CDS 270-2: Lecture 6-3
Optimum Receiver Design for Estimation over Wireless Links

Yasamin Mostofi

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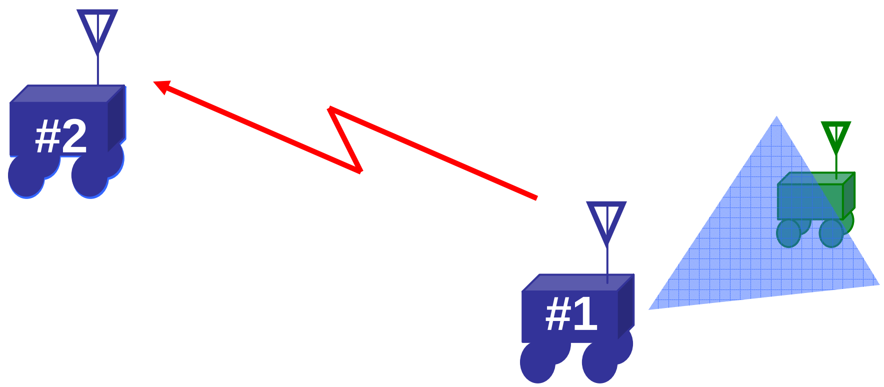
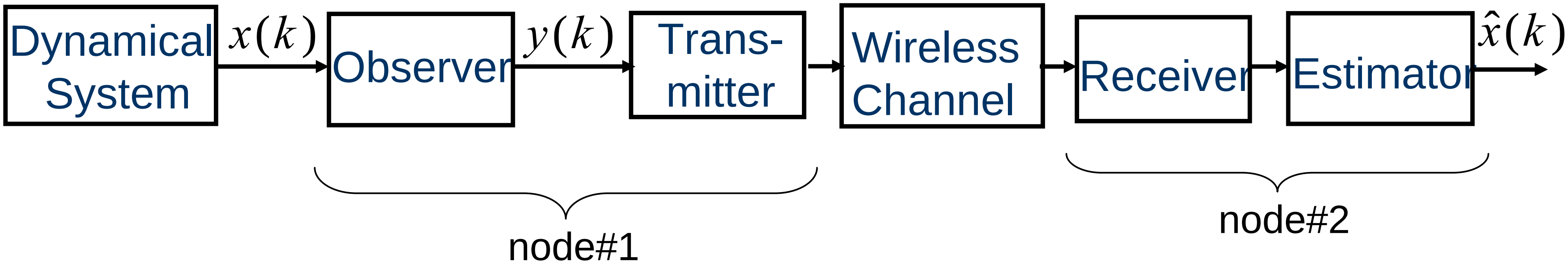
Goals:

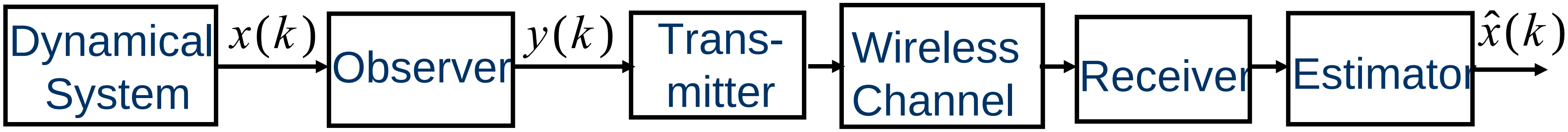
- To understand impact of wireless communication impairments on estimation over wireless
- To learn non-traditional designs for estimation over wireless applications
  - Optimization of packet drop
  - Use of cross-layer information paths

System Model (review)





System Model (review)



