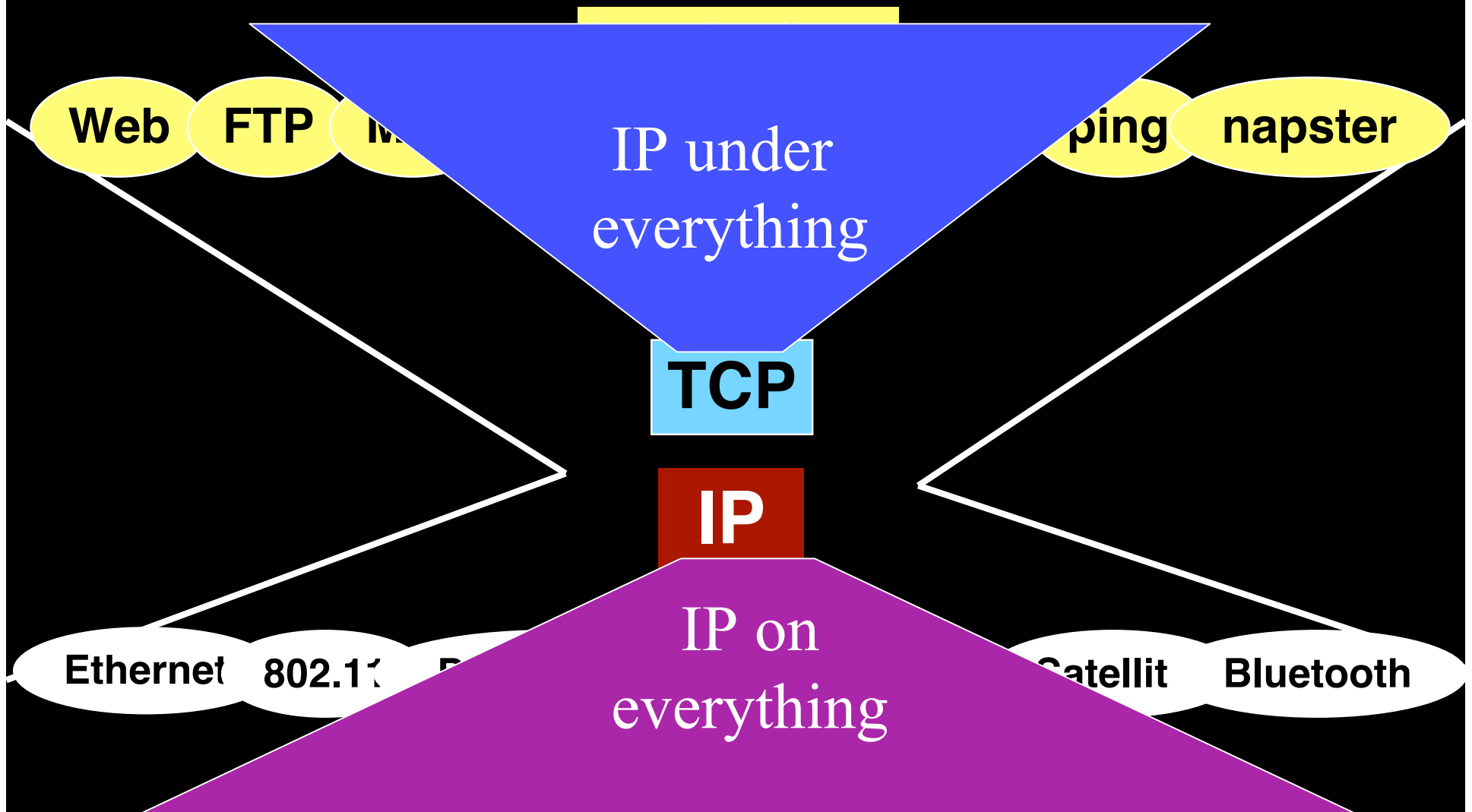
Satellit

Bluetooth

Link technologies



# The Internet hourglass





The diagram illustrates the relationship between network layers and robustness. On the left, a vertical stack of four elements is shown: a blue inverted triangle at the top labeled 'Applications', a light blue square labeled 'TCP/AQM', a red square labeled 'IP', and a purple triangle at the bottom labeled 'Link'. To the right of this stack are two large, horizontal, arrow-shaped boxes pointing left. The top box is teal and contains the text 'Top of “waist” provides robustness to variety and uncertainty above'. The bottom box is red and contains the text 'Bottom of “waist” provides robustness to variety and uncertainty below'. The entire diagram is set against a black background.

