

Optimization, Dynamics, and Layering in Complex Networked Systems: From the Internet to the Smart Grid

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Networked systems

Complexity is ever increasing

- ❑ Large in size and scope
- ❑ Enormous heterogeneity
- ❑ Incomplete information
- ❑ Uncertain environments
- ❑ Emerging technologies
- ❑ New applications
- ❑ New design dimensions
- ❑

Design (& understanding) is increasingly dominated by

- ❑ Efficiency (optimality)
- ❑ Manageability
- ❑ Reliability & Security
- ❑ Economic viability
- ❑ Scalability
- ❑ Evolvability
- ❑

emerging, collective properties

The diagram consists of three main parts. At the top, a dark blue downward-pointing arrow contains the text 'Systems requirements: functional, efficient, robust, secure, evolvable, ...'. In the center, the word 'architecture' is written in bold red text. At the bottom, a dark blue upward-pointing arrow contains the text 'Components'. To the right of the 'architecture' text is a bulleted list with two items, each preceded by a small blue square icon.

Systems requirements:
functional, efficient,
robust, secure,
evolvable, ...

architecture

Components

- ❑ Organizational principles, including abstractions and interfaces
- ❑ Highly conserved core resource allocation, control, and management mechanisms

Systems requirements:
functional, efficient,
robust, secure,
evolvable, ...

architecture

Components

Constraints that deconstrain

- ❑ Certain fixed points and structure under which the network can expand/evolve
- ❑ Can be constraining for the issues that the network was originally not designed for

Good architecture enables innovation, bad one freezes it

Systems requirements:
functional, efficient,
robust, secure,
evolvable, ...

architecture

Components

Architectural design

- ❑ Remains an art, primarily empirical, reasoning-based
- ❑ Good architecture easy to recognize in retrospect but elusive to forward-engineer
- ❑ No formal theory nor systematic design method

Goal

Systems requirements:
functional, efficient,
robust, secure,
evolvable, ...

architecture

Components

- ❑ **Mathematical underpinning** of network architecture
- ❑ **Systematic methods** to develop and evaluate design choices and algorithms

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Systems requirements:
functional, efficient,
robust, secure,
evolvable, ...

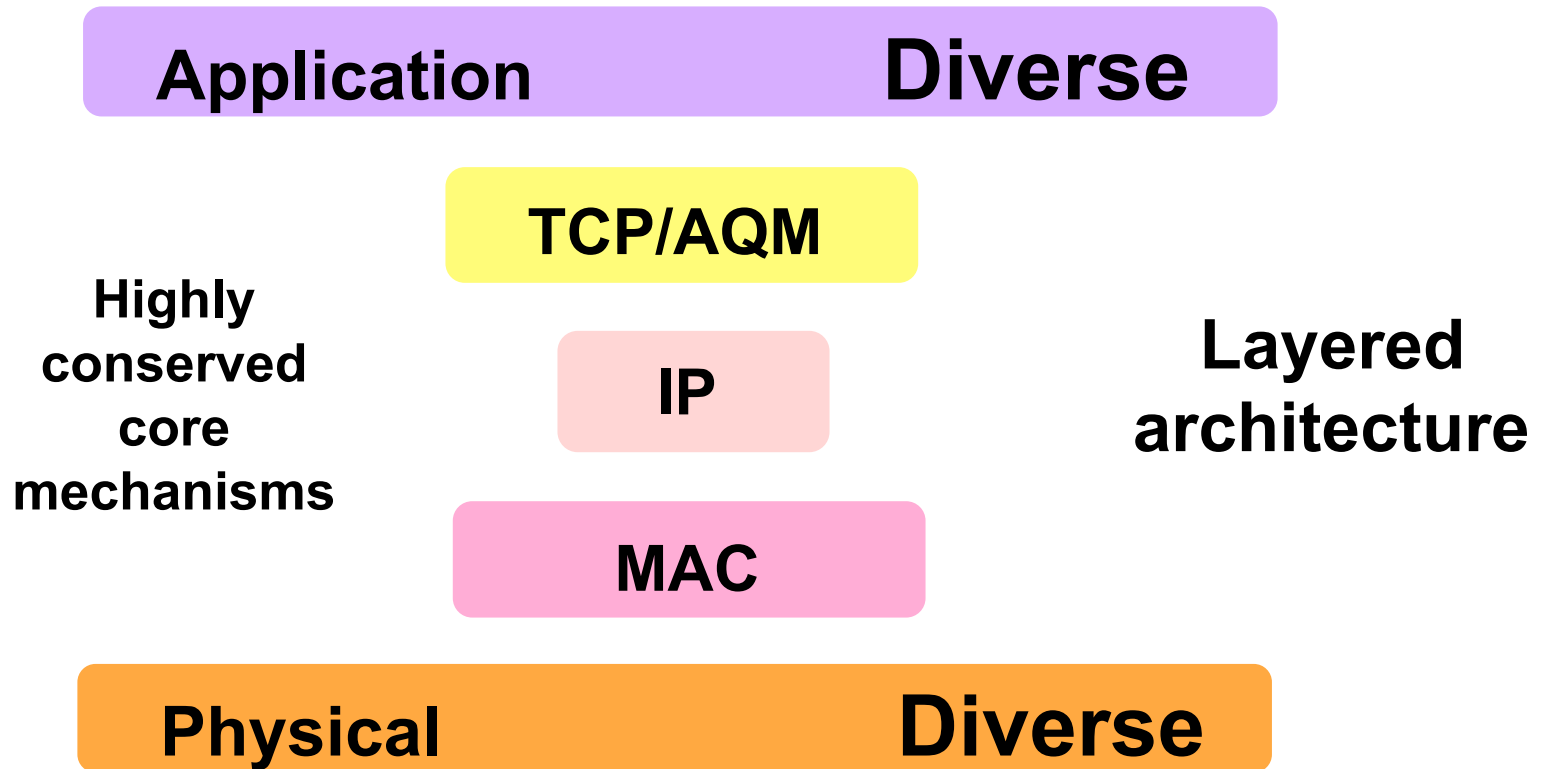
architecture

Components

Goal

- ❑ Understand architecture and main mechanisms of existing networks (**reverse engineering**)
- ❑ Design architecture and main mechanisms for emerging networks (**forward engineering**)

Internet



Internet architecture

Application

Little quantitative understanding

❑ Optimal? In what sense?

TCP/AQM

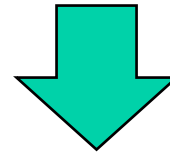
Lots of problems

IP

❑ Efficiency, security, mobility, accountability, ...

MAC

Physical



fixes (middle boxes
& overlays & underlays)

Emerging networked systems

future internet



energy-efficient
data center



smart grid



architectures are being designed now ...

Future Internet architecture

Clean slate Internet design that aims to build in

- ❑ security
- ❑ mobility
- ❑ new communication paradigms
- ❑ ...



not clear what the right architecture is and how to best design different components and their interactions

Energy-efficient data center

- ❑ How to decompose & coordinate energy management decisions spatially and temporally
- ❑ How to interact with other resource allocation algorithms
- ❑ How to interconnect servers to balance performance and energy usage

