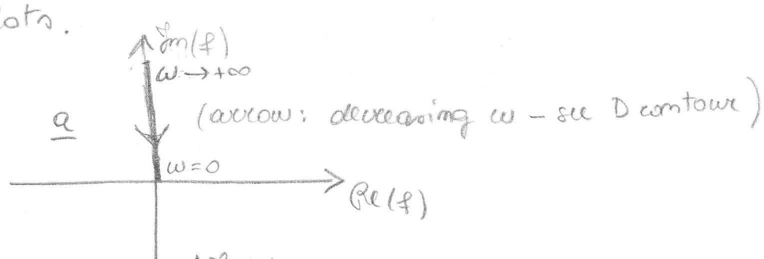
WEEK 7, RECITATION SECTION

1) WHAT IS A NYQUIST DIAGRAM?
2) NYQUIST CRITERION: OPEN LOOP VS CLOSED LOOP
3) HOW TO SKETCH NYQUIST DIAGRAMS BY HAND
4) PERFORMANCE SPECIFICATIONS: g_m, ϕ_m, S.S. ERROR AND BANDWIDTH, FINAL VALUE THM.
5) EX # 2 of HW 6 - TEMPLATE SOLUTION
6) Bode plots w/ RHP poles or zeros

1) The Nyquist plot is the mapping of a complex function into the complex plane; the resulting curve is parameterized in ω.
If we start from a transfer function $f(s)$: $s = j\omega \to f(j\omega)$
Let's see some simple Nyquist plots.

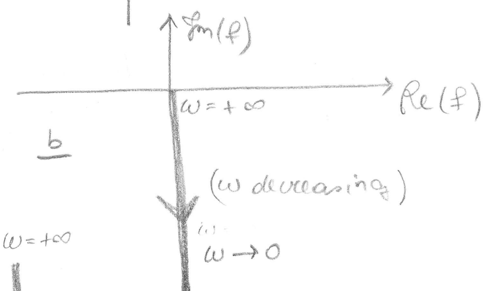
a $f(s) = s \Rightarrow f(j\omega) = j\omega$

a



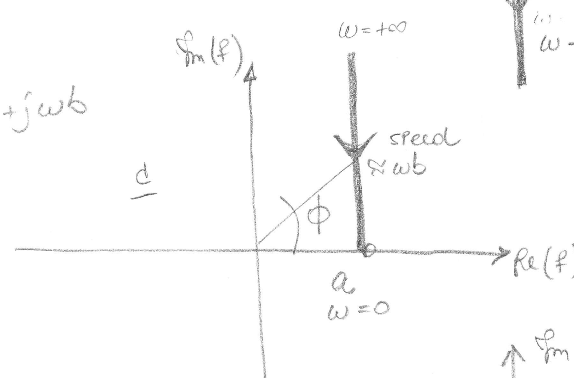
b $f(s) = \frac{1}{s} \Rightarrow f(j\omega) = \frac{1}{j\omega} = -\frac{j}{\omega}$

b



c $f(s) = a + s \cdot b \Rightarrow a + j\omega b$
$a, b > 0$

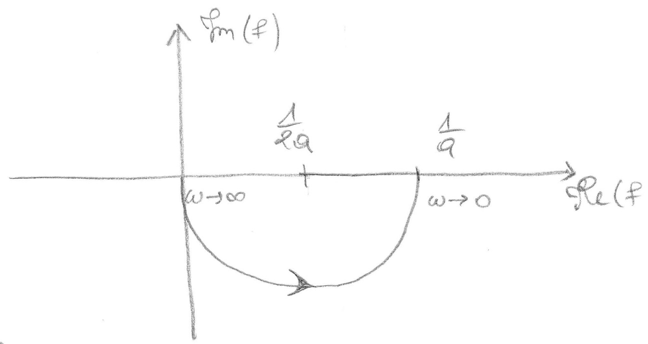
d



$\phi = \operatorname{atan}\left(\frac{\omega b}{a}\right)$

d $f(s) = \frac{1}{a+sb} \Rightarrow \frac{1}{a+j\omega b}$

$a>0,\ b>0$



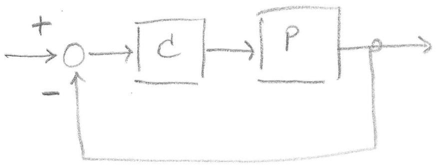
$f(j\omega) = \frac{1}{a+j\omega b} \cdot \frac{a-j\omega b}{a-j\omega b} = \frac{a}{a^2+\omega^2 b^2} - \frac{j\omega b}{a^2+\omega^2 b^2}$