Applications

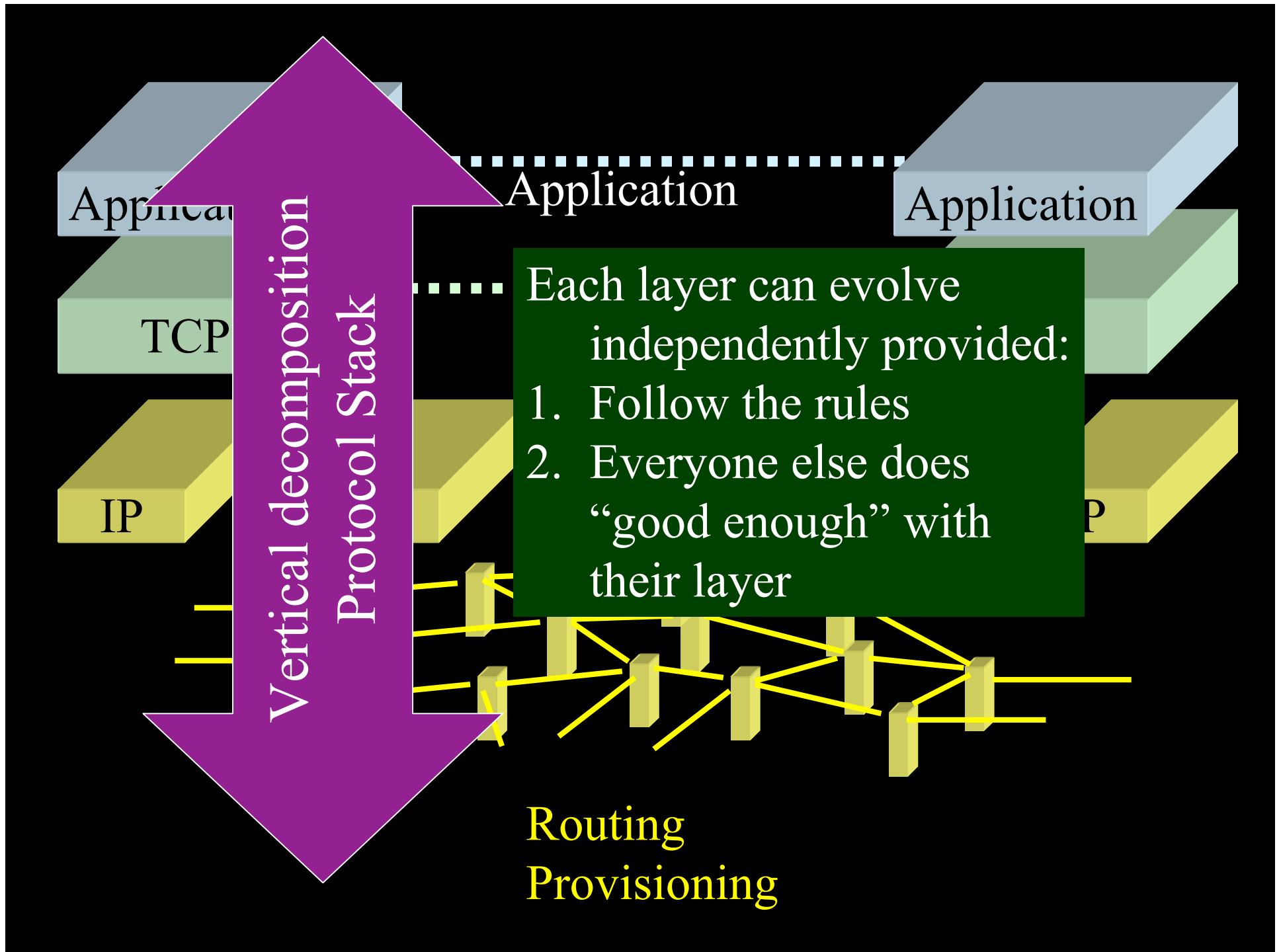
**TCP/  
AQM**

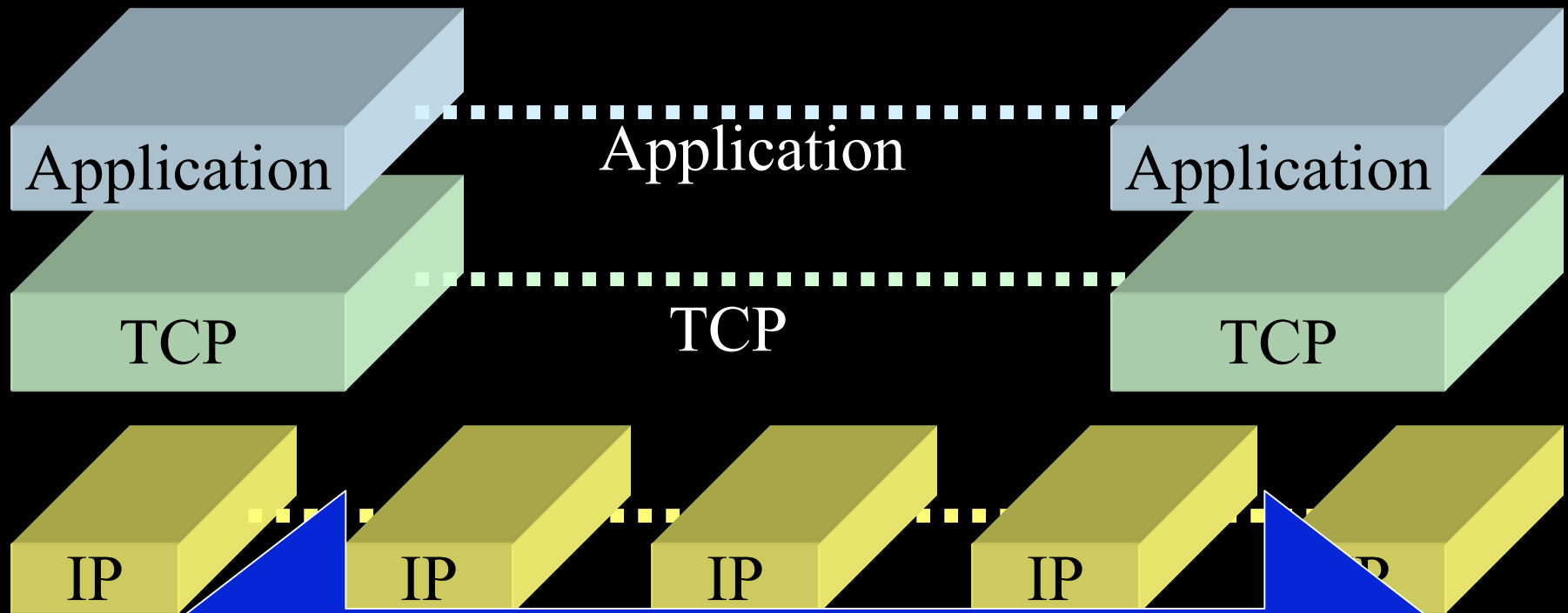
**IP**

Link

Top of “waist” provides  
robustness to variety and  
uncertainty above

Bottom of “waist” provides  
robustness to variety  
and uncertainty below



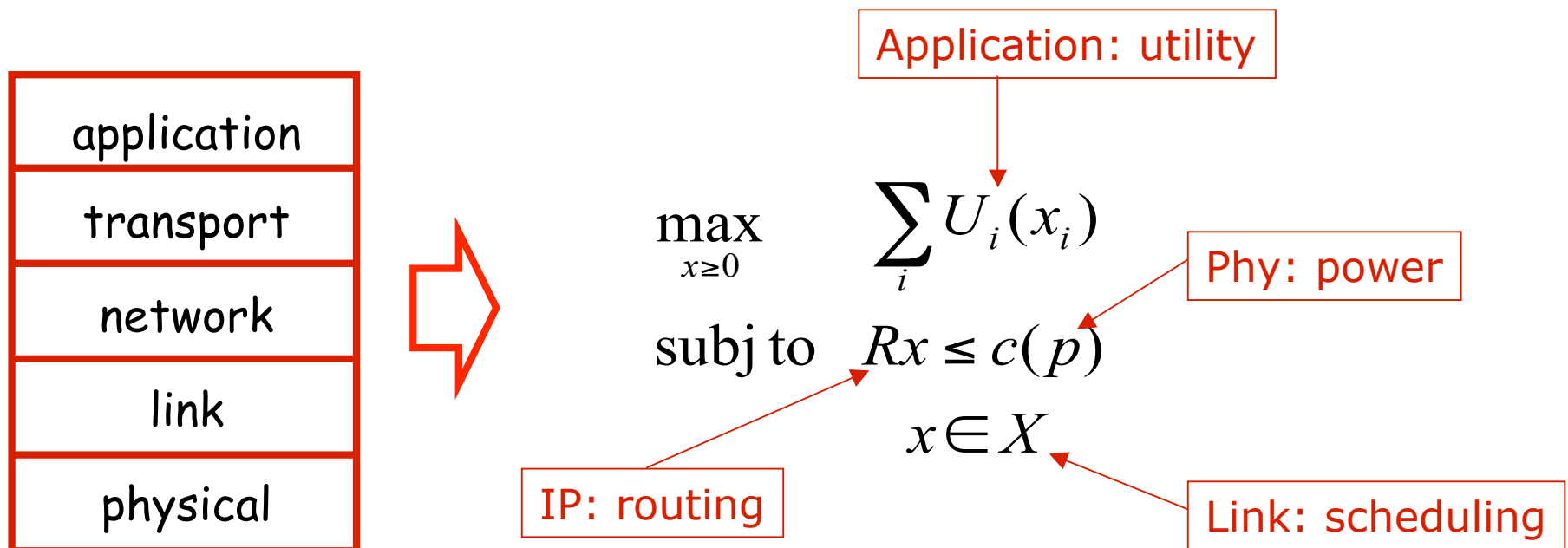


Horizontal decomposition  