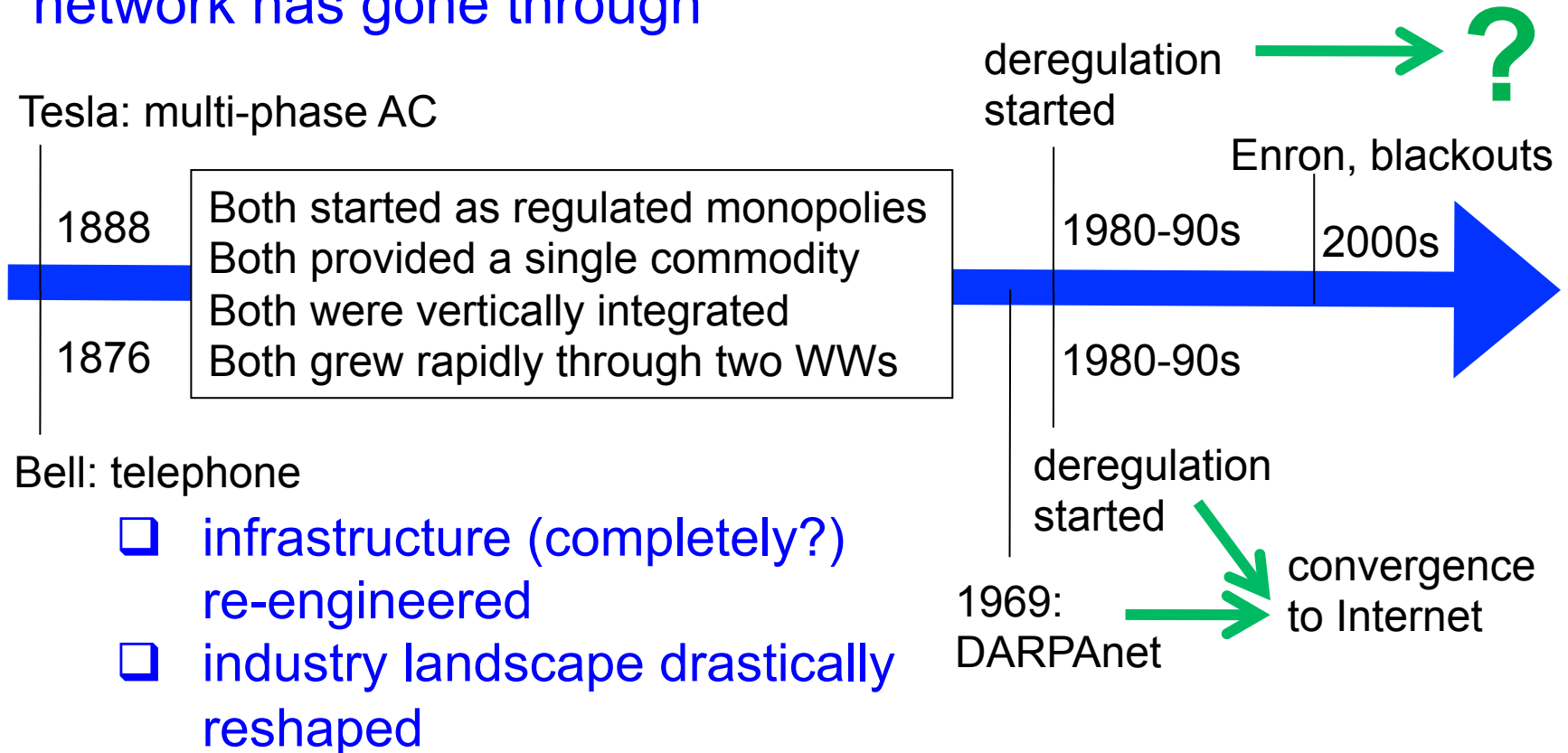


3-5% of total US energy use

theories and models are needed to guide architecture
and algorithm design

Smart grid

Power network will go through similar **architectural transformation** in the next few decades that telephone network has gone through



Smart grid

... to become more interactive, more distributed, more open, more autonomous, and with greater user participation



what is an architecture theory to help guide the transformation?

... while maintaining security & reliability

Research

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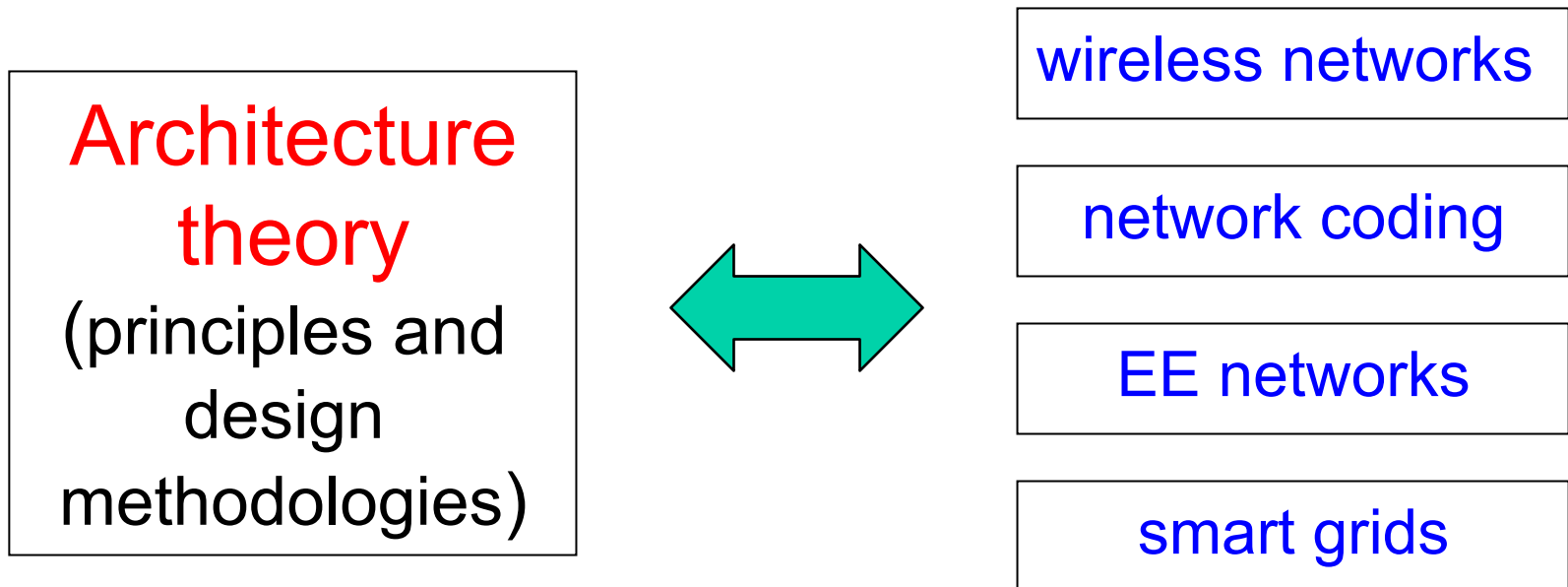
Systems requirements:
functional, efficient,
robust, secure,
evolvable, ...

architecture

Components

- ❑ Rigorous foundations and new methodologies for understanding & designing architecture and various mechanisms
- ❑ Employ and develop techniques in
 - ❑ optimization theory/algorithm
 - ❑ distributed control
 - ❑ game theory
 - ❑ systems theory

Approach



- ❑ must be **foundational** and **practical**
- ❑ must be **abstract** and **concrete**

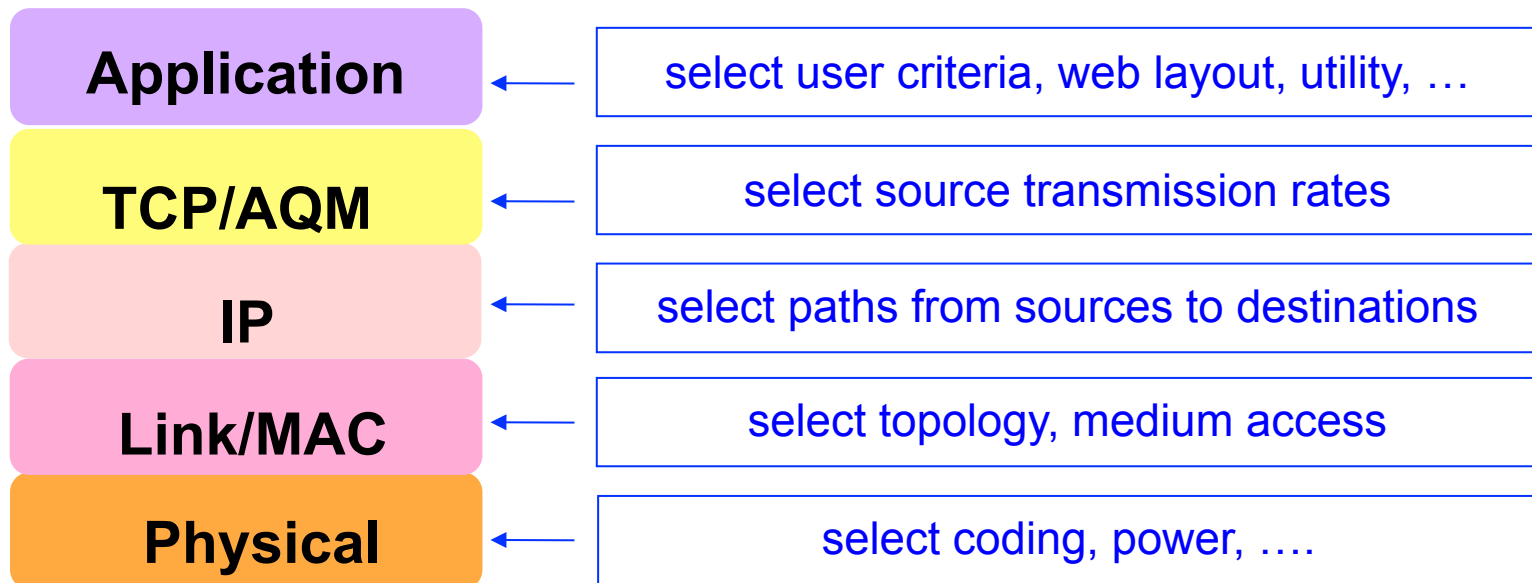
Outline

- ❑ Layering and constrained optimization
- ❑ Network dynamics as optimization algorithms
- ❑ Look into future