To focus on communication noise, assume scalar quantities

Linear dynamical system: $x(k+1) = Ax(k) + w(k)$

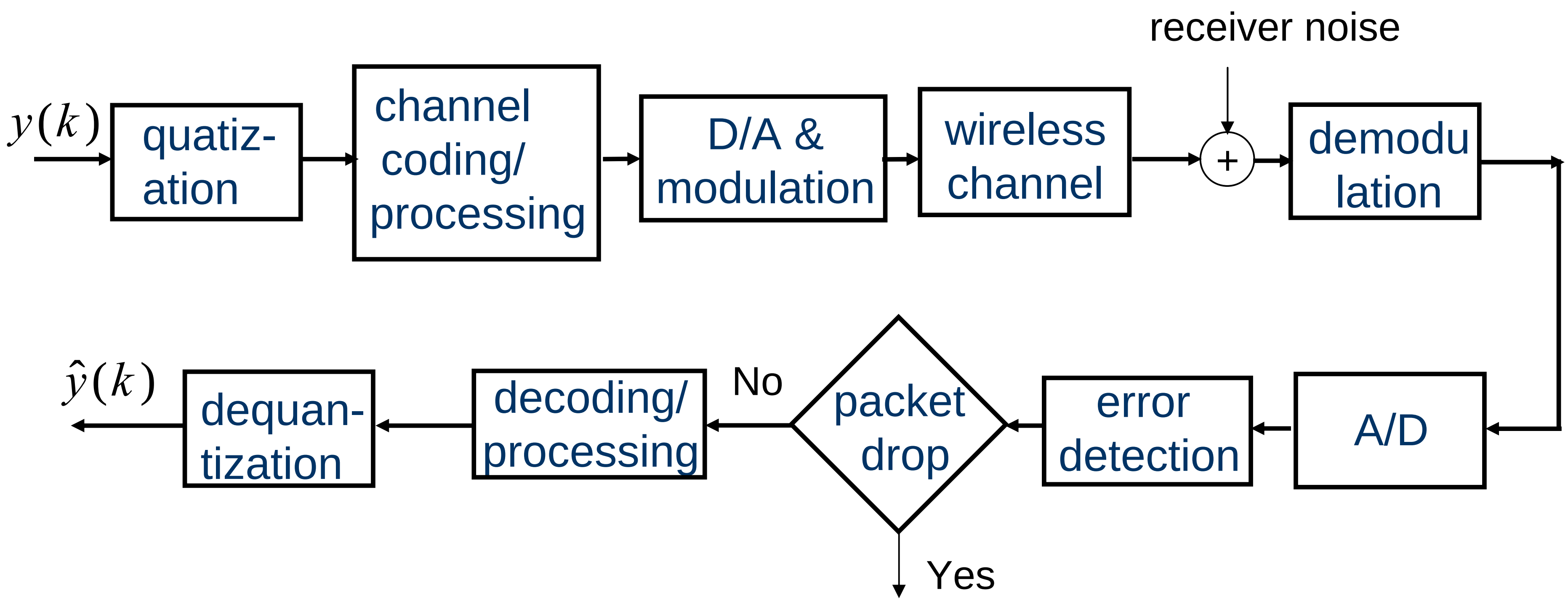
Observation: $y(k) = Cx(k) + v(k)$

$w(k)$: Zero mean noise with variance of Q

$v(k)$: Zero mean noise with variance of R

$\hat{x}(k)$: Kalman filter estimate of $x(k)$

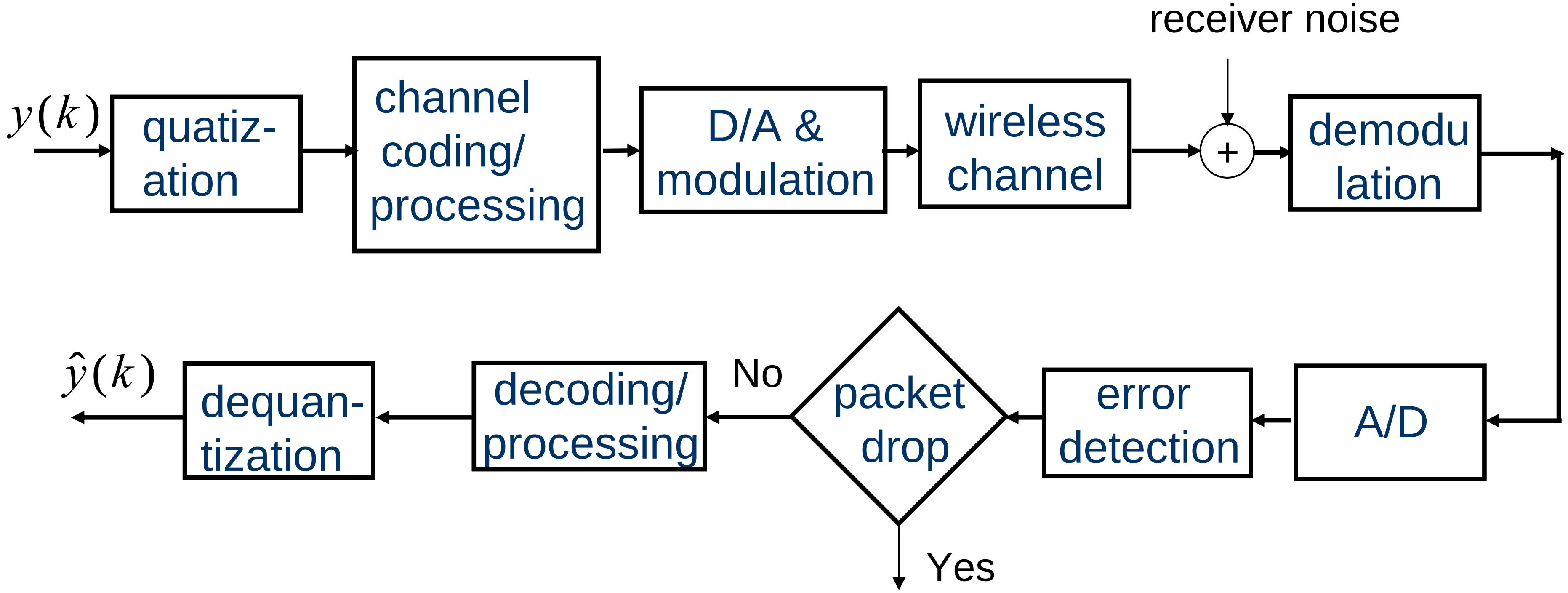
Wireless Transmission (review)