- $Re(f) = 0 \Leftrightarrow \omega \to +\infty \Rightarrow f(j\omega) = 0$
- $Im(f) = 0 \Leftrightarrow \omega = 0 \Rightarrow f(j\omega) = \frac{1}{a}$
- $\phi = \operatorname{atan}\left(-\frac{j\omega b}{a}\right)$

- $\frac{Re(f)}{Im(f)} = \frac{R}{I} = -\frac{a}{\omega b} \Rightarrow \omega = -\frac{I}{R} \cdot \frac{a}{b} \Rightarrow$

$\Rightarrow R = \frac{aR^2}{aR^2 + a^2 I^2} \Rightarrow R^2 + I^2 - \frac{R}{a} = 0$: eqn of a CIRCLE

2) NYQUIST CRITERION:

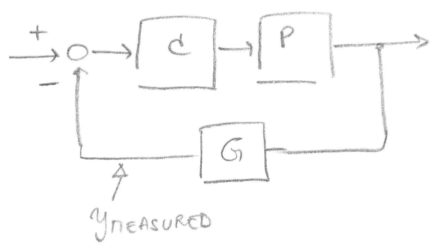


OPEN LOOP T.F.: $L = PC$ (OLTF)

CLOSED LOOP T.F.: $H = \dfrac{PC}{1+PC} = \dfrac{L}{1+L}$ (CLTF)

Matlab: feedback(L, 1) (assumes neg feedb)

→ WITH SENSOR DYNAMICS G



OPEN LOOP: $L = PCG$

CLOSED LOOP: $H = \dfrac{PC}{1+PCG}$

- TESTS ON OPEN LOOP: gain margin, phase margin, Bode plot, Nyquist plot } tell us about stability of closed loop sys.

- NYQUIST: P = # of RHP poles of $L(s)$
  N = # of CCW encirclements of -1
  Z = # of RHP zeros of $1+L(s)$ = # of RHP poles of $H = \dfrac{L}{1+L}$

PoA:

$Z = N + P$

How to use this: SKETCH NYQUIST FOR OLTF. DOES IT HAVE RHP poles "P"? HOW MANY TIMES "N" DOES IT ENCIRCLE -1?

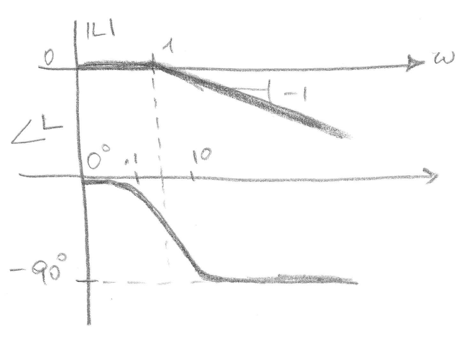
if "N" + "P" = 0 ⇒ Z = 0 ⇒ CLTF STABLE

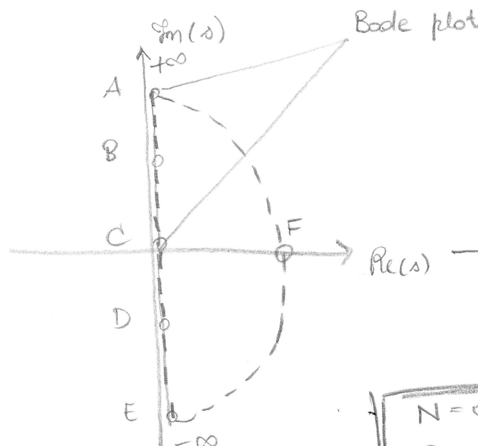
3) SKETCH NYQUIST FROM BODE w/ D-CONTOUR

