CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 202

Problem Set #7

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Winter 2009

Issued: 19 Feb 09
Due: 26 Feb 09

Reading: Abraham, Marsden, and Ratiu (MTA), Section 5.3; and Kelly and Murray (1994) [from web site]

Problems:

1. [Boothby, page 151, #4]
   Find the one-parameter subgroups of $GL(2,\mathbb{R})$ generated by

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

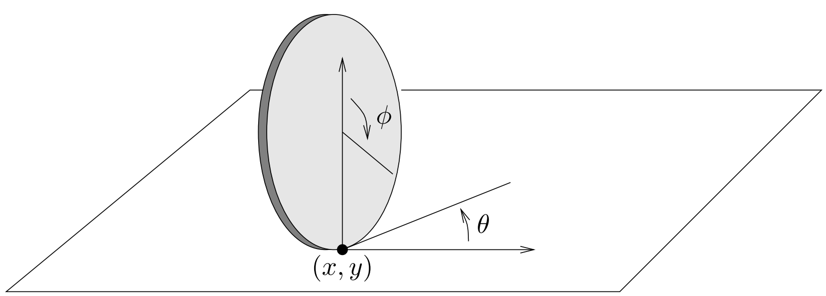
   Find the corresponding actions on $\mathbb{R}^2$ and their infinitesimal generators, starting from the natural action of $GL(2,\mathbb{R})$ on $\mathbb{R}^2$.

2. MTA 5.3-1: semidirect product groups

3. MTA 5.3-2: SE(3)

4. MTA 5.3-4: tangent bundle of a Lie group

5. Consider the locomotion system given by a disk rolling on the plane,



   where we assume we can control the angles θ and ϕ.

   (a) Let $Q = SE(2) \times S^1$ represent the configuration space for the system. Compute the Lagrangian for the sytem and show that is invariant under the action of $SE(2)$ given by translation and rotation as well as the subgroup of actions given just by translation.

   (b) Compute the kinematic connection for the system $A : TQ \to \mathfrak{g}$ corresponding to the system rolling without slipping.

   (c) Determine if the system is totally controllable and/or fiber controllable.

   (d) (Optional) Construct an explicit trajectory that moves the system from an arbitrary initial configuration $q = (x_0, y_0, \theta_0, \phi_0)$ to the origin.