$$\hat{y}(k) = y(k) + n(k)$$

$n(k)$ is communication noise with variance of $\sigma_n^2(k)$

Past Lectures

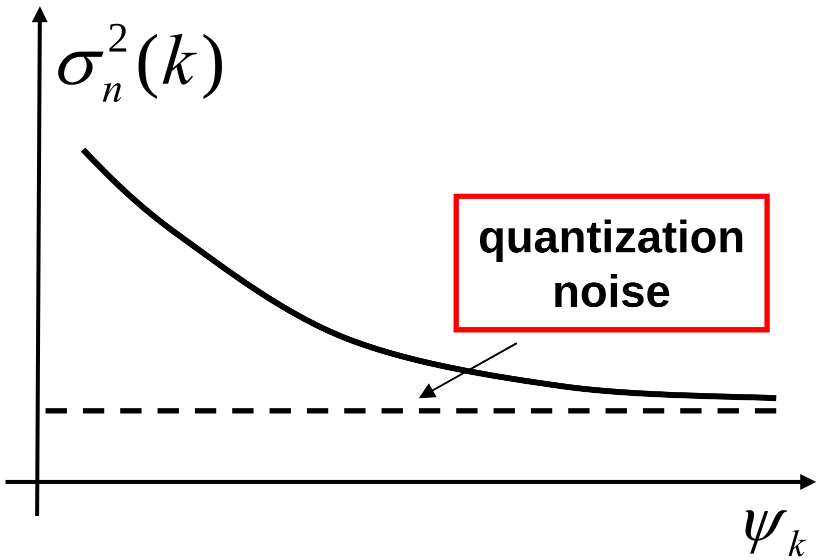


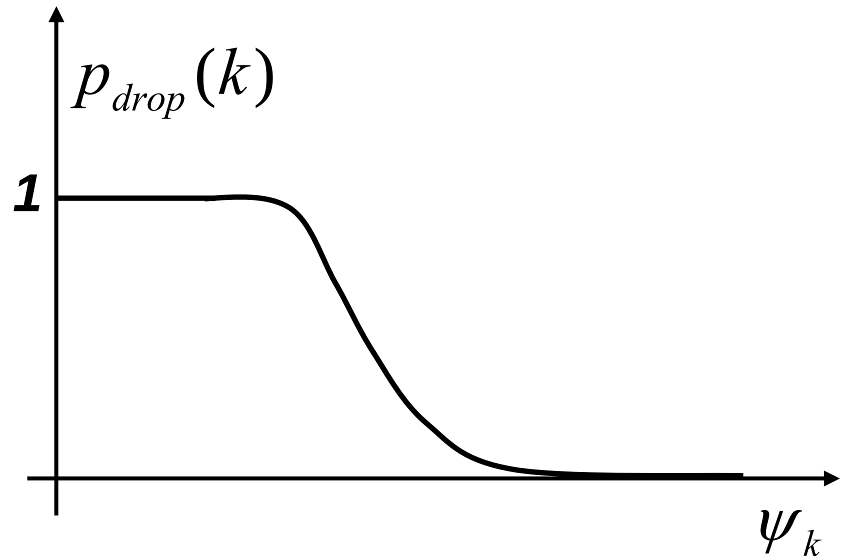
- Last week's lectures: Only allow noise-free samples
- Last lecture:
  – We looked at a receiver that keeps all the packets and uses a cross-layer information path
  – We derived analytical expression to evaluate performance
  – We showed that the design is always stable

Today: Optimum Design

- What is the optimum packet drop for estimation and control over wireless links?
- What are the benefits of using channel knowledge in the estimator?
  - Stability & performance
- Consider general cases: general ψ and σ_n^2 and system parameters
- Ideal noise profile: keeping only noise-free samples
  - Suitable for non delay-sensitive applications