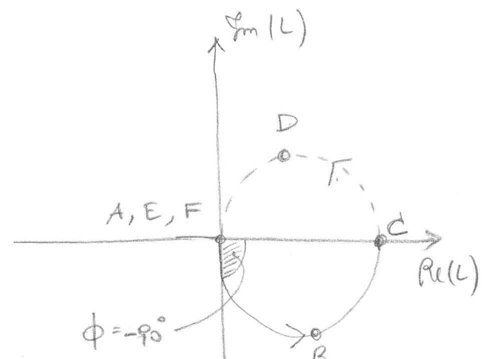
EX 1

$L(s) = \dfrac{1}{s+1}$

Bode





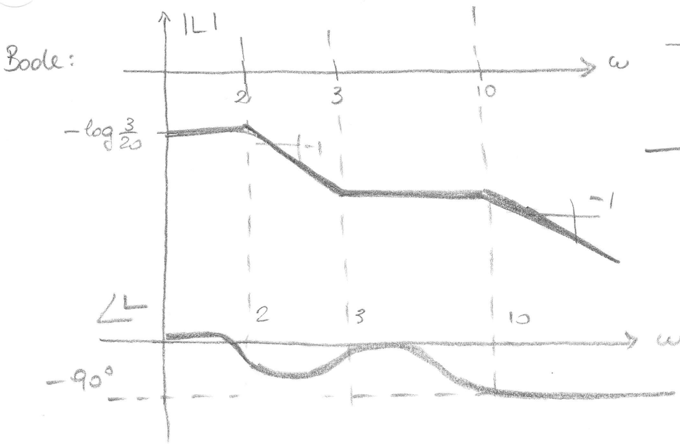


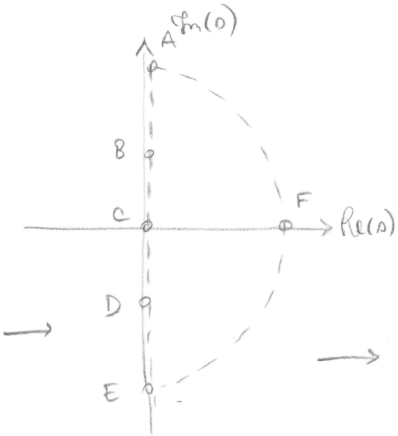
$\boxed{N=0 \quad P=0}$ ⇒ Z = 0

CLTF stable!!

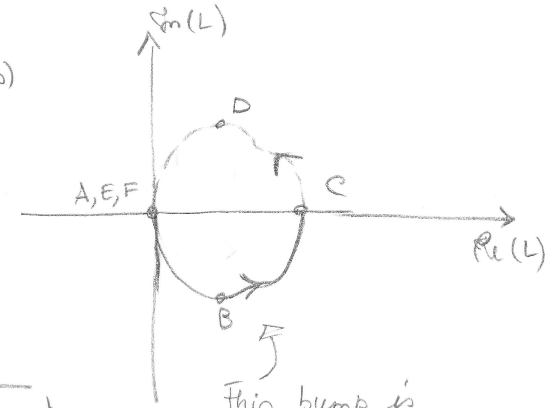
EX 2

$$L(s) = \frac{s+3}{(s+2)(s+10)}$$





NYQUIST



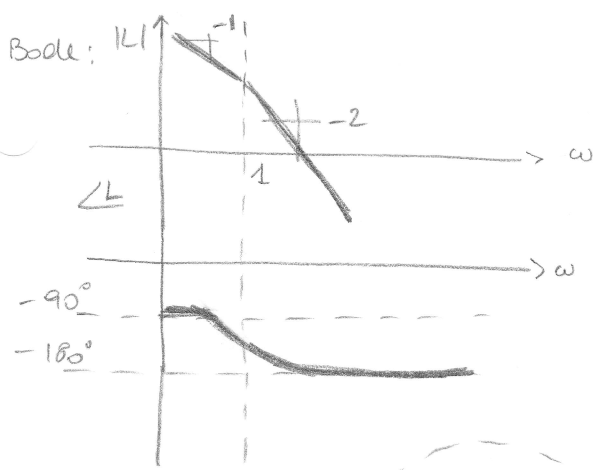
This bump is almost invisible using matlab!

