1. **Perko, Section 3.4, problem 1**: Show that $\gamma(t) = (2\cos 2t, \sin 2t)$ is a periodic solution of the system

$$
\dot{x} = -4y + x\left(1 - \frac{x^2}{4} - y^2\right)
$$

$$
\dot{y} = x + y\left(1 - \frac{x^2}{4} - y^2\right)
$$

that lies on the ellipse $(x/2)^2 + y^2 = 1$ (i.e., $\gamma(t)$ represents a cycle $\Gamma$ of this system). Then use the corollary to Theorem 2 in Section 3.4 to show that $\Gamma$ is a stable limit cycle.

2. **Perko, Section 3.4, problem 3a**: Solve the linear system

$$
\dot{x} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} x
$$

and show that any at point $(x_0, 0)$ on the $x$-axis, the Poincare map for the focus at the origin is given by $P(x_0) = x_0 \exp(2\pi a/b)$. For $d(x) = P(x) - x$, compute $d'(0)$ and show that $d(-x) = -d(x)$.

3. **Perko, Section 3.5, problem 1**: Show that the nonlinear system

$$
\dot{x} = -y + xz^2 \\
\dot{y} = x + yz^2 \\
\dot{z} = -z(x^2 + y^2)
$$

has a periodic orbit $\gamma(t) = (\cos t, \sin t, 0)$. Find the linearization of this system about $\gamma(t)$, the fundamental matrix $\Phi(t)$ for the autonomous system that satisfies $\Phi(0) = I$, and the characteristic exponents and multipliers of $\gamma(t)$. What are the dimensions of the stable, unstable and center manifolds of $\gamma(t)$?

4. **Perko, Section 3.5, problem 5a**: Let $\Phi(t)$ be the fundamental matrix for $\dot{x} = A(t)x$ satisfying $\Phi(0) = I$. Use Liouville's theorem, which states that

$$
\det \Phi(t) = \exp \int_0^t \text{trace} A(s)ds,
$$

to show that if $m_j = e^{\lambda_j T}, j = 1, \ldots, n$ are the characteristic multipliers of $\gamma(t)$ then
\[
\sum_{j=1}^{\infty} m_j = \text{trace} \Phi(T)
\]
\[
\prod_{j=1}^{n} m_j = \exp \int_{0}^{T} \text{trace} A(t) \, dt.
\]

5. **Perko, Section 3.9, problem 4a**: Show that the limit cycle of the van der Pol equation
\[
\begin{align*}
\dot{x} &= y + x - x^3 / 3 \\
y &= -x
\end{align*}
\]
must cross the vertical lines \( x = \pm 1 \).


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