Abstraction in the Higher Layer





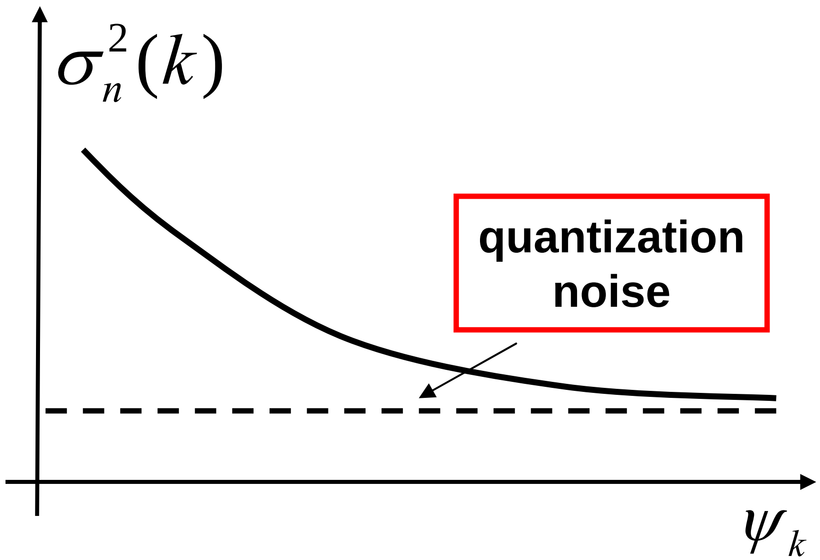
$\sigma_n^2(k)$ & $p_{drop}(k)$ **:**

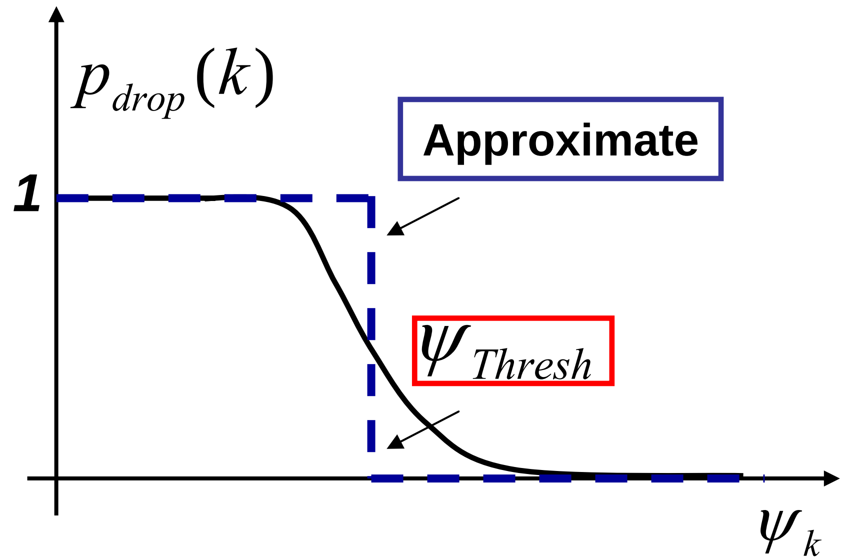
- Function of TX/RX technologies like quantization, noise figure, modulation, channel coding, ..

Distribution of ψ_k:

- Function of environment

Abstraction in the Higher Layer





$\sigma_n^2(k)$ & $p_{drop}(k)$:

- Function of TX/RX technologies like quantization, noise figure, modulation, channel coding, ..

Distribution of ψ_k:

- Function of environment

New Design Paradigms

Take impact of noisy samples into account → Non-ideal noise profile

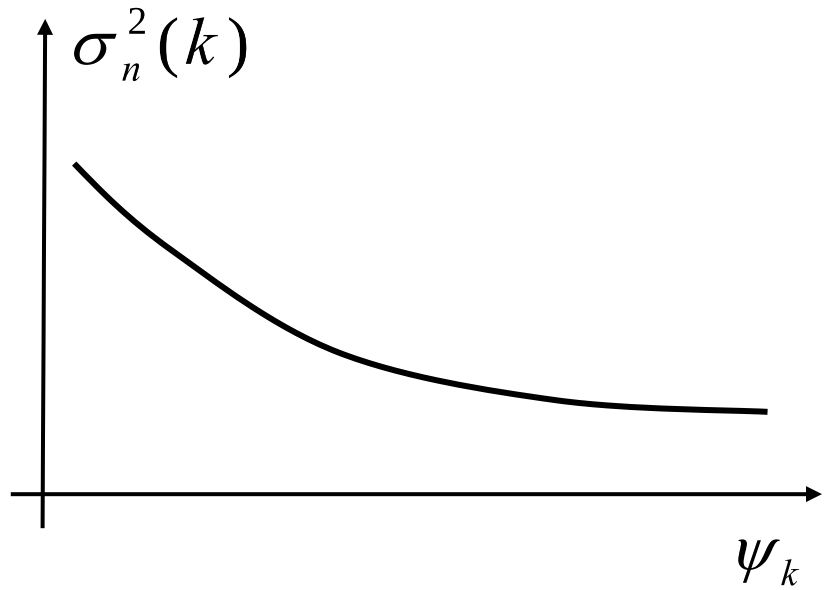
Only keep noise-free samples → Ideal noise-profile