Layered Internet protocol stack

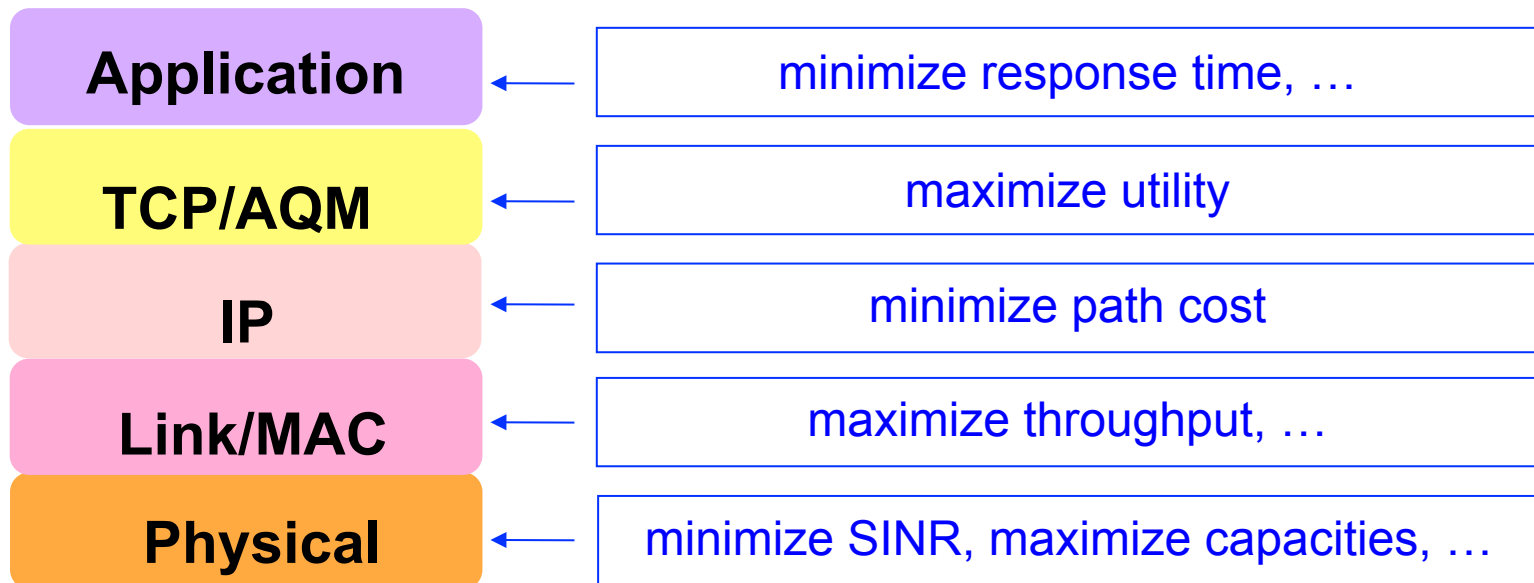
Each layer

- ❑ controls a subset of decision variables
- ❑ hides the complexity of the layer below
- ❑ provides a service to the layer above
- ❑ designed separately and evolves asynchronously



Optimization and layering

- ❑ Each layer is abstracted as an optimization problem
- ❑ Operation of a layer is a distributed solution
- ❑ Results of one problem (layer) are parameters of others
- ❑ Operate at different timescales



Optimization and layering

Networks as optimizers

- integrate various protocol layers, by regarding them as carrying out distributed computation over the network to implicitly solve a certain global optimization problem
- different layers iterate on different subsets of the decision variables using local information to achieve individual optimality
- taken together, these local algorithms achieve a global optimality

Protocol decomposition: TCP/AQM

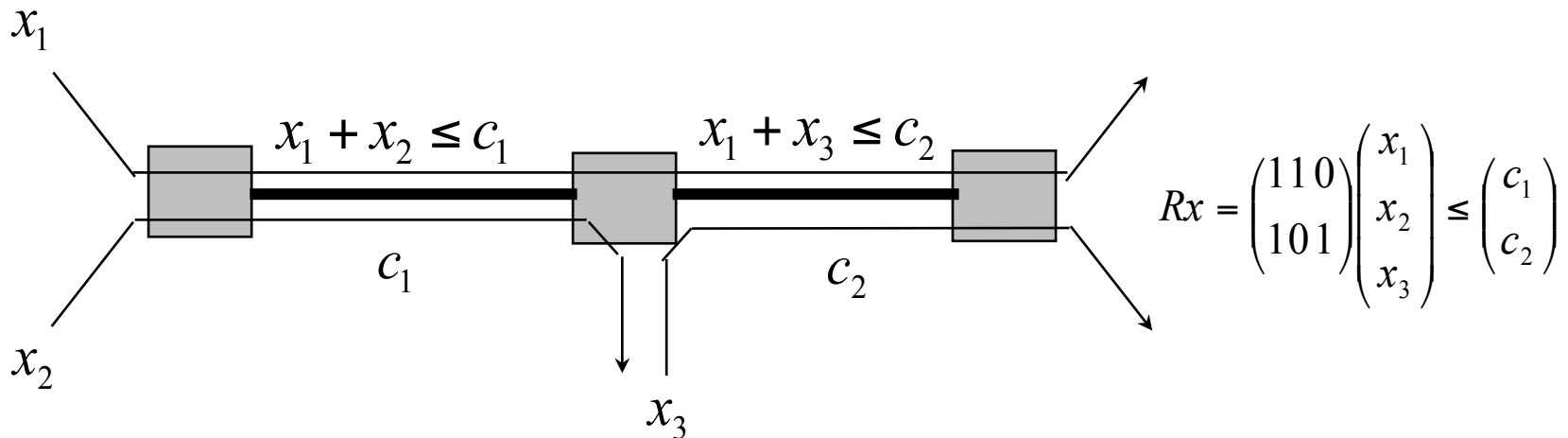
Duality model: TCP/AQM as distributed primal-dual algorithm over network to maximize aggregate utility (Kelly '98 , Low '99, '03)

Primal:

$$\begin{aligned} \max_{x \geq 0} \quad & \sum_s U_s(x_s) \\ \text{s.t.} \quad & Rx \leq c \end{aligned}$$

Dual

$$\min_{p \geq 0} \left(\sum_s \max_{x_s \geq 0} \left(U_s(x_s) - x_s \sum_l R_{ls} p_l \right) + \sum_l p_l c_l \right)$$



Protocol decomposition: TCP/AQM

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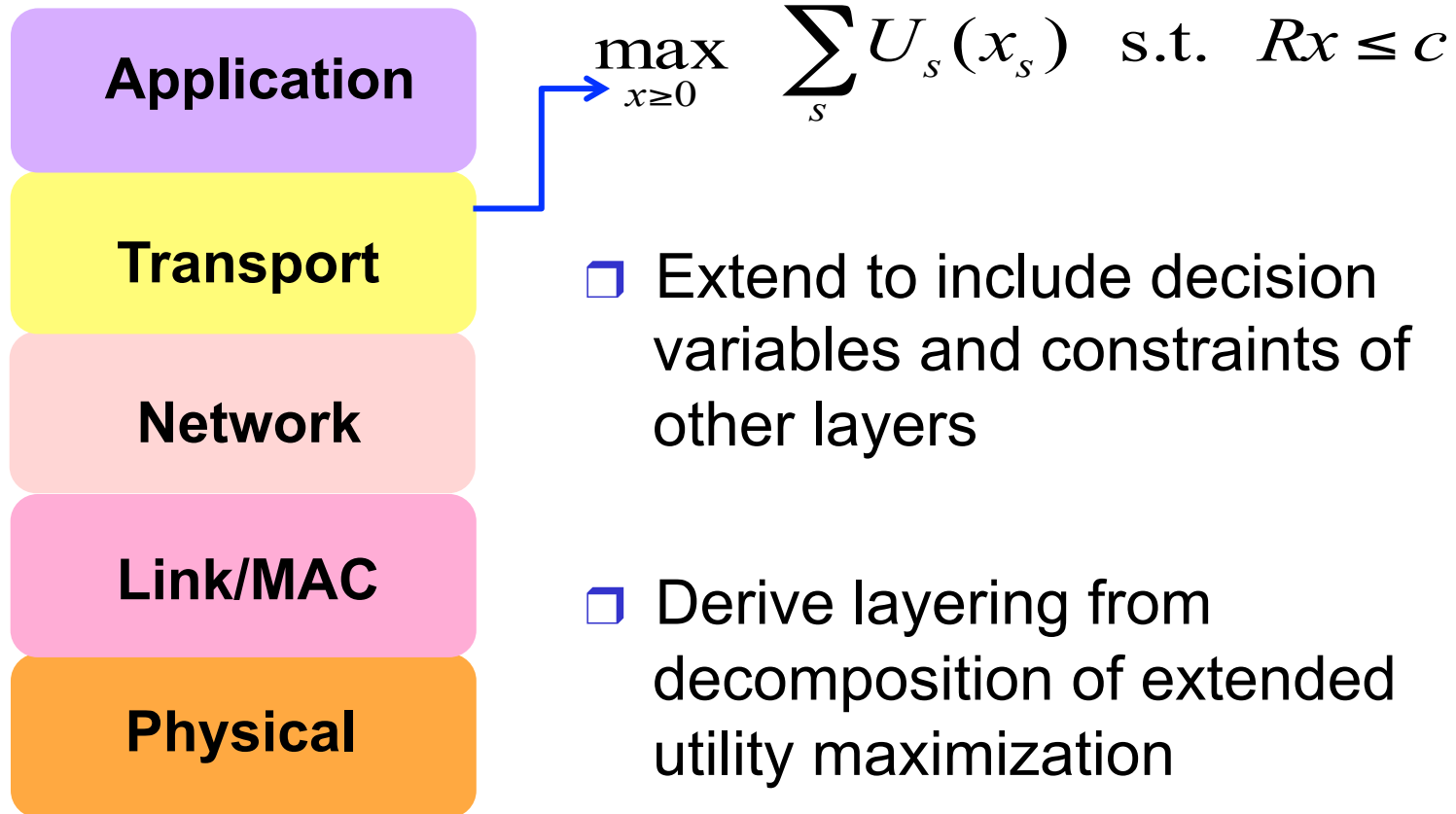
Dual

$$\min_{p \geq 0} \left(\sum_s \max_{x_s \geq 0} \left(U_s(x_s) - x_s \sum_l R_{ls} p_l \right) + \sum_l p_l c_l \right)$$

$$\begin{cases} x_s(t) = U_s'^{-1} \left(\sum_l R_{ls} p_l \right) \\ p_l(t+1) = [p_l(t) + \gamma \left(\sum_s R_{ls} x_s(t) - c_l \right)]^+ \end{cases}$$