Each level is decentralized and asynchronous

Routing  
Provisioning

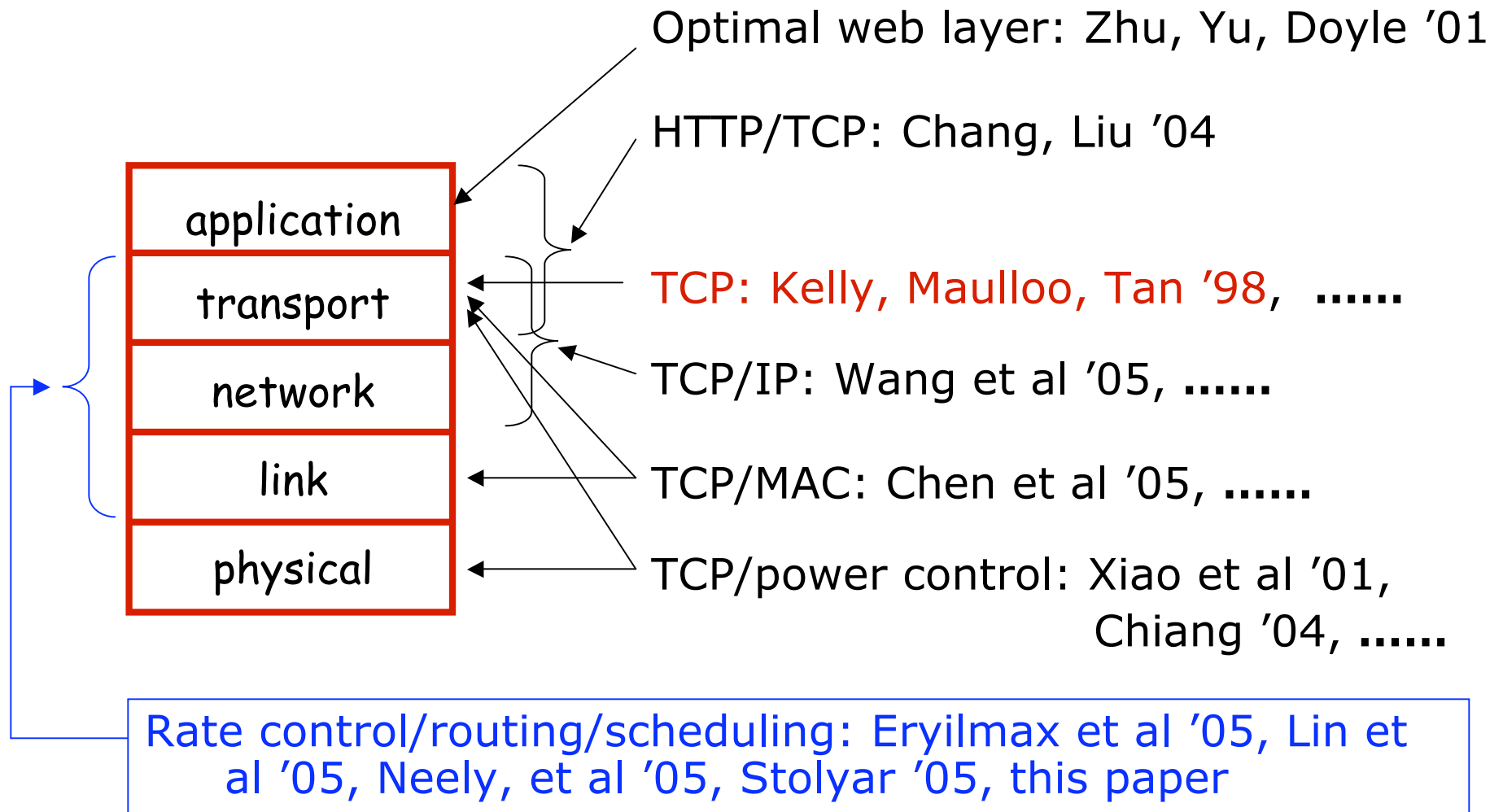
# Layering as optimization decomposition

- Each layer is abstracted as an optimization problem
- Operation of a layer is a distributed solution
- Results of one problem (layer) are parameters of others
- Operate at different timescales