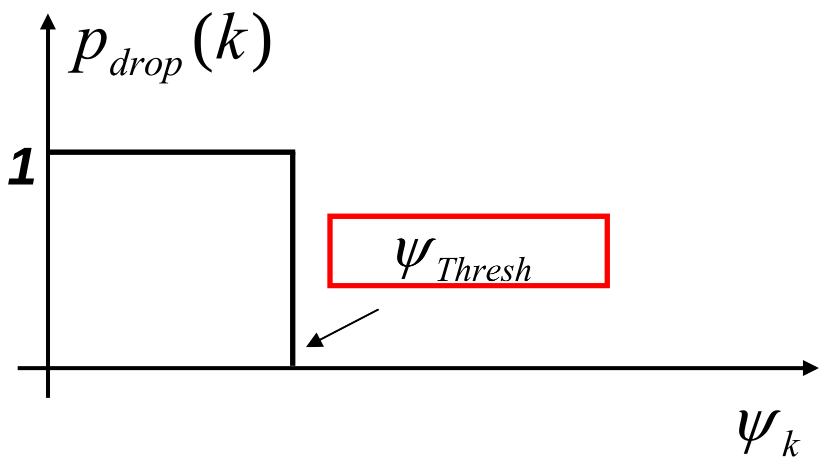
	Non-ideal noise profile: *cross-layer*	Non-ideal noise profile: *no cross-layer*	Ideal noise-profile
drop	**Scenario#3** ?	**Scenario#2** ?	**Scenario#1** Sinopoli et al. & Liu et al.
keep all	?	?	✗

Scenario#1: Sinopoli et al. (TAC 04), to maintain stability:

$$\text{for non-mobile nodes}: p_{drop,scenario\#1} < A^{-2}$$

Scenario#2: No Channel Info Available





- What is the optimum packet drop design?
- If ψ_{Thresh} is too high, many packets are dropped => information loss
- If ψ_{Thresh} is too low, estimation will be too noisy
- Intuitively, there should be an optimum ψ_{Thresh}

Scenario#2

- In Scenaior#2, KF does not know anything about the quality of the communication link
- From the point of view of the Kalman filter, the link was perfect
- To focus on communication noise, we assume that observation noise is negligible in the derivations of scenario#2

Scenario#2

- Then $\hat{x}(k+1) = \begin{cases} A\hat{x}(k) & \text{if } k^{th} \text{ sample is dropped} \\ AC^{-1}\hat{y}(k) & \text{if } k^{th} \text{ sample is kept} \end{cases}$

- We will have the following recursion for estimation error variance:

$$P(k+1) = A^2 P(k) + Q - \frac{A^2 P(k) - A^2 C^{-2} \sigma_n^2 (\psi(k))}{S(k)}$$

where $S(k) = \begin{cases} 1 & \psi(k) \geq \psi_{Thresh} \\ \infty & \text{else} \end{cases}$

Scenario#2

- Averaging over Signal to Noise Ratio distribution, $f(\psi)$,will result in the following:

$$\overline{P}(k+1) = A^2 p_L(\psi_{Thresh})\overline{P}(k) + A^2 C^{-2} p_N(\psi_{Thresh}) + Q$$

where

$$p_L(\psi_{Thresh}) = \overline{p_{drop}} = \int_0^{\psi_{Thresh}} f(\psi)\,d\psi$$

$$p_N(\psi_{Thresh}) = \int_{\psi_{Thresh}}^{\infty} \sigma_n^2(\psi) f(\psi)\,d\psi$$

Stability Condition for Scenario#2

To keep average estimation error variance bounded :

$$\overline{p_{drop}} = \int_0^{\psi_{Thresh}} f(\psi)d\psi < A^{-2}$$

Remark: Define **stability range** as the range of average Signal to Noise Ratios or A matrices for which estimation is stable. Having lower ψ_{Thresh} will increase the stability range.

Scenario#2: Optimum Performance

- Theorem1: Balance of Information Loss & Communication Noise
  - Consider asymptotic average estimation error variance:
    $$\overline{P(\infty)} = \frac{A^2 C^{-2} p_N(\psi_{Thresh}) + Q}{1 - A^2 p_L(\psi_{Thresh})} \quad \text{for } p_L(\psi_{Thresh}) < A^{-2}$$
  - Optimum ψ_{Thresh} that minimizes asymptotic average estimation error variance will be as follows:

$$\psi_{Thresh,opt} = \begin{cases} \psi^*_{Thresh} & \psi^*_{Thresh} \geq 0 \\ 0 & \text{else} \end{cases}$$

Scenario#2: Optimum Performance

- Where ψ^*_{Thresh} balances information loss and communication noise as follows:

$$\underbrace{p_L(\psi^*_{Thresh})}_{\text{Information loss}} + \underbrace{p_{N,normalized}(\psi^*_{Thresh})}_{\text{Communication noise}} + \frac{C^2 Q}{A^2 \sigma_n^2(\psi = \psi^*_{Thresh})} = A^{-2}$$

Eq. #1

where $$p_{N,normalized}(\psi^*_{Thresh}) = \frac{p_N(\psi^*_{Thresh})}{\sigma_n^2(\psi = \psi^*_{Thresh})}$$

Proof of Theorem 1

- Let ψ^{*}_{Thresh} represent any solution to Eq#1. Let ψ^{c}_{Thresh} represent the critical stability Threshold: $1 - A^2 p_L(\psi^{c}_{Thresh}) = 0$
- We have $\psi^{*}_{Thresh} < \psi^{c}_{Thresh}$
- It is easy to verify that

$$\frac{\partial \overline{P(\infty)}}{\partial \psi_{Thresh}} = 0 \text{ at } \psi_{Thresh} = \psi^{*}_{Thresh}$$

- We have to prove that Eq#1 has a unique solution

Proof of Theorem 1 (cont.)