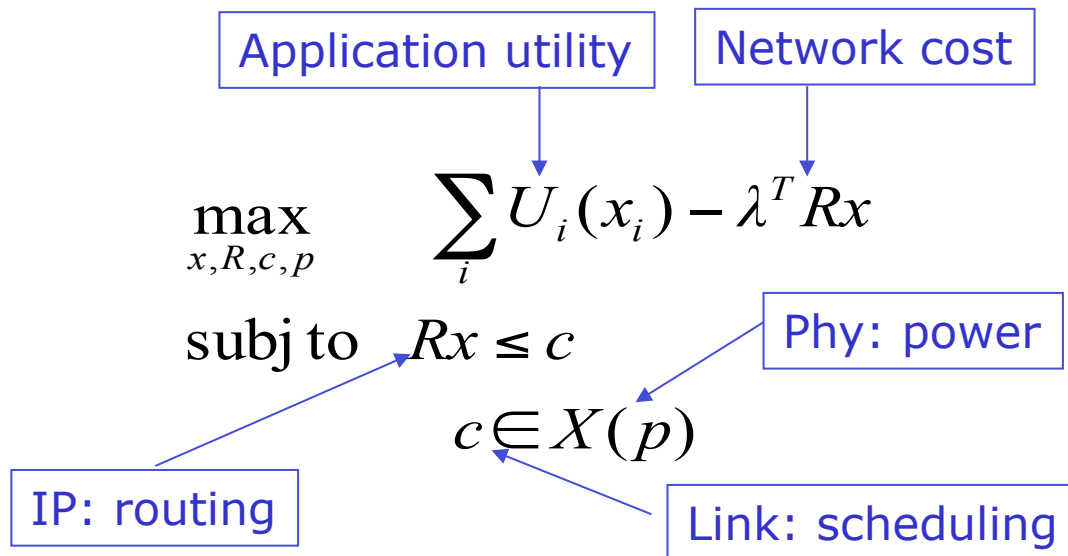
horizontal decomposition

Optimization and layering



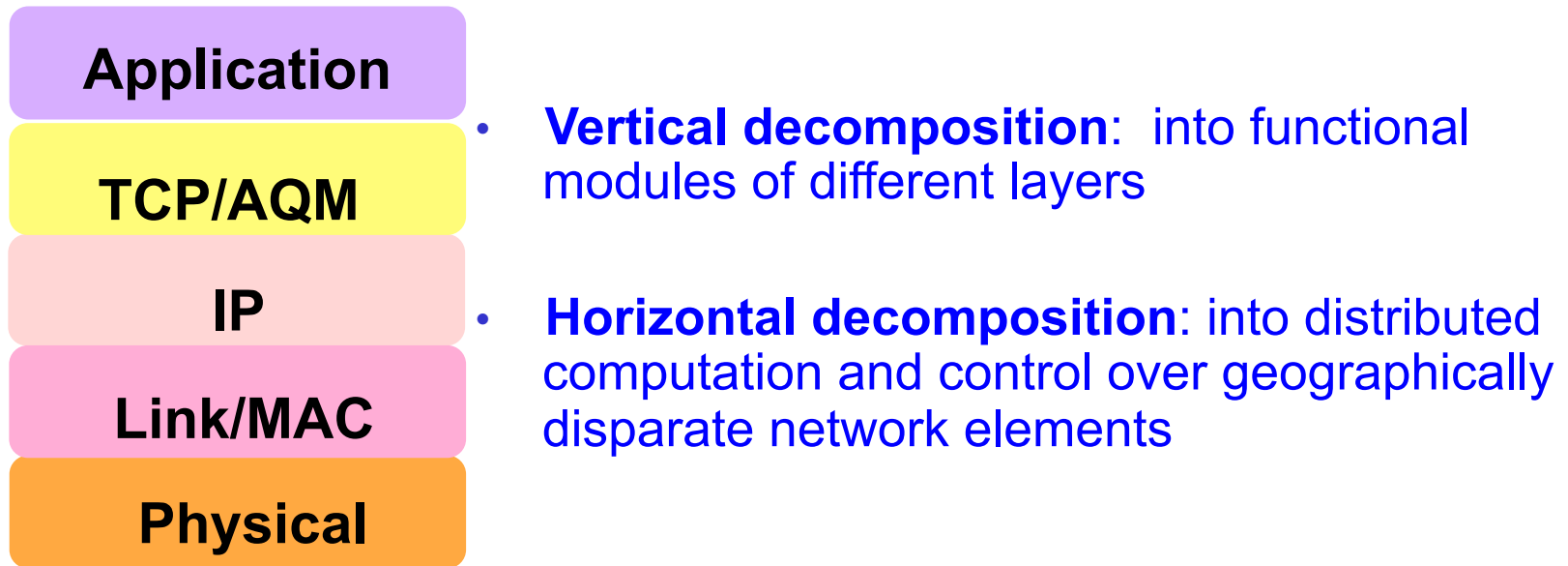
Generalized utility maximization

- ❑ Objective function: user application needs and network cost
- ❑ Constraints: restrictions on resource allocation (could be physical or economic)
- ❑ Variables: Under the control of this design
- ❑ Constants: Beyond the control of this design



Layering as optimization decomposition

- ❑ Network generalized NUM
- ❑ Layers sub-problems
- ❑ Interface functions of primal/dual variables
- ❑ Layering decomposition methods



Layering as optimization decomposition

- ❑ Network generalized NUM
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Application

Provides a top-down approach to design protocol stack

TCP/AQM

- ❑ explicitly tradeoff design objective

IP

- ❑ explicitly model constraints and effects of, e.g., new technologies

Link/MAC

- ❑ provide guidance on how to structure and modularize different functions

Physical

- ❑ make transparent the interactions among different components and their global behaviors

Cross-layer design in ad hoc wireless networks

Application

TCP/AQM

IP

MAC

Physical

- ❑ Network performance can be improved if network layers are jointly designed
- ❑ Most works
 - ❑ design based on intuition, evaluated by simulations
 - ❑ unintended consequences

Cross-layer design in ad hoc wireless networks

Application

A principled/holistic approach

TCP/AQM

- Capture global structure of the problem

IP

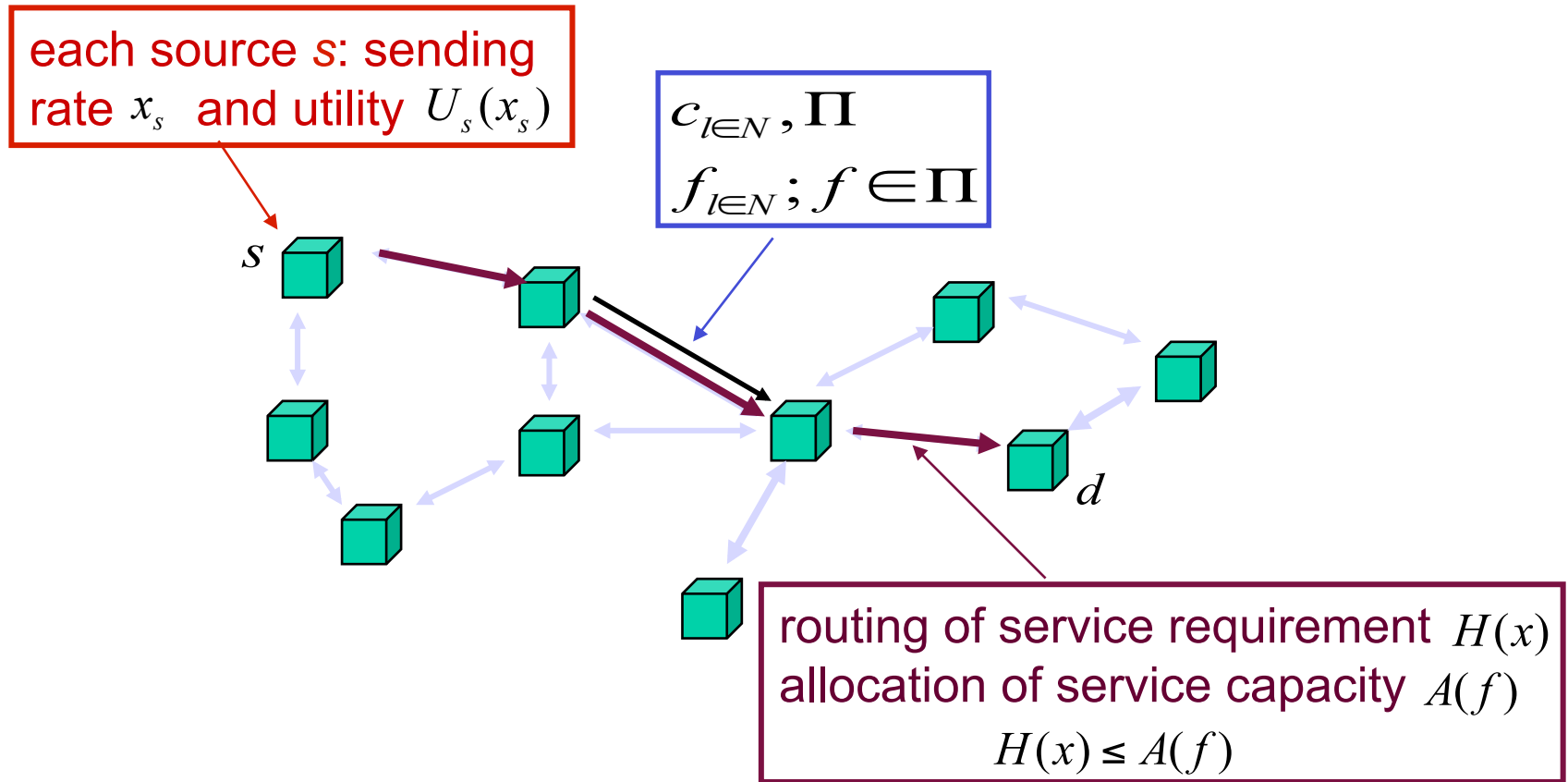
- design objective
- constraints

MAC

- Derive the design from the distributed decomposition of certain optimization problem