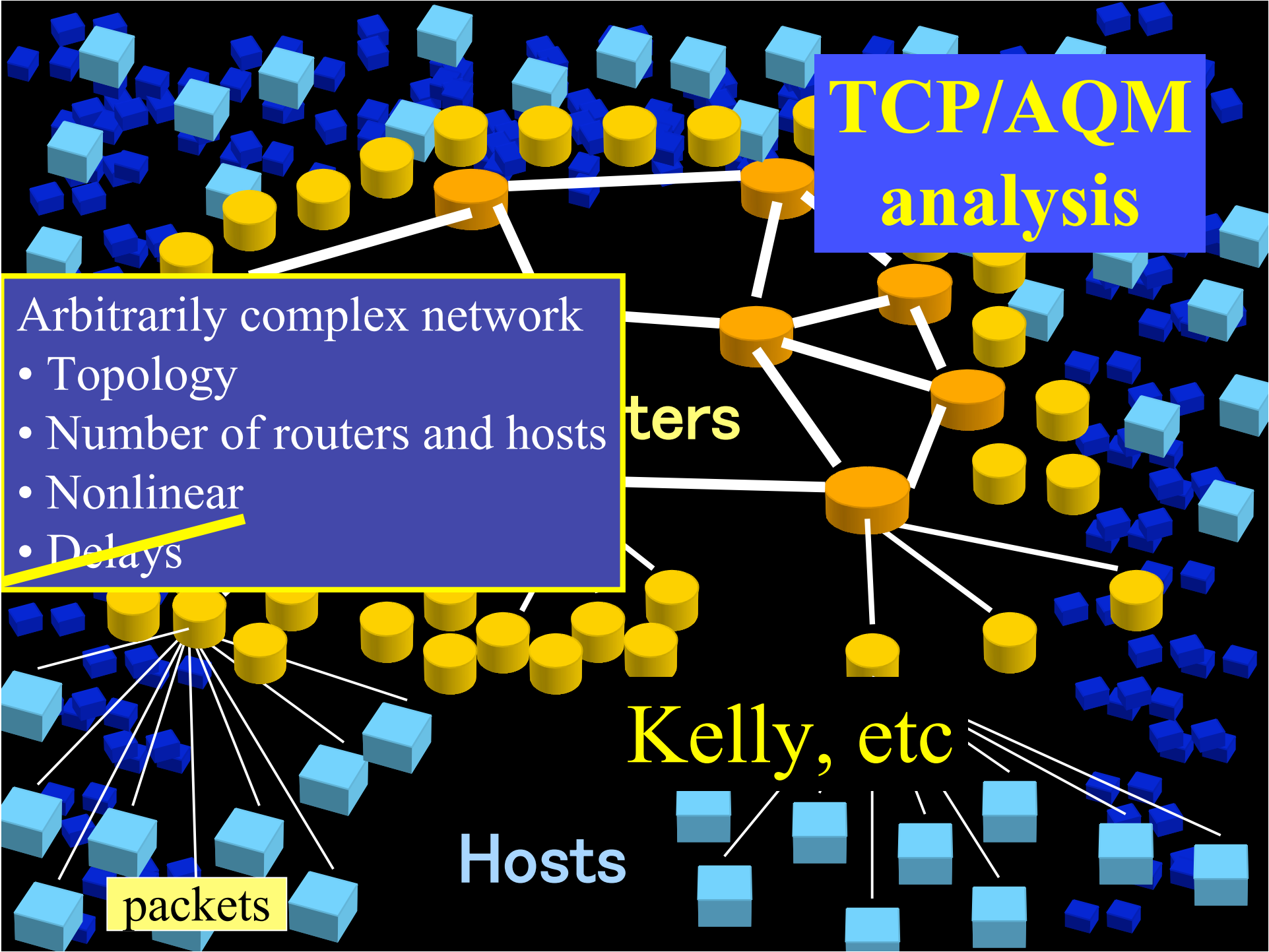


# Examples



detailed survey in Proc. of IEEE, 2006

# TCP/AQM analysis



Arbitrarily complex network

- Topology
- Number of routers and hosts
- Nonlinear
- Delays

routers

Kelly, etc

Hosts

packets

# TCP/AQM analysis

Arbitrarily complex network

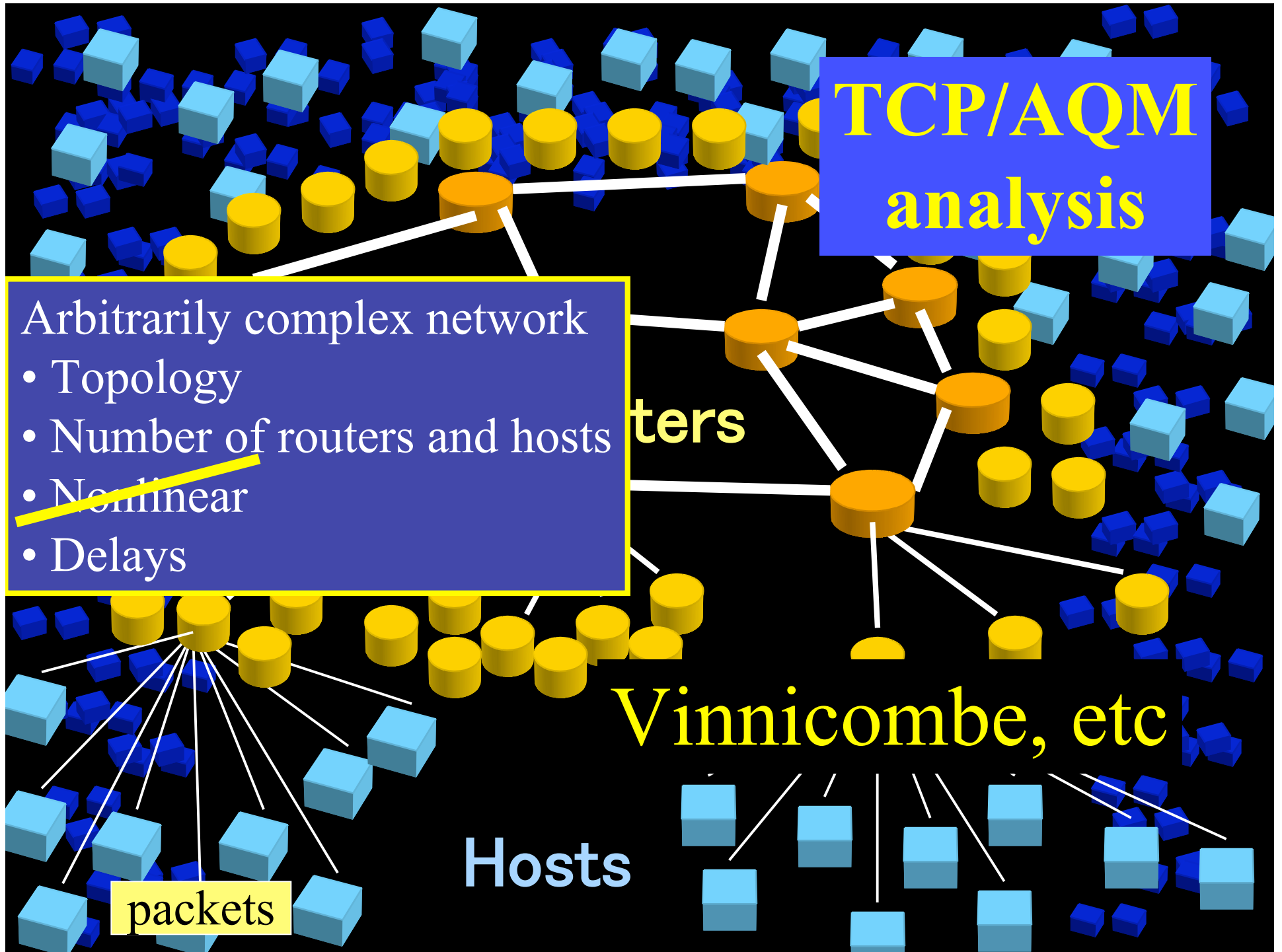
- Topology
- Number of routers and hosts
- ~~Nonlinear~~
- Delays