Assume that Eq#1 has two solutions: $\psi^*_{Thresh,1}$ and $\psi^*_{Thresh,2} > \psi^*_{Thresh,1}$

Since σ_n^2 is a non-increasing function of ψ, we will have

$$\left[p_L(\psi^*_{Thresh,1}) + p_{N,normalized}(\psi^*_{Thresh,1}) + \frac{C^2 Q}{A^2 \sigma_n^2(\psi = \psi^*_{Thresh,1})} \right] -$$

$$\left[p_L(\psi^*_{Thresh,2}) + p_{N,normalized}(\psi^*_{Thresh,2}) + \frac{C^2 Q}{A^2 \sigma_n^2(\psi = \psi^*_{Thresh,2})} \right] =$$

$$\int_{\psi^*_{Thresh,2}}^{\psi^*_{Thresh,1}} f(\psi) d\psi + \int_{\psi^*_{Thresh,1}}^{\psi^*_{Thresh,2}} \frac{\sigma_n^2(\psi) f(\psi)}{\sigma_n^2(\psi = \psi^*_{Thresh,1})} d\psi$$

$$+ \left[\frac{1}{\sigma_n^2(\psi = \psi^*_{Thresh,1})} - \frac{1}{\sigma_n^2(\psi = \psi^*_{Thresh,2})} \right] \int_{\psi^*_{Thresh,2}}^{\infty} \sigma_n^2(\psi) f(\psi) d\psi +$$

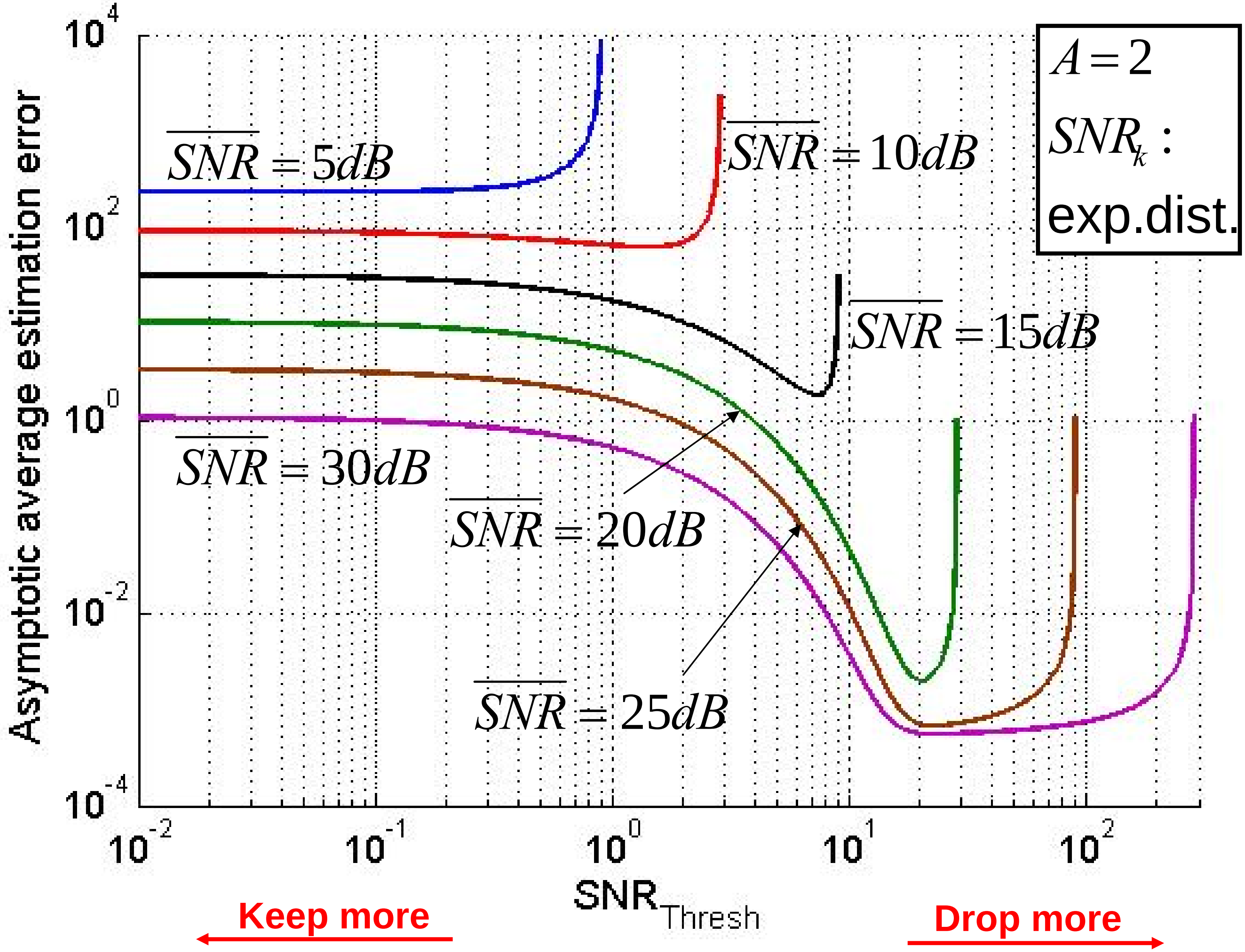
$$\frac{C^2 Q}{A^2} \left[\frac{1}{\sigma_n^2(\psi = \psi^*_{Thresh,1})} - \frac{1}{\sigma_n^2(\psi = \psi^*_{Thresh,2})} \right] < 0$$