Physical

Cross-layer design/optimization



Network model

Problem formulation

Network resource allocation:

$$\begin{aligned} \max_{x, f} \quad & \sum_s U_s(x_s) \\ \text{s.t.} \quad & H(x) \leq A(f) \\ & f \in \Pi \end{aligned}$$

constraint for routing

constraint from wireless interference

Protocol decomposition

rate constraint schedulability constraint

Primal: $\max_{x,f} \sum_s U_s(x_s) \text{ s.t. } H(x) \leq A(f), f \in \Pi$

Dual: $\min_{p \geq 0} \left\{ \max_x \left(\sum_s U_s(x_s) - p^T H(x) \right) + \max_{f \in \Pi} p^T A(f) \right\}$

Rate control Routing Scheduling

The diagram illustrates the decomposition of a network protocol into three main components: Rate control, Routing, and Scheduling. These components are mapped to specific parts of the primal and dual optimization problems. The primal problem is a maximization over flow variables x and f , with a constraint $H(x) \leq A(f)$. The dual problem is a minimization over Lagrange multipliers p , involving two maximization sub-problems. Arrows indicate the following mappings: 'rate constraint' points to $H(x) \leq A(f)$; 'schedulability constraint' points to $f \in \Pi$; 'Rate control' points to the inner maximization over x ; 'Routing' points to the summation over s in the inner maximization; and 'Scheduling' points to the outer maximization over $f \in \Pi$.

Cross-layer implementation

Dual: $\min_{p \geq 0} \{ \max_x (\sum_s U_s(x_s) - p^T H(x)) + \max_{f \in \Pi} p^T A(f) \}$

Rate control
Routing
Scheduling

Application

Transport

□ Rate control:

→ $x(t) = x(p(t)) = \arg \max_x \sum_s U_s(x_s) - p^T(t) H(x)$

Network

□ Routing:

→ solved with rate control or scheduling

Link/MAC

□ Scheduling:

→ $f(t) = f(p(t)) = \arg \max_{f \in \Pi} p^T(t) A(f)$

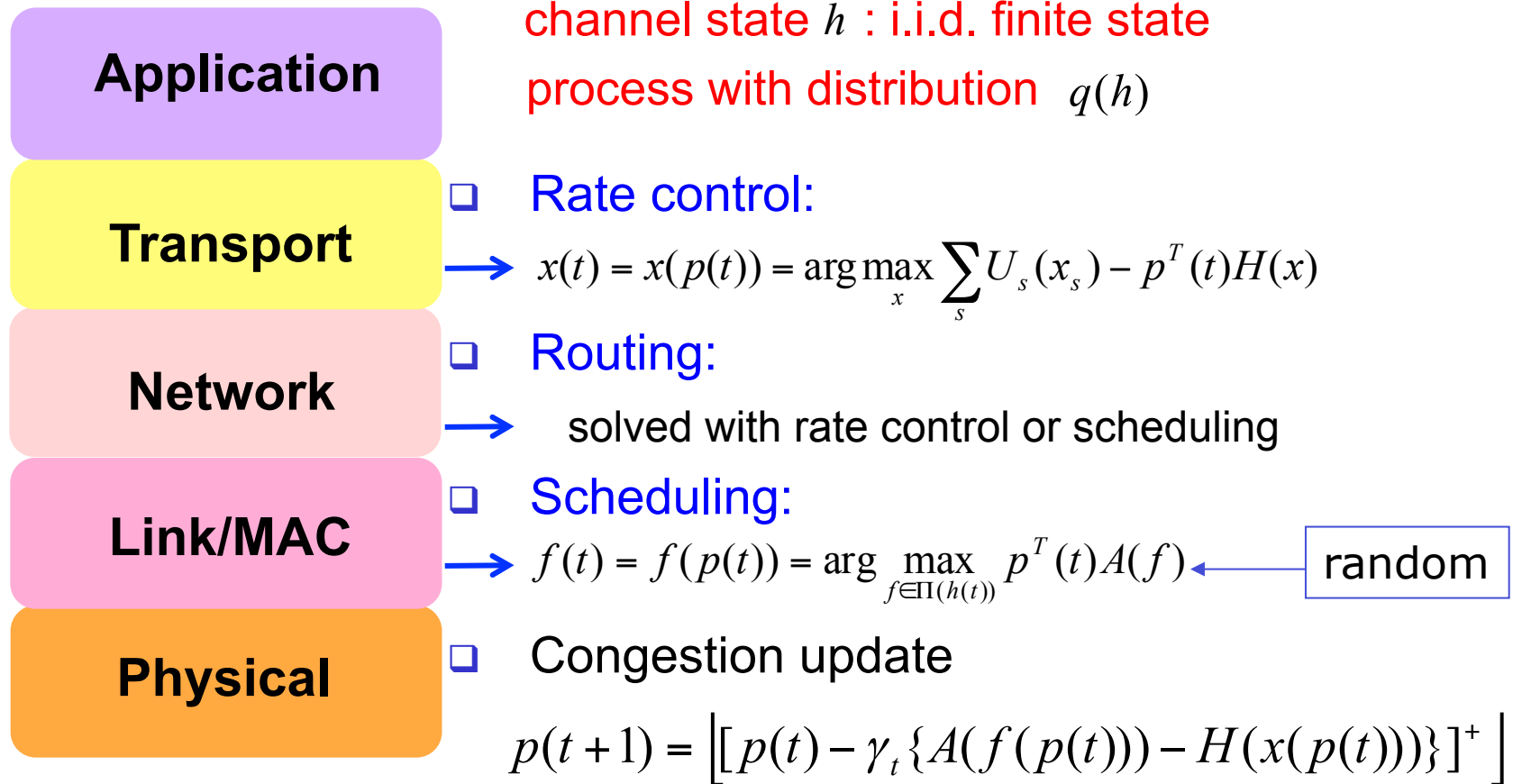
Physical

□ Congestion update:

vertical decomposition

$$p(t+1) = \left[p(t) - \gamma_t \{ A(f(p(t))) - H(x(p(t))) \} \right]^+$$

Extension to time-varying channel



Stability and optimality

Theorem (Chen-Low-Chiang-Doyle '06, '11): The Markov chain is stable. Moreover, the cross-layer algorithm solve the following optimization problem

$$\begin{aligned} \max_{x, f} \quad & \sum_s U_s(x_s) \\ \text{s.t.} \quad & H(x) \leq A(f) \\ & f \in \overline{\Pi} \\ & \overline{\Pi} = \{\bar{r} : \bar{r} = \sum_h q(h)r(h), r(h) \in \Pi(h)\} \end{aligned}$$

