CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

ACM 101/AM 125a/CDS 140a

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Problem Set #5

Issued: 1 Feb 11
Due: 10 Feb 11

1. Perko Section 2.9 Problem 3

2. Perko Section 2.9 Problem 4

3. (Khalil 4.8) Consider
$$\dot{x}_1 = \frac{-6x_1}{u^2} + 2x_2, \qquad \dot{x}_2 = \frac{-2(x_1+x_2)}{u^2}$$
where $u = 1 + x_1^2$. Let $V(x) = \frac{x_1^2}{1+x_1^2} + x_2^2$.

   (a) Show that $V(x) > 0$ and $\dot{V}(x) < 0$ for all $x \in \mathbb{R}^2 - \{0\}$.

   (b) Consider the hyperbola $x_2 = \frac{2}{x_1 - \sqrt{2}}$. Show, by investigating the vector field on the boundary of this hyperbola, that trajectories to the right of the branch in the first quadrant cannot cross the branch.

   (c) Show that the origin is not globally asymptotically stable.

   Hint: In part (b), show that
$$\frac{\dot{x}_2}{\dot{x}_1} = \frac{-1}{1 + 2\sqrt{2}x_1 + 2x_1^2}$$
on the hyperbola, and compare with the slope of the tangents to the hyperbola.

4. Perko Section 2.12 Problem 2

5. Perko Section 2.12 Problem 7

6. Consider the following system in $\mathbb{R}^2$:
$$\begin{aligned} \dot{x} &= -\frac{3}{2}\left(x^2 + y^2\right) + 3\left(x + y\right) - 3 \\ \dot{y} &= -3xy + 3\left(x + y\right) - 3 \end{aligned}$$

   (a) Determine the stable, unstable and center manifold of the equilibrium point at $x = 1$, $y = 1$.

   (b) Determine the stability of the equilibrium point at $x = 1$, $y = 1$.
   Hint: There are two 1-dimensional invariant linear manifolds of $(1, 1)$ (i.e., two invariant manifold of the form $M = \{(a_1, a_2) \in \mathbb{R}^2 | a_2 = k a_1\}$). Determine the flow on these invariant manifolds.