Therefore $\psi^*_{Thresh,1} = \psi^*_{Thresh,2}$

Remarks on Optimum Performance

- Theorem 1 shows that as long as Eq#1 has a positive solution, then the optimum way of dropping packets is the one that balances loss of information and the amount of communication noise that enters the estimation process
- If process noise is the dominant factor compared to the communication noise, Eq#1 may not have a positive solution. Then keeping all the packets is optimum

Example: Optimum Packet Drop for Scenario#2



Scenario#3: Knowledge of channel available for KF

$$P(k+1) = A^2 P(k) + Q - \frac{A^2 C^2 P^2(k)}{C^2 P(k) + \sigma_z^2(\psi(k))}$$

where

$$\sigma_z^2(\psi(k)) = \begin{cases} \sigma_n^2(\psi(k)) + R & \psi(k) \geq \psi_{Thresh} \\ \infty & \text{else} \end{cases}$$

Scenario#3: Stability Condition

Lemma 1: Stability region of scenario#1 includes that of scenario#3

Proof: consider a special case of scenario#1 with $R=0$. Let g(k) and P(k) represent estimation error variances of scenario#1 with R=0 and scenario#3 respectively. We will have,

$$\overline{g}(k+1) = A^2 p_L \overline{g}(k) + Q$$

Scenario#3: Stability Condition

$$P(k+1) \geq A^2 P(k) + Q - \frac{A^2 C^2 P^2(k)}{C^2 P(k) + S(k)}$$

where $S(k) = \begin{cases} 0 & \psi(k) \geq \psi_{Thresh} \\ \infty & \text{else} \end{cases}$

We will have $\overline{P}(k+1) \geq A^2 p_L \overline{P}(k) + Q$.

Then, if $\overline{P}(k) \geq \overline{g}(k) \Rightarrow \overline{P}(k+1) \geq \overline{g}(k+1)$ ■

Scenario#3: Stability Condition

Lemma 2: Stability region of scenario#3 includes that of scenario#2

Proof: Let $q(k)$ represent estimation error variance of scenario#2 with $R \neq 0$, where no knowledge of R is available at the KF. We will have,

$$\overline{q}(k+1) = A^2 p_L \overline{q}(k) + Q + A^2 C^{-2} p_{N,R}$$

where $p_{N,R} = p_N + (1 - p_L)R$

Scenario#3: Stability Condition

$$E(P(k+1) \mid P(k)) =$$
$$(1-p_L)E(P(k+1) \mid P(k), \psi(k) > \psi_{Thresh}) +$$
$$p_L E(P(k+1) \mid P(k), \psi(k) < \psi_{Thresh})$$

$P(k+1)$ is a concave function of σ_z^2.

Then using conditional Jensen's Inequality,

$$E(P(k+1) \mid P(k), \psi(k) > \psi_{Thresh}) \leq$$
$$A^2 P(k) + Q - \frac{A^2 C^2 P^2(k)}{C^2 P(k) + E(\sigma_z^2(\psi(k)) \mid \psi(k) > \psi_{Thresh})}$$

Scenario#3: Stability Condition

Then,

$$E(P(k+1)\,|\,P(k)) \le A^2 P(k) + Q + \frac{(p_L - 1)A^2 C^2 P^2(k)}{C^2 P(k) + E(\sigma_z^2(\psi(k))\,|\,\psi(k) > \psi_{Thresh})}$$

The third term on the right hand side is a concave function of $P(k)$.

Appying Jensen's Inequality,

$$E(P(k+1)) \le A^2 E(P(k)) + Q + \frac{(p_L - 1)A^2 C^2 E^2(P(k))}{C^2 E(P(k)) + E(\sigma_z^2(\psi(k))\,|\,\psi(k) > \psi_{Thresh})}$$