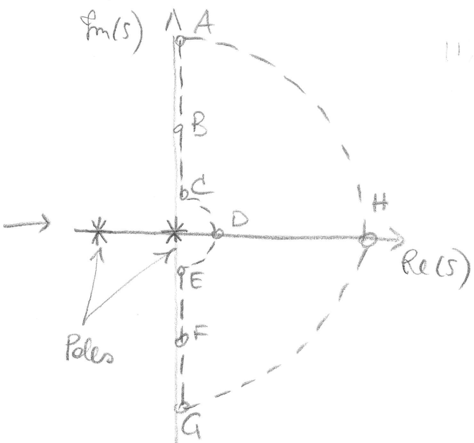
N=0 P=0	⇒ Z=0 CLTF STABLE

EX 3

$$L(s) = \frac{1}{s(s+1)}$$

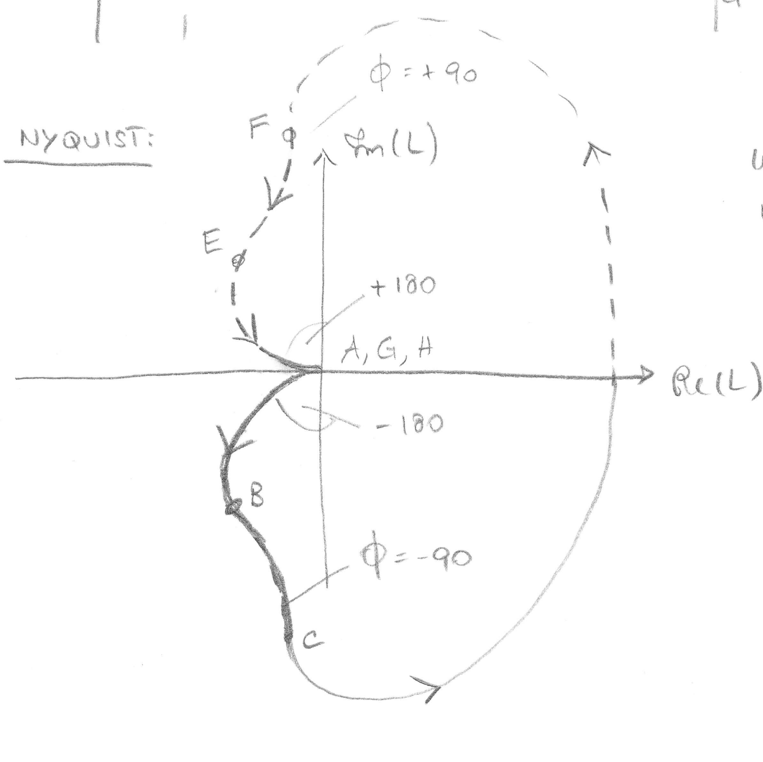
POLE @ ORIGIN!





The origin is a singularity for L(s) therefore must be eliminated by the contour

NYQUIST:



WHICH SIDE IS THIS CLOSING?

evaluate $s = \varepsilon + j0$ = point D in the contour

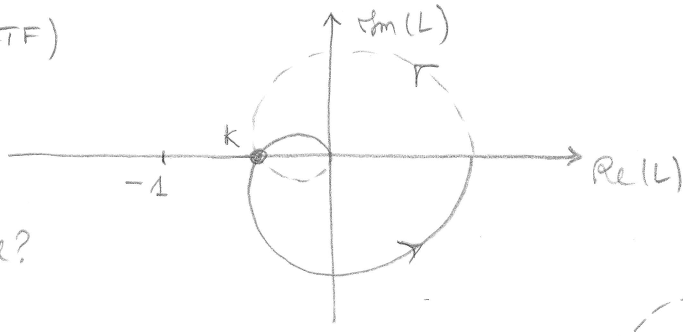
$$L(\varepsilon + j0) = \frac{1}{\varepsilon^2 + \varepsilon} > 0$$

Closes to the right!

4) PERFORMANCE SPECS:

- **GAIN MARGIN** (OLTF)

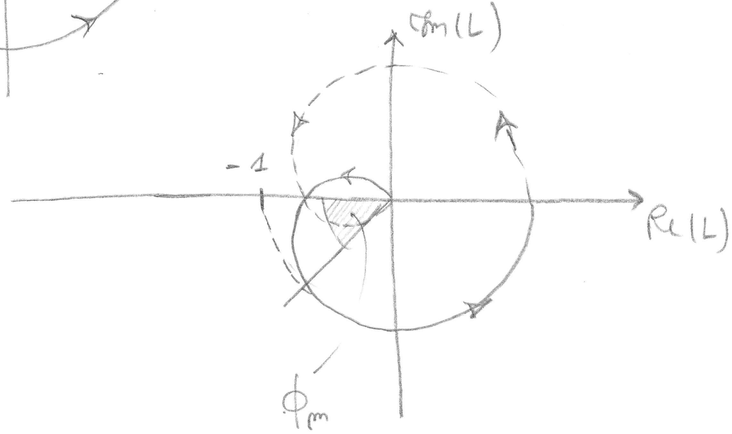
How much gain can we add in the loop before we get unstable?