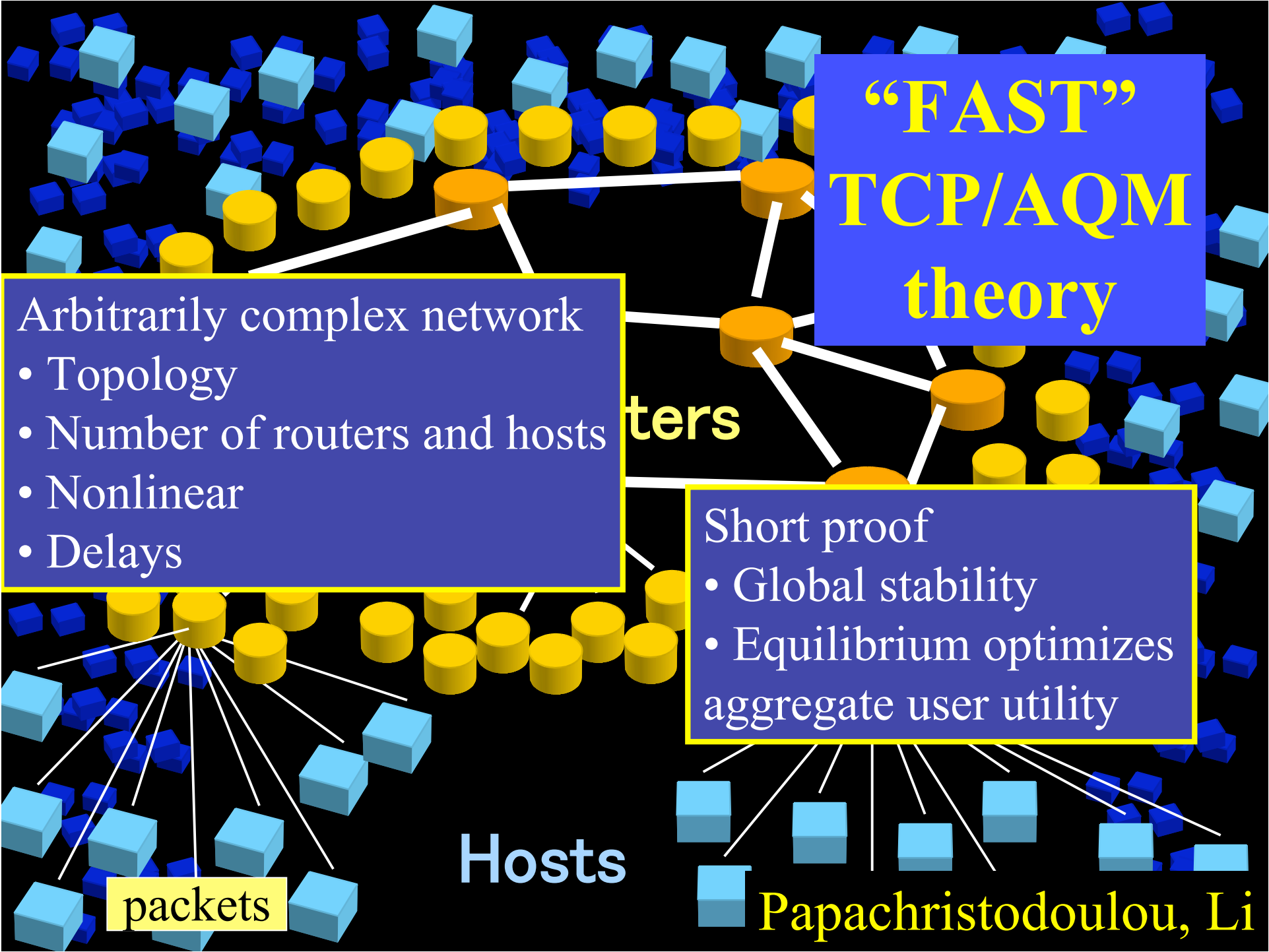
routers

Vinnicombe, etc

Hosts

packets





# “FAST” TCP/AQM theory

Arbitrarily complex network

- Topology
- Number of routers and hosts
- Nonlinear
- Delays

Short proof

- Global stability
- Equilibrium optimizes aggregate user utility

Hosts

packets

Papachristodoulou, Li

# Architecture?

- *“The bacterial cell and the Internet have
  - architectures
  - that are robust and evolvable”*
- For the Internet, we know how all the parts work, and we can ask the architects
- For biology, we know how some parts work, and evolution is the “architect” (another source of confusion)

Bio: Huge variety of environments, metabolisms  
Internet: Huge variety of applications

Huge variety of components



Bio: Huge variety of environments, metabolisms  
Internet: Huge variety of applications

Both components  
and applications are  
highly uncertain.

Huge variety of components

Bio: Huge variety of environments, metabolisms  
Internet: Huge variety of applications

Huge variety of genomes  
Huge variety of physical networks

Huge variety of components



# Hourglass architectures

Bio: Huge variety of environments, metabolisms  
Internet: Huge variety of applications