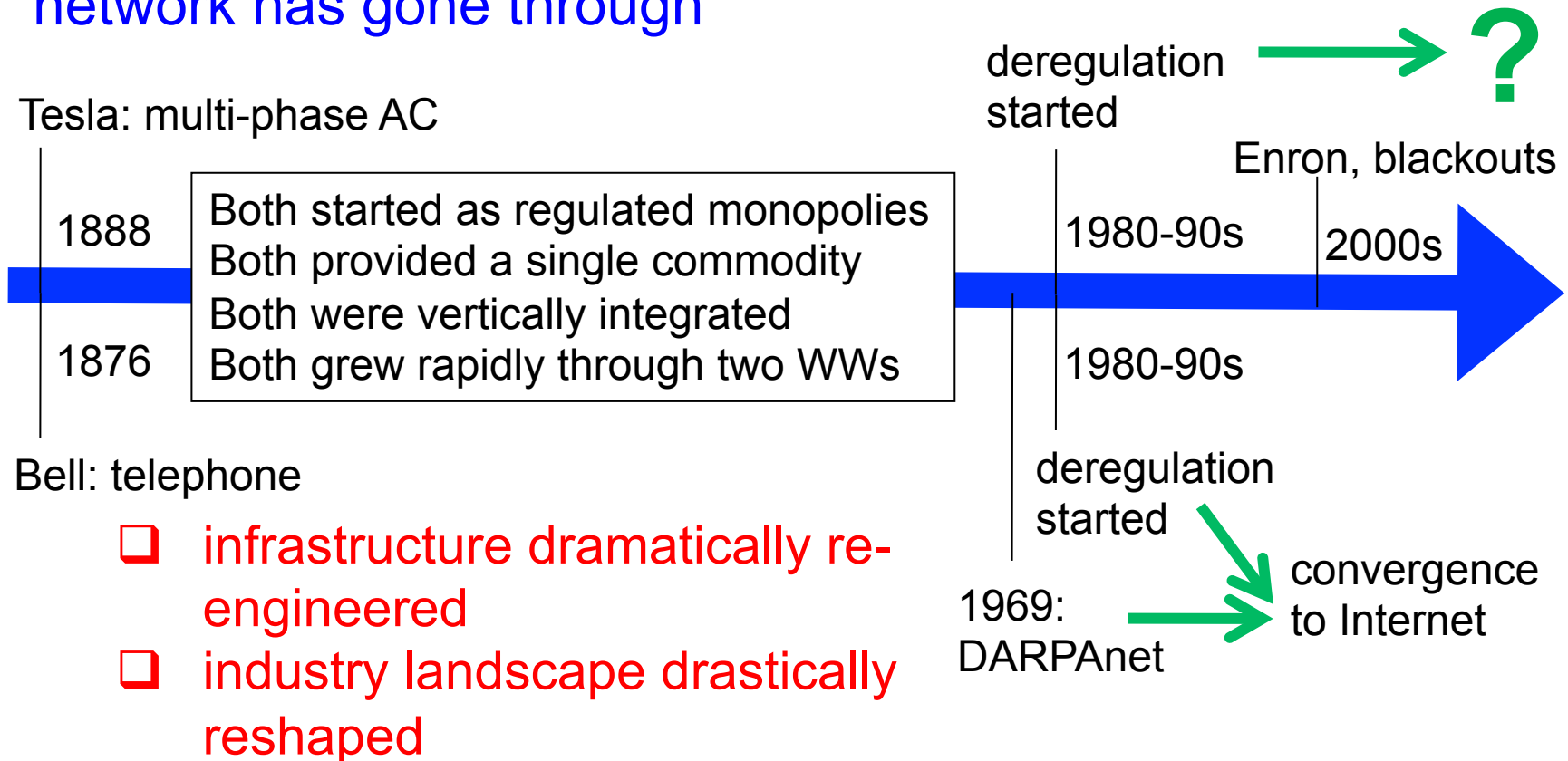
- ❑ Applicable to any queueing network with interdependent, time-varying, parallel servers
 - ❑ optimality holds even with time-varying topologies
 - ❑ throughput-optimal when flow-level dynamics is considered
- ❑ A Wi-Fi implementation by Rhee's group at NCSU shows significantly better performance than existing system

Outline

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- Network dynamics as optimization algorithms
- Look into future

Smart grid

Power network will go through a similar **architectural transformation** in the next few decades that telephone network has gone through



Emerging trends

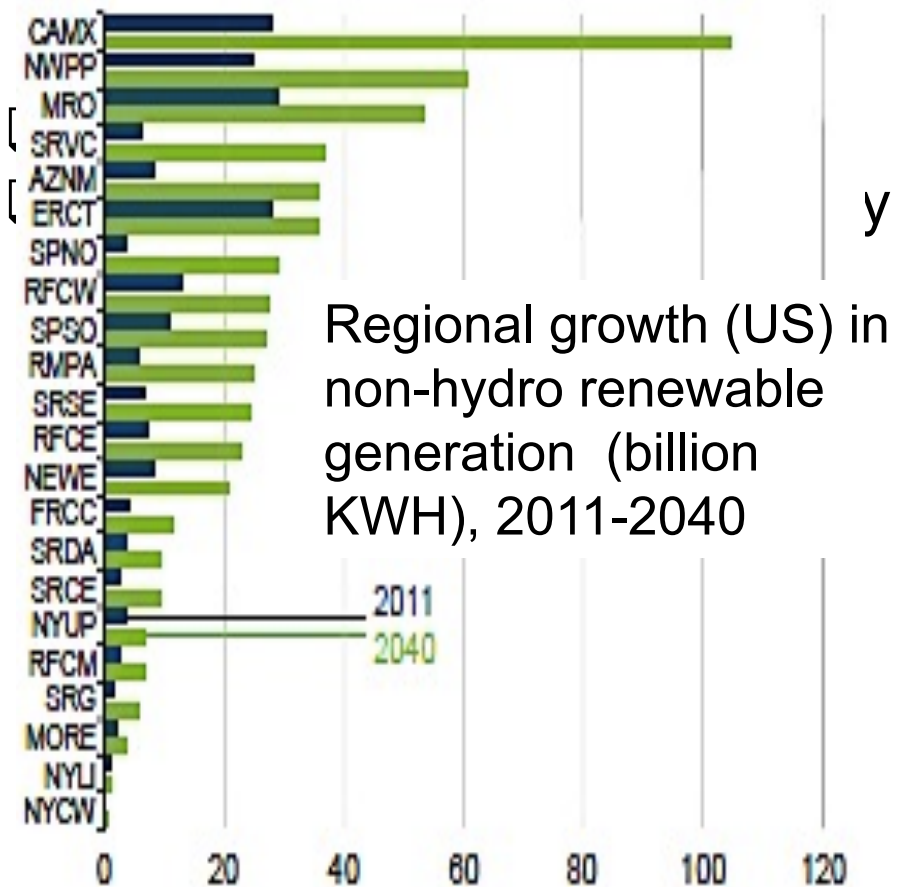
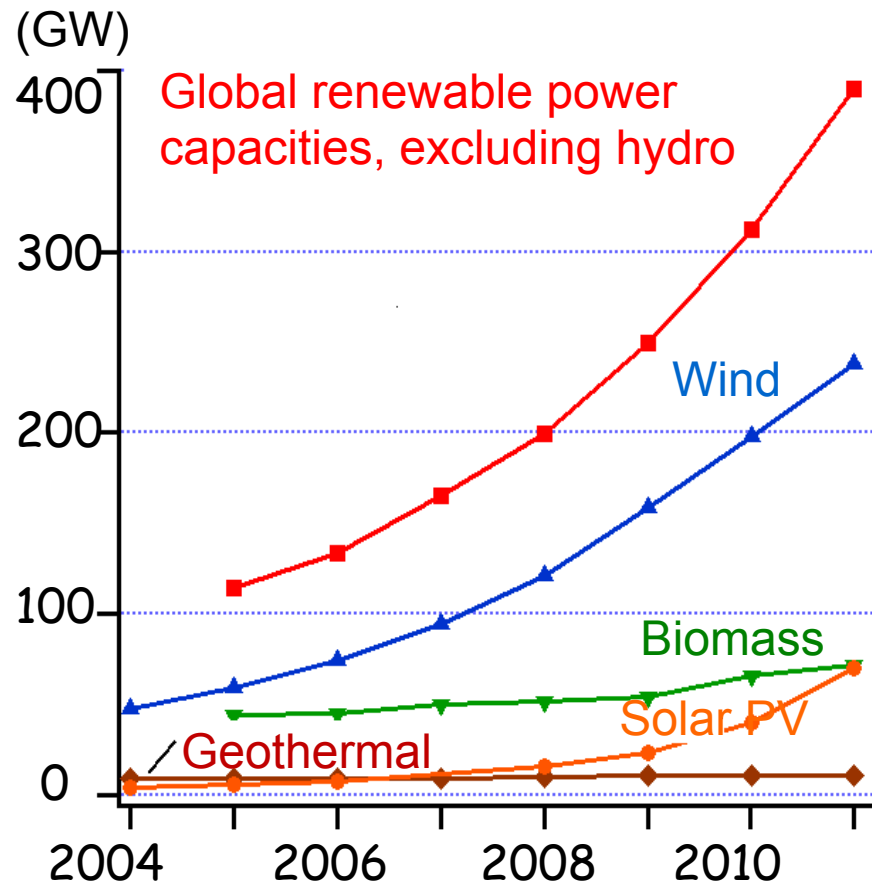
- ❑ Proliferation of renewable and distributed generation
- ❑ Electrification of transportation
- ❑ Participation of end users

} drivers

- ❑ Advances in power electronics
- ❑ Deployment of sensing, communication, computation infrastructure

} enablers

Proliferation of renewables



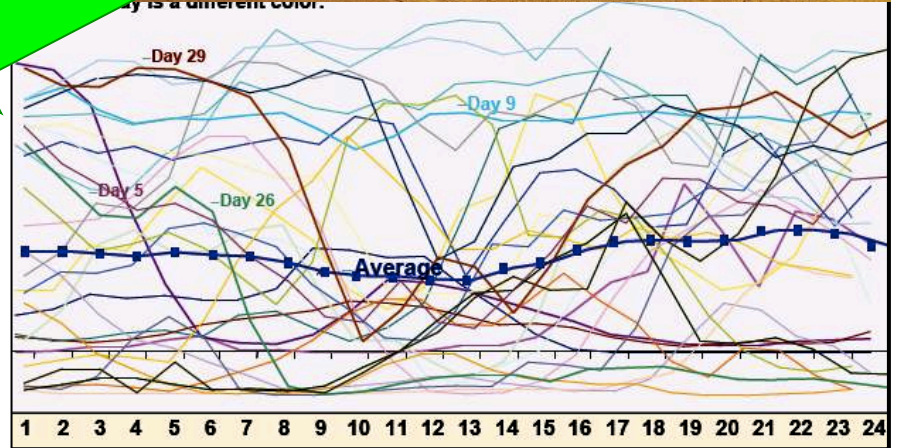
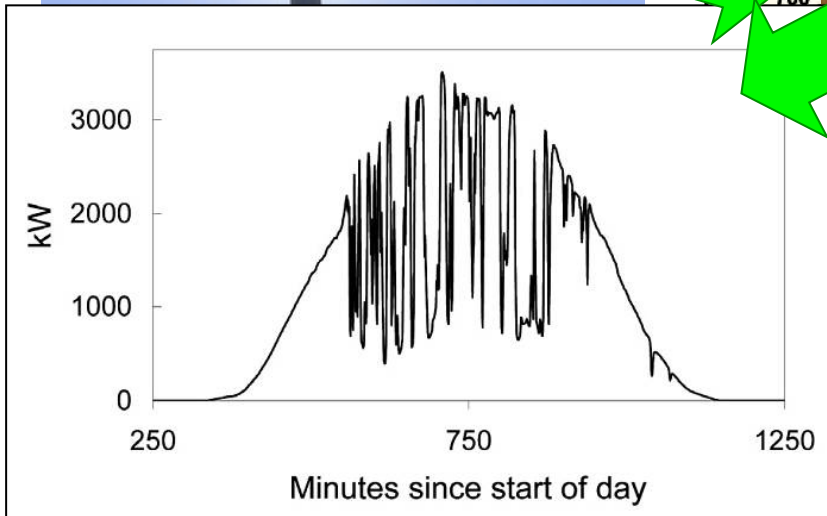
real opportunity for sustainability

