CALIFORNIA INSTITUTE OF TECHNOLOGY
BioEngineering

BE 250C

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Problem Set #7

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1. *From Alon 8.1. Diffusion from both sides.* A morphogen is produced at both boundaries of a region of cells that ranges from x=0 to x=L. The morphogen diffuses into the region and is degraded at rate α. What is the steady state concentration of the morphogen as a function of position? Assume that the concentration at the boundaries is $M(0) = M(L) = M_o$. Under what conditions is the concentration of morphogen at the center of the region very small compared to M_o?

   Hint: The morphogen concentration obeys the diffusion-degradation equation at steady state:

   $$D\frac{d^2M}{dx^2} - \alpha M = 0$$

   The solutions of this equation are of the form:

   $$M(x) = Ae^{-x/\lambda} + Be^{x/\lambda}$$

   Fine λ, A, and B that satisfy the diffusion-degradation equation and the boundary conditions.

2. *Alon 8.3. Polynomial self-enhanced degradation.* Find the steady state concentration profile of a morphogen produced at x=0. The morphogen diffuses into a field of cells, with nonlinear self-enhanced degradation described by:

   $$\frac{dM}{dt} = D\frac{d^2M}{dx^2} - \alpha M^n$$

   When is patterning with this profile robust to the level of M at the boundary, M_o? *Hint:* Try a solution of the form $M(x) = a(x+b)^m$ and find the parameters a and b in terms of D, M_o, and α.

3. *Alon 8.4. Robust Timing.* A signaling protein X inhibits pathway Y. At time t=0, X production stops and its concentration decays due to degradation. The pathway Y is activated when X levels drop below a threshold T. The time at which Y is activated is t_Y. Our goal is to make t_Y as robust as possible to the tinitial level of X, $X(t=0) = X_o$.

   a) Compare the robustness of t_Y in two mechanisms, linear degradation and self-enhanced degradation (note that in this problem, all concentrations are spatially uniform).

   $$\frac{dX}{dt} = -\alpha X$$

   $$\frac{dX}{dt} = -\alpha X^n$$

   Which mechanism is more robust to fluctuations in X_o? Explain.

   b) Explain why a robust timing mechanism requires a rapid decay of X at times close to t=0.