Feedback  
control

**Identical  
control  
architecture**

Huge variety of genomes  
Huge variety of physical networks

Huge variety of components

TCP/IP

Metabolism/biochem

5:Application/function: variable supply/demand

4:TCP

Feedback  
Control

4:Allosteric

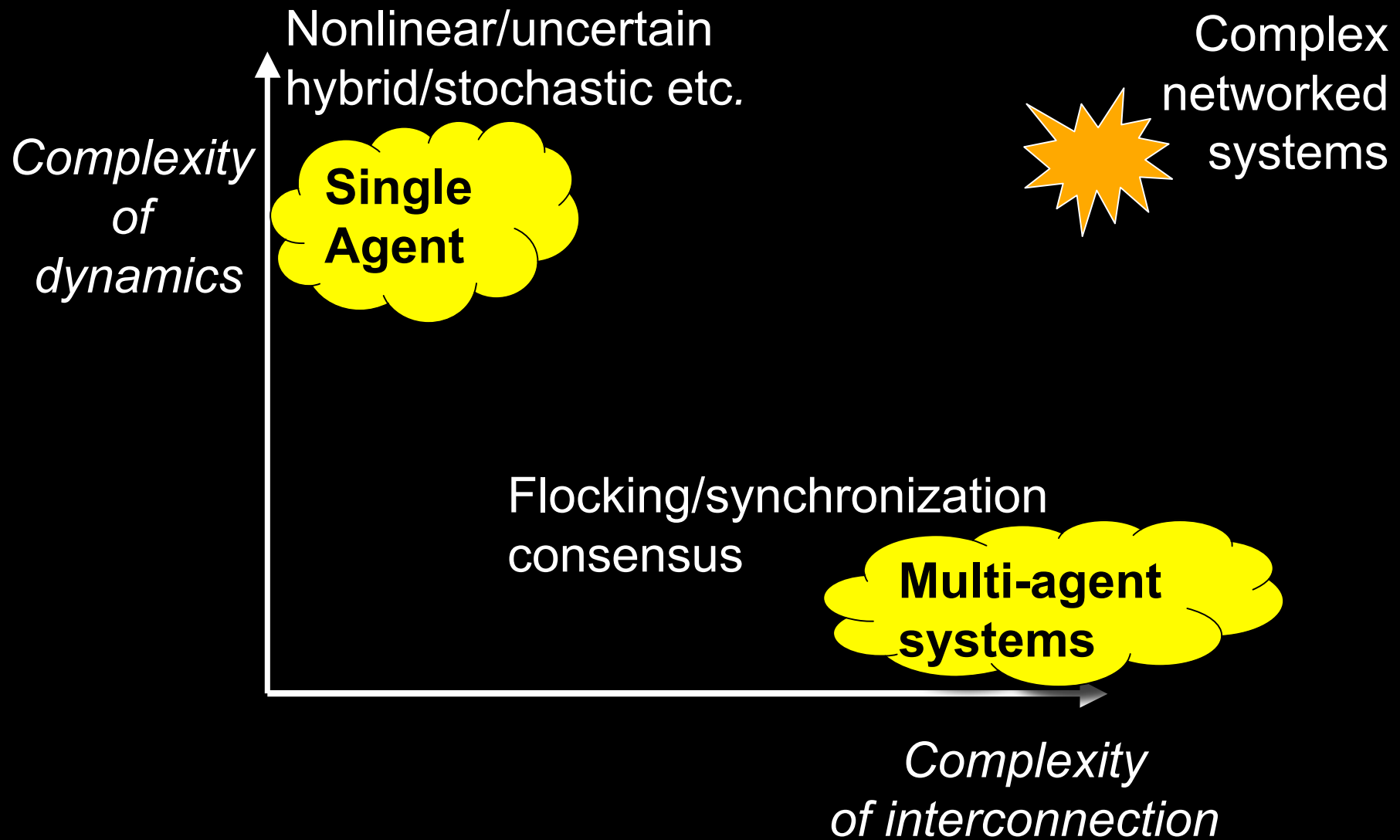
3:IP

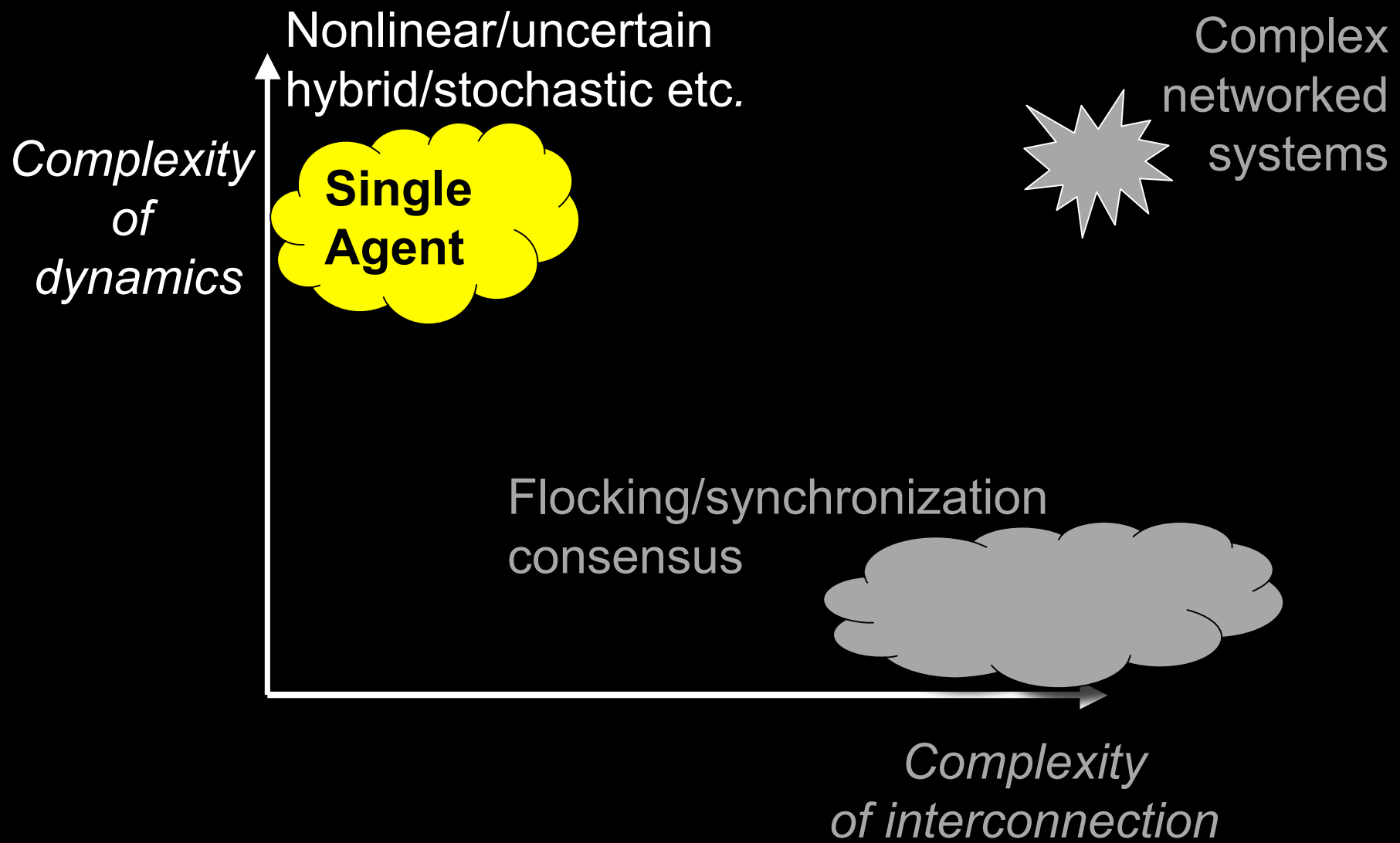
3:Transcriptional

2:Potential physical network

1:Hardware components

# Networked dynamical systems





# “Emergent” complexity

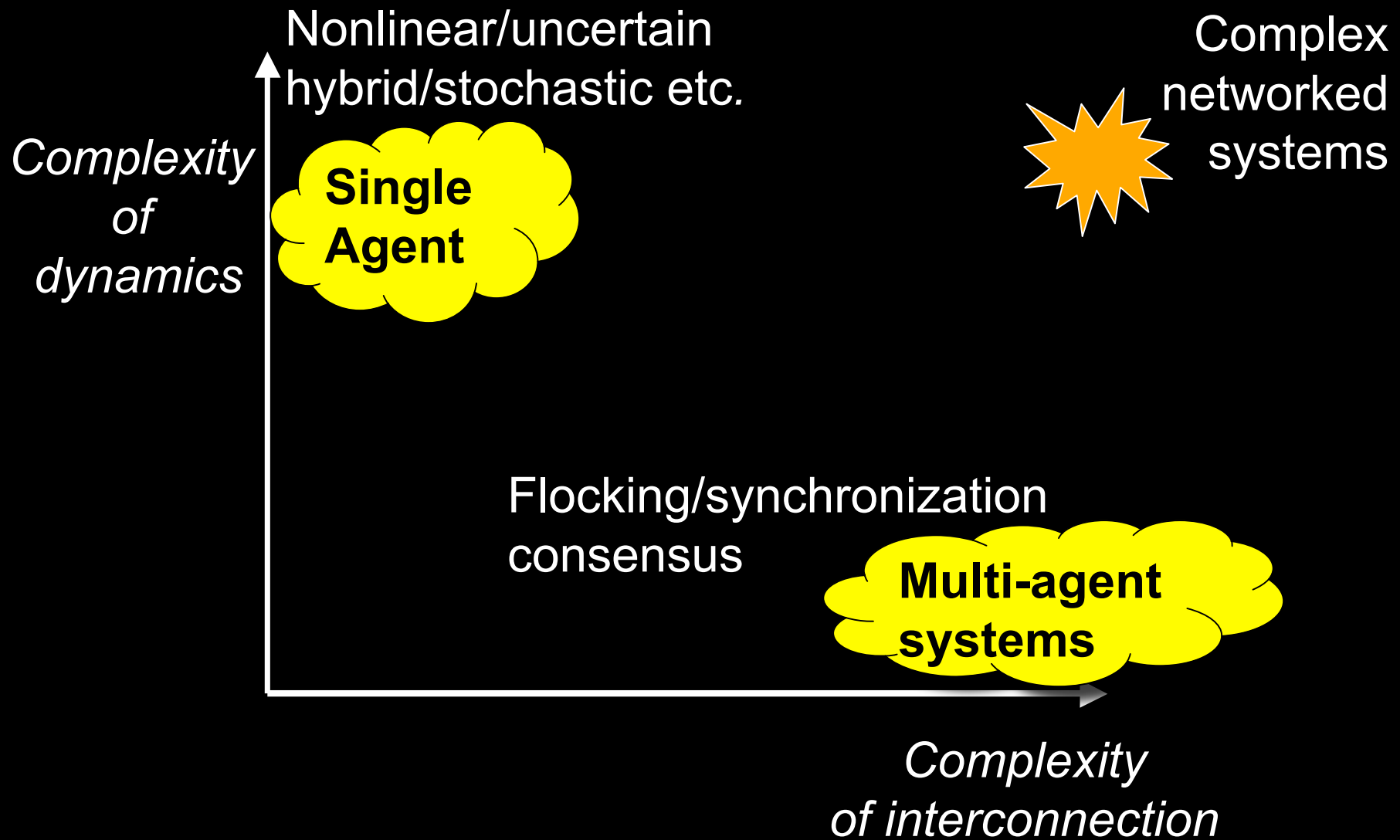