Sources: REN21, Renewables global status report (2006-2012); DOE/EIA-0383 (2013)

Random/rapid fluctuations



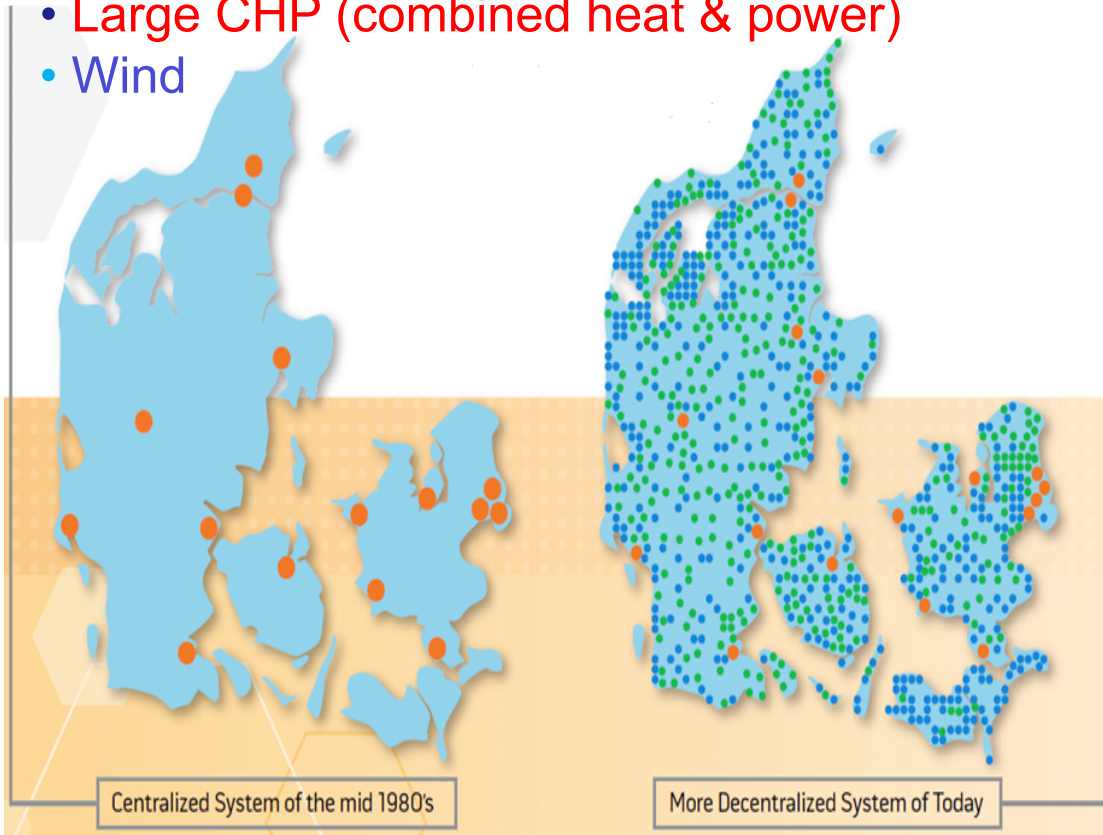
Tehach



Source: Rosa Yang, EPRI

Migration to distributed architecture

- Small CHP (combined heat & power)
- Large CHP (combined heat & power)
- Wind



- ❑ 2-3x generation efficiency
- ❑ relieve demand on grid capacity
- ❑ also in control and management

Denmark's experience

Large-scale network of DERs

Distributed energy resources (DERs): PVs, wind turbines, smart loads, inverters, storages, EVs



millions of **active** endpoints that may
**generate, sense, compute, communicate,
and control**

industrial plant

storage

central power plant

microgrid

management

Source: ITERES

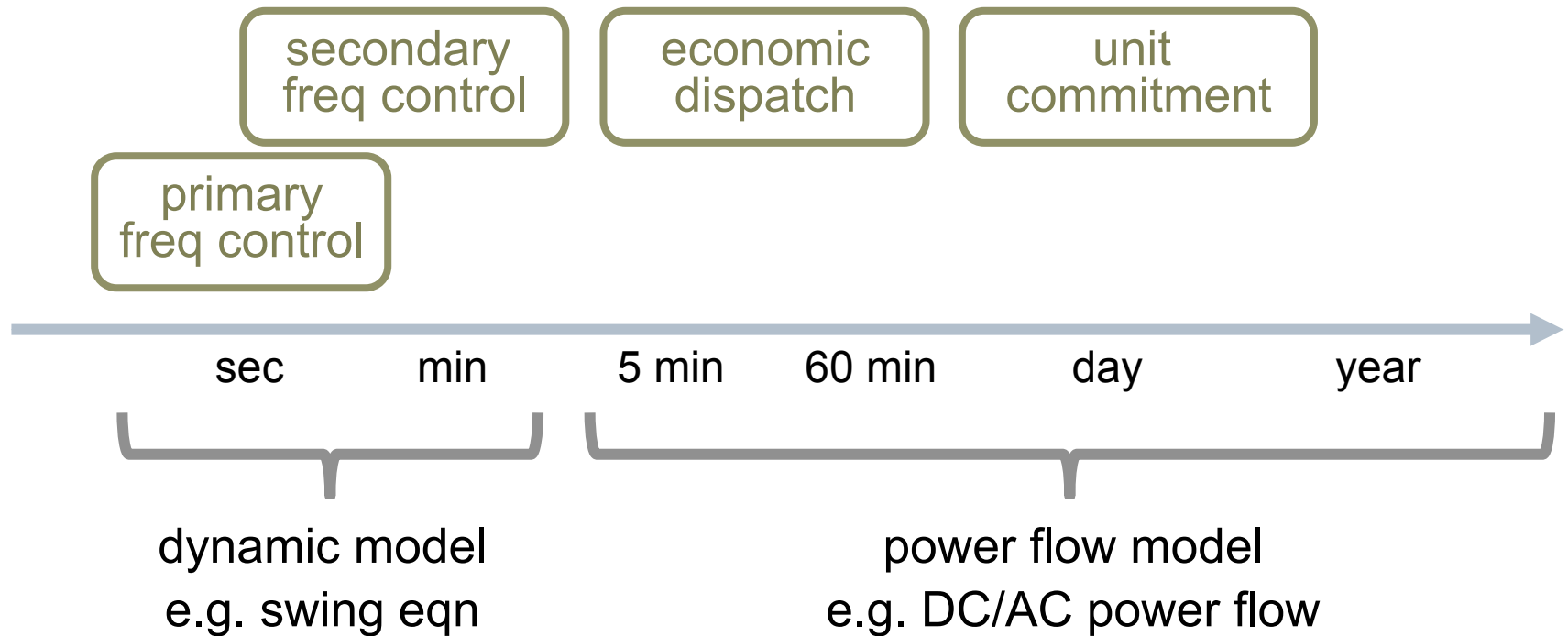
Large-scale network of DERs

- ❑ **Challenge**: an interconnected system of millions of DERs introducing rapid, large, and random fluctuations in supply, demand, voltage, and frequency
- ❑ **Opportunity**: increased capability to coordinate and optimize their operation for unprecedented efficiency and robustness

but how?

Current control paradigm

- Hierarchical control structure spanning multiple timescales from subseconds to hours and up



Frequency control as an example

Current control paradigm

- ❑ Centralized, open-loop, worst-case preventive, and often human-in-the-loop at slow timescales
 - ❑ cope with slow/predictable but often large variations
 - ❑ economic efficiency and system security are the key (optimization model)
- ❑ Local and automatic at fast timescales
 - ❑ cope with fast but relatively small variations
 - ❑ (local) stability is the key (dynamical model)
 - ❑ oblivious of system-wide properties or global perspective
- ❑ Sufficient for today's power system
 - ❑ relatively low uncertainties, few active assets, mainly to match controllable supply to passive load
 - ❑ the lack of ubiquitous sensing, control and communication

Current control paradigm

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- ❑ Sufficient for today's power system
 - ❑ relatively low uncertainties, few active assets, mainly to match controllable supply and passive load
 - ❑ the lack of ubiquitous sensing, control and communication

all are changing

Future control needs

- ❑ Real-time and close-loop
 - ❑ with rapid, large, and random fluctuations, feedback control based on real-time information is needed
- ❑ Distributed to ensure scalability
 - ❑ with large number of control points, information must be decentralized and decisions must be made locally
- ❑ Fast/local controls actively and globally coordinated
 - ❑ local controls must be bridged with the global situation, to ensure system-wide efficiency and robustness
- ❑ Enabled by the deployment of sensing, control, and communication infrastructure and the advances in power electronics

New control paradigm

- ❑ Autonomous DERs for distributed real-time control
 - ❑ Each DER made autonomous through local sensing, computing, communication, and control
 - ❑ intelligence embedded everywhere
- ❑ Local algorithms with global perspective
 - ❑ algorithm design starts with global objectives, which will be decomposed into local algorithms
- ❑ Layered architecture
 - ❑ control as a service: the network should provide a set of common control services to various applications
 - ❑ applications call and synthesize the control services to meet performance specifications

New control paradigm

- ❑ Autonomous DERs for distributed real-time control
- ❑ Local algorithms with global perspective
- ❑ Layered architecture

What are fundamental challenges motivated?