$g_m = \frac{1}{k}$ (sometimes take $\log_{10}$)

- **PHASE MARGIN** (OLTF)

How much phase (delay) can we add in the loop and still be stable?

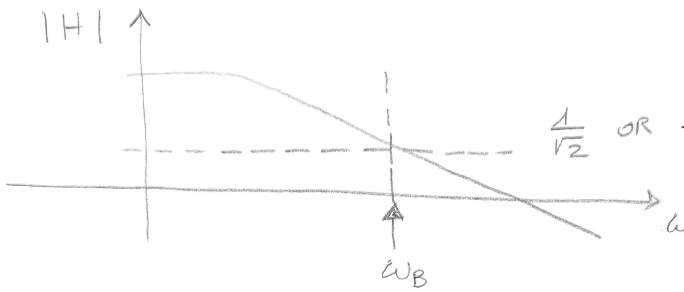


MATLAB COMMAND: margin (L)

- **BANDWIDTH** (CLTF)

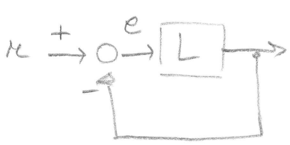
$$H = \frac{L}{1+L}$$

MATLAB: bandwidth (H)



$\frac{1}{\sqrt{2}}$ OR $-3dB = 20 \log_{10} \frac{1}{\sqrt{2}}$

- **STEADY STATE ERROR** (CLTF) TO A STEP INPUT



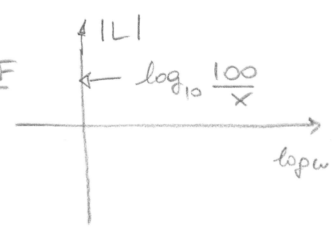
$$e(s) = \frac{1}{1+L(s)} \cdot r(s) \Rightarrow H_{er} = \frac{1}{1+L(s)}$$

IF SPECIFICATION IS: $|e_{ss}| < X\%$

(1) $$|H_{er}(0)| = \left|\frac{1}{1+L(0)}\right| \underset{\text{APPROX}}{\cong} \left|\frac{1}{L(0)}\right| \le \frac{X}{100}$$

ON BODE: USE OLTF

$$|L(0)| \ge \frac{100}{X}$$



(2) INITIAL VALUE THEOREM (LAPLACE)

$$\lim_{s \to 0} s F(s) = \lim_{t \to \infty} f(t)$$

ASSM:
F(s) has at most one pole at the origin

EXAMPLE

$$L(s) = \frac{1}{(s+1)(s+2)}$$

FIND S.S. ERROR

- $H_{er} = \frac{1}{1+L}$ MATLAB: dcgain(Her)

$$\lim_{s\to 0} \frac{s}{(s+1)(s+2)+1} \cdot \frac{1}{s} = \lim_{s\to 0} \frac{\cancel{s}}{s^2+3s+3} \cdot \frac{1}{\cancel{s}} = \frac{1}{3}$$

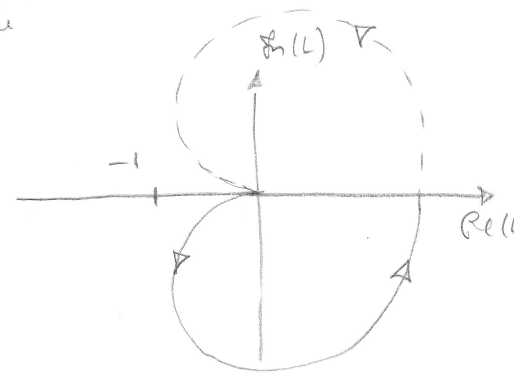
- Approx:

$$|L(0)| = \frac{1}{2}$$

if we want $e_{ss} < 10\%$, with the approximated criterion:

$|L(0)| \geq 10$ not true $\Rightarrow$ we should increase the gain

$$L(s) = \frac{k}{(s+1)(s+2)} \qquad |L(0)| = \frac{k}{2} \geq 10 \iff k \geq 20$$