- Simple question
- Undecidable

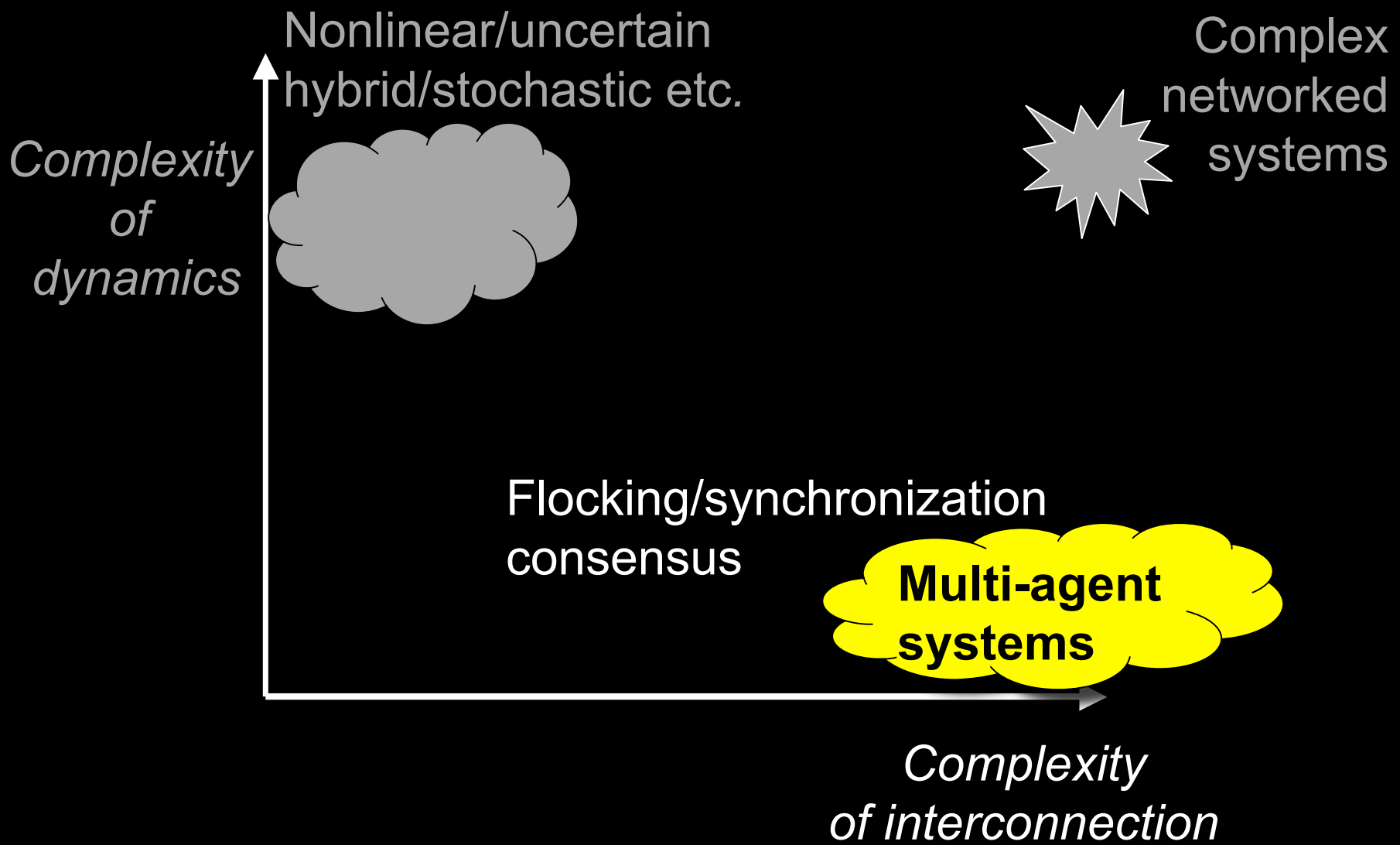
Simulations and conjectures but no “proofs”

- Chaos
- Fractals

Mandelbrot

# Networked dynamical systems





# Statistical Physics and emergence of collective behavior

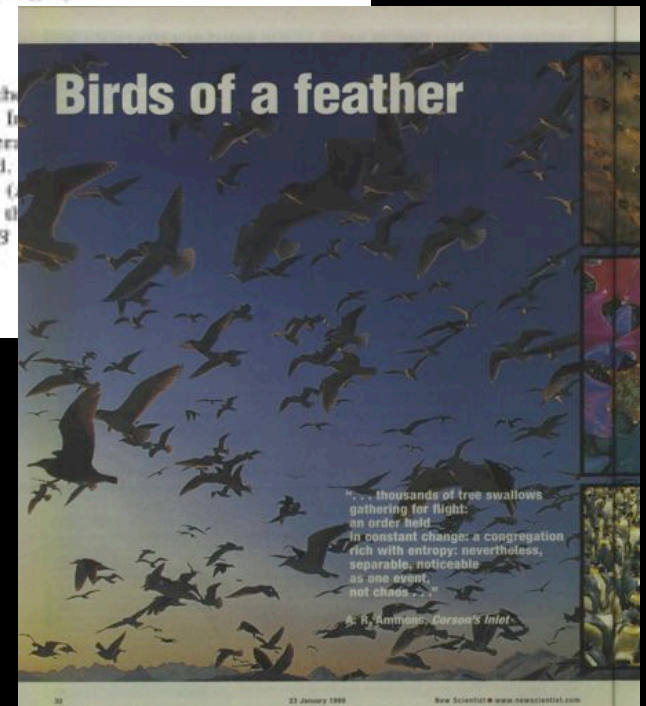
VOLUME 75, NUMBER 6

PHYSICAL REVIEW LETTERS

Novel Type of Phase Transition in a System of Self-Driven Particles

Tamás Vicsek,<sup>1,2</sup> András Czirók,<sup>1</sup> Eshel Ben-Jacob,<sup>3,4</sup> et al.