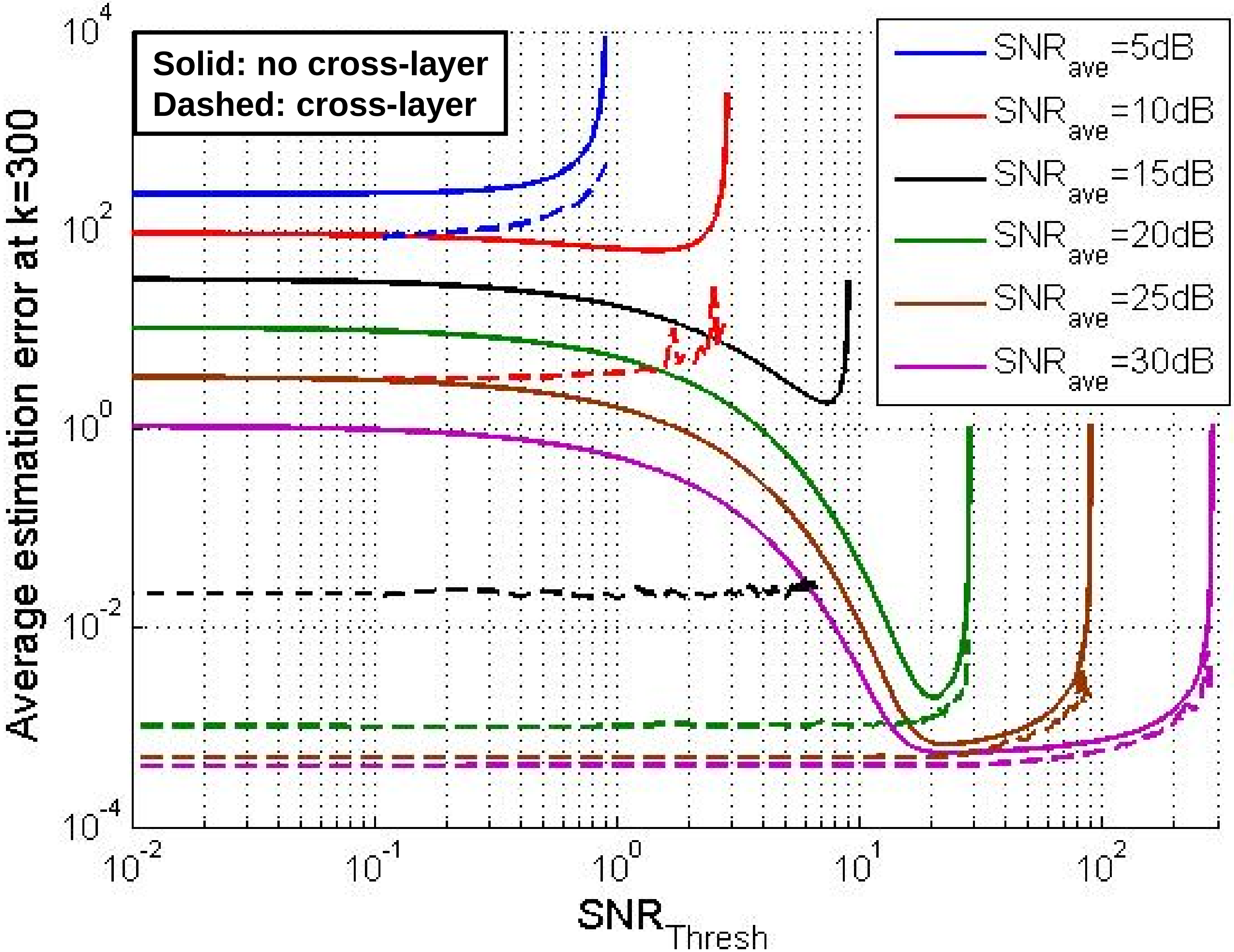
Noting that $E(\sigma_z^2(\psi(k))\,|\,\psi(k) > \psi_{Thresh}) = \dfrac{p_{N,R}}{1 - p_L} \Rightarrow$

if $E(P(k)) \le E(q(k)) \Rightarrow E(P(k+1)) \le E(q(k+1))$ ■

Scenario#3: Cross-layer Design

- **Theorem 2:** Cross-layer path on channel quality does NOT impact stability region
  - Proof: Lemma 1 and Lemma 2 proved that stability region of scenario#1 includes that of scenario#3 and stability region of scenario#3 includes that of scenario#2. We proved that stability region of scenario#2 is the same as scenario#1. Therefore, scenario#3 will have the same stability condition.
- Keeping all packets minimizes average estimation error variance
  - Easy to prove: see CDC05 in the reference list
- Cross-layer available => keep all packets for stability & performance

Effect of Cross-Layer Design



Estimation Over Wireless: Summary

- We studied optimum packet drop mechanism
- We proved that stability condition is the SAME independent of cross-layer or shape of communication noise variance
- Cross-layer on channel knowledge available:
  - keep packets for both stability & performance
- Cross-layer on channel knowledge not available:
  - Stability range main factor: keep packets
  - Estimation error main factor: packet drop to **balance information loss and comm. noise**

Possible Projects: Study Impact of Communication Impairments on Estimation and Control over Wireless

- Have a node estimate/control a dynamical system over a wireless link
- study the impact of channel variations, channel correlation
- explore redesigning the communication side: keeping packets, using cross-layer design, control packet drop, use of SNR info in estimation/control
- Try other communication protocols like analog communication, sensor network protocols like 802.15.4 (ZigBee),
- Compare different protocols (ZigBee, 802.11b, Bluetooth, Analog, …)
- Survey of suitable networking protocols for these applications