Does this gain guarantee stability? Nyquist

check g_m finding $\Im(L) = 0$

$$L(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)} = \frac{A}{1+j\omega} + \frac{B}{2+j\omega} = \frac{2A + j\omega A + B + j\omega B}{\dots} = \frac{j\omega(A+B) + 2A+B}{\dots}$$

$$\begin{cases} A = -B \\ B - 2B = 1 \end{cases} \qquad \begin{matrix} B = -1 \\ A = 1 \end{matrix}$$

$$\Rightarrow L(j\omega) = \frac{1}{1+j\omega} - \frac{1}{2+j\omega} = \frac{1-j\omega}{(1+\omega^2)} - \frac{j\omega-2}{(4+\omega^2)}$$

$$\Im(L) = \frac{\omega}{1+\omega^2} - \frac{\omega}{4+\omega^2} = 0 \iff 4\omega + \cancel{\omega^3} - \omega - \cancel{\omega^3} = 0 \quad \text{only for } \omega = 0$$

$\Rightarrow g_m = \infty$! OK to increase the gain!

5) HW 6, EX 2

We will solve a "template" of ex 2 using Matlab
Consider:

$$P(s) = \frac{g}{(s+a)(s+b)} \qquad \text{plant dynamics}$$

a) Controller: $C(s) = k_p$

$$\Rightarrow L = \frac{g \cdot k_p}{(s+a)(s+b)}$$

- g_m, ϕ_m → margin (gives Bode plot for OLTF)
- ss error & poles/zeros : This has to be done for the CLTF

$$H_{er} = \frac{1}{1+L} \qquad H_{yr} = \frac{L}{1+L}$$

Matlab : dcgain (Her) ⇒ ss err = 1 − dcgain (Her)
pzmap (Hyr)

or you can find them by hand!

b) $C(s) = k_p + \frac{k_i}{s} = \frac{s k_p + k_i}{s}$

$L = P \cdot C \Rightarrow H_{er} = (1/1+L) \qquad H_{yr} = (L/1+L)$
Fill in the table for different values of k_p, k_i

- Stability : check poles of Hyr (or Her)
- g_m, ϕ_m : margin (L)
- ss err : dcgain (Her) - or 1 − dcgain (Hyr)
- Bandwidth : bandwidth (Hyr)

plot pzmap & step responses of Hyr

//

6) BODE w/ RHP poles or zeros

Negative sign causes $180°$ shift in phase, no change in magnitude

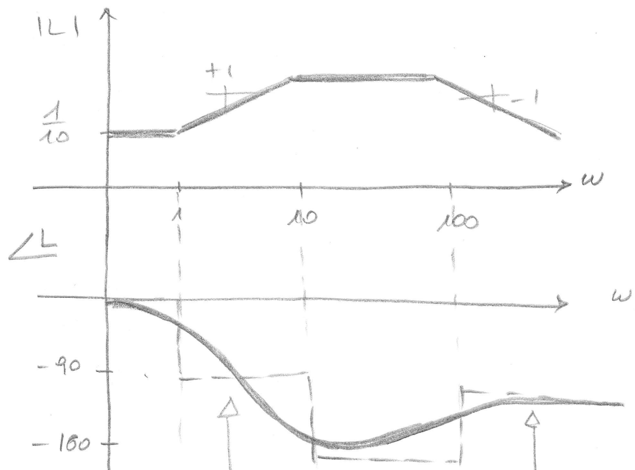
EXAMPLES

① $L(s) = 100 \dfrac{(s-1)}{(s+10)(s-100)}$

poles: -10, $+100$

zeros: $+1$

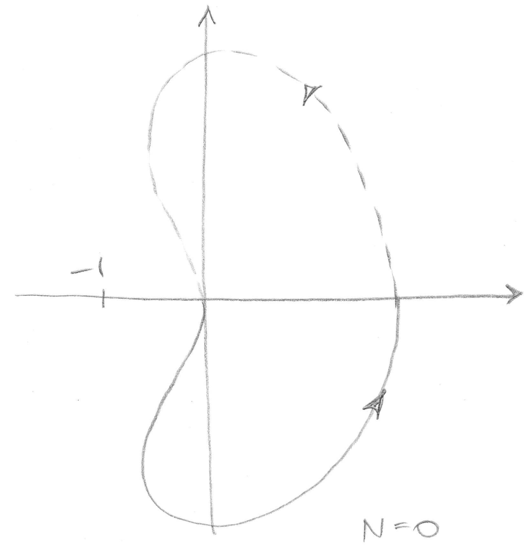
$L(0) = \frac{1}{10}$



RHP zero
-90
instead of $+90$

RHP pole
$+90$
instead of -90

Nyquist
⟶



$N = 0$

$P = 1$

$\Rightarrow Z = 1$

CLTF UNSTABLE