CALIFORNIA INSTITUTE OF TECHNOLOGY
Bioengineering and Biology

BE 150

M. Elowitz and R. M. Murray | **Problem Set #1** | Issued: Jan 9
Winter 2012 | | Due: Jan 18

1. *Negative auto regulation.*

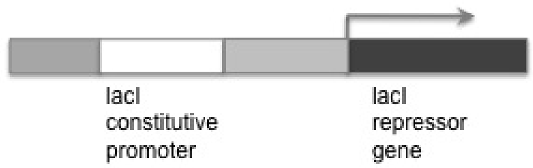

Figure 1: Unregulated *lacI* expression

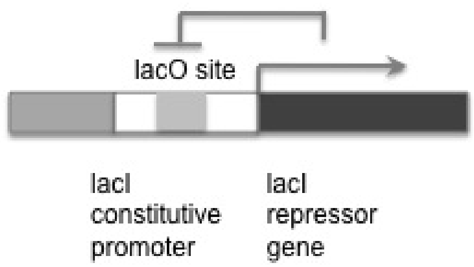

Figure 2: Negative autoregulation *lacI* expression

a) Write down an ODE model for the *lacI* repressor circuit in figure 1, including dynamics for transcription and translation of *lacI*. Find the steady state.

b) Rewrite the equations for the *lacI* negative autoregulated circuit in figure 2.

c) Pick parameters such that the open loop and closed loop systems have the same steady state. You can use the BioNumbers website: http://bionumbers.hms.harvard.edu/ as reference for picking parameters and be careful with units.

d) Plot concentrations of for both mRNA and protein concentrations for unregulated and regulated circuits as a function of time.

e) What is the response time of the *lacI* protein for unregulated and regulated circuits? Define the response time as the time it takes to reach 90% of the steady state.

f) Comment on differences between unregulated and regulated gene expression.

2. *Toggle switch.*

Consider a positive transcriptional feedback loop composed of two negative interactions $X \dashv Y$ and $Y \dashv X$.

a) Write the ODEs for the system above. Assume that the two transcription/repression mechanisms have the same dynamics and both genes are degraded at the same rate 0.2. Let the basal transcription rate be 1, K=2, n=2.

b) Find an analytical expression for steady states. Verify by plotting nullclines.

c) Plot the time response of X and Y using the following two initial conditions: (X(0),Y(0))=(1, 4) and (4, 1). Plot the phase plane of the system using *pplane* in MATLAB.

d) How do the responses change with initial conditions? Describe a situation where this type of interaction would be useful and give another example.

3. Consider the following network $X \to Y$ and $X \to X$.

a) Write the ODEs for the system above. Use basal expression $\beta_X = \beta_Y = 2$ and activation coefficients $K_X = 1$, $K_Y = 2$, $n_1 = n_2 = 2$. The degradation coefficients for X and Y are both 0.5.

b) Plot the vector field using pplane. How many steady states do you observe?

c) Analytically describe the stability of the equilibria.

d) Describe the relevance of having a positive feedback loop in a biological system.