<sup>1</sup>Department of Atomic Physics, Eötvös University, Budapest, Hungary



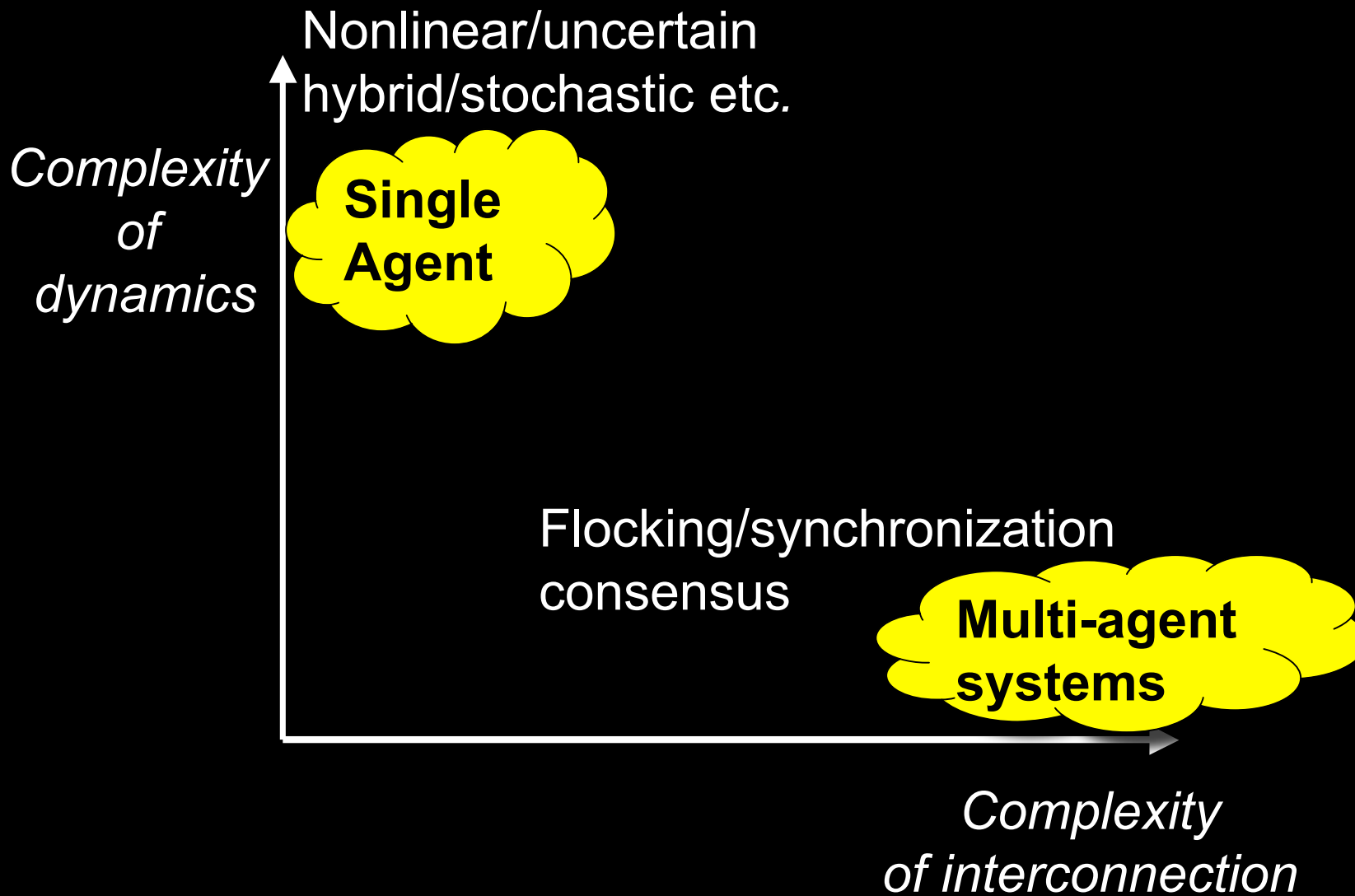
Simulations and conjectures but no “proofs”



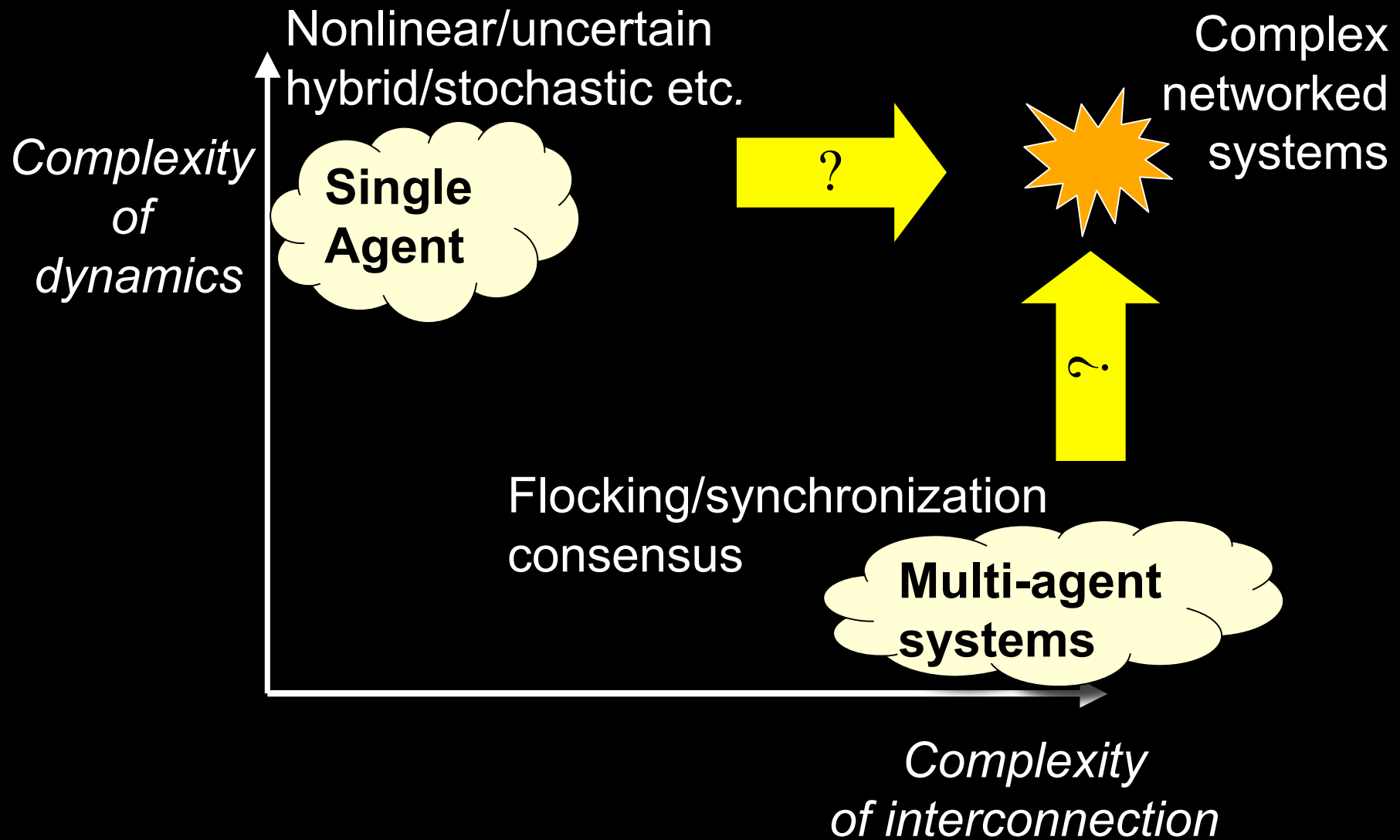


Working systems but no “proofs”

# Spectacular progress



# Open questions



# Core theory challenges

- Hard limits
- Short proofs
- Small models
- Architecture

# Today's tutorial

- Hard limits (morning)
- Short proofs (afternoon)
- Small models
- Architecture
- Common background, standard results