Comparison with the Internet

- ❑ Motivated by the Internet

- ❑ power flow versus information flow
- ❑ how it is managed and controlled
- ❑ theories for architecture and protocol design

the precedence on the Internet lends hope to a
much bigger scale and more dynamic and
distributed control architecture

- ❑ The physics of electricity cuts through all power system functionalities and operation

- ❑ nonconvexity of power flow
- ❑ dynamics cannot be “designed”

Fundamental Challenges

Convexification of power flow

- ☐ for fast computation for real-time optimization
- ☐ for distributed algorithm

Distributed decomposition under dynamics constraints

- ☐ for distributed real-time control with global perspective
- ☐ exploit or implemented as power system dynamics

Integrating sensing, communication, and control

- ☐ fundamental limits on control performance arising from sensing constraints and communication constraints
- ☐ communication/networking for distributed control

Architecture and layering

- ☐ mathematical underpinning of smart grid architecture
- ☐ systematic methods to develop/evaluate design choices

Fundamental Challenges

Convexification of power flow

- ☐ convex relaxation

Distributed decomposition under dynamics constraints

- ☐ reverse and forward engineering
- ☐ network dynamics as optimization algorithms

Integrating sensing, communication, and control

- ☐ fundamental limits on control performance arising from sensing constraints and communication constraints
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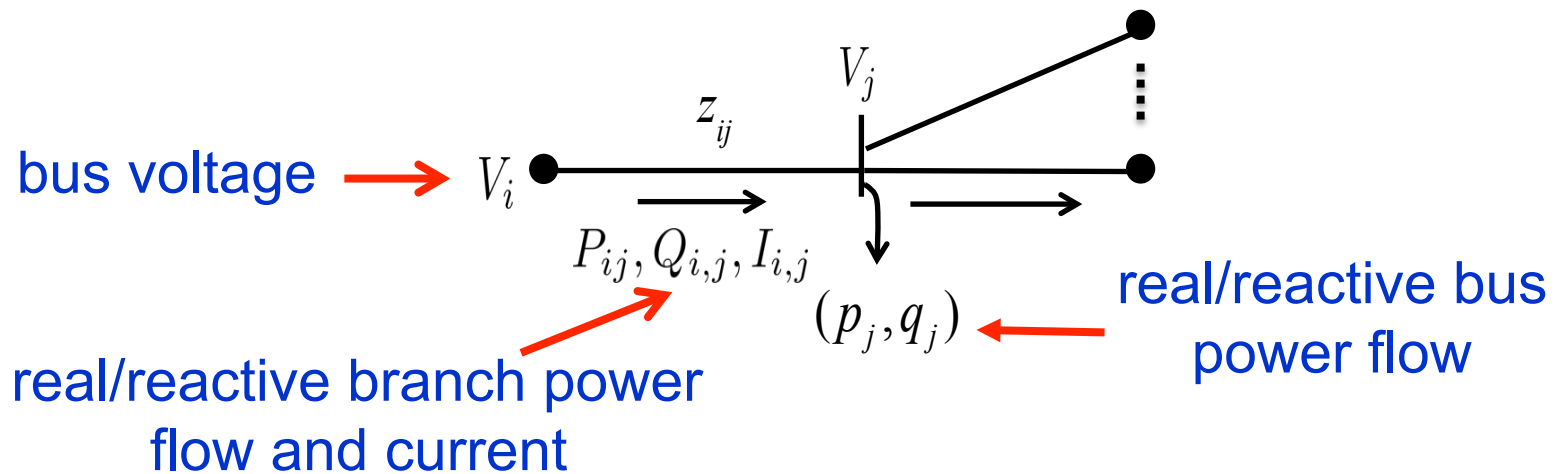
Architecture and layering

- ☐ mathematical underpinning of smart grid architecture
- ☐ systematic methods to develop/evaluate design choices

Convexification of OPF

- ❑ Optimal power flow (OPF) problem
 - ❑ a fundamental problem underlying power system controls and operations
 - ❑ huge literature since first formulated in 1962, focusing on approximate algorithms and solutions
- ❑ Convexity critical to the development of efficient, distributed, and robust algorithms
 - ❑ for **real-time** computation **at scale**
 - ❑ for **distributed** algorithms
 - ❑ for efficient market, as **foundation** for pricing schemes such as LMP
 - ❑ for **global optimality**, critical for new/enhanced application

Branch flow model



Power flow constraints

$$V_i - V_j = z_{ij} I_{ij}$$

Kirchhoff law

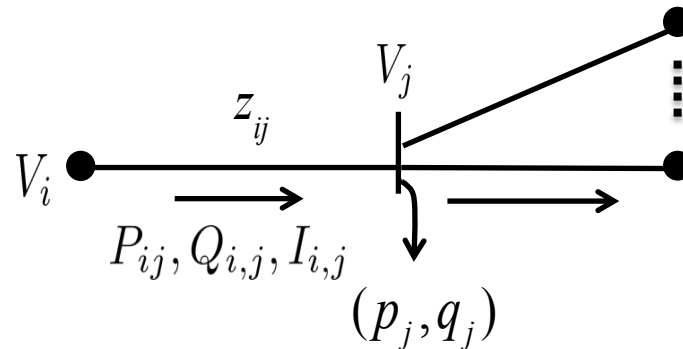
$$S_{ij} = V_i I_{ij}^*$$

power definition

$$\sum_{i \rightarrow j} \left(S_{ij} - z_{ij} |I_{ij}|^2 \right) - \sum_{j \rightarrow k} S_{jk} = s_j$$

power balance

Optimal power flow



OPF:

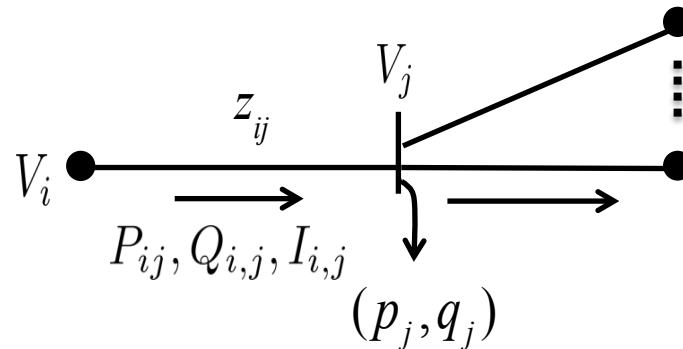
$$\max \sum_i U_i(p_i) - C\left(\sum_{(0,j)} P_{0j}\right) - r_{i,j} |I_{i,j}|^2 \quad \leftarrow \text{social welfare}$$

over $x := (P, Q, I, V, p, q)$

s. t. $x \in PFC(x)$ \leftarrow power flow constraints

$(v_i, p_i, q_i) \in OC(x)$ \leftarrow operation constraints,
e.g., in safe range

Convexity structure



OPF:

$$\max \sum_i U_i(p_i) - C\left(\sum_{(0,j)} P_{0j}\right) - r_{i,j} |I_{i,j}|^2$$

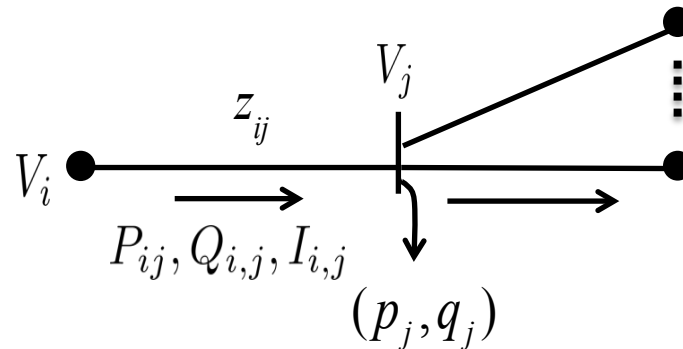
over $x := (P, Q, I, V, p, q)$

s. t. $x \in PFC(x)$

$(v_i, p_i, q_i) \in OC(x)$

Convex ?

Convexity structure



OPF:

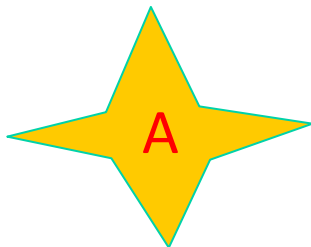
$$\max \sum_i U_i(p_i) - C\left(\sum_{(0,j)} P_{0j}\right) - r_{i,j} |I_{i,j}|^2$$

over $x := (P, Q, I, V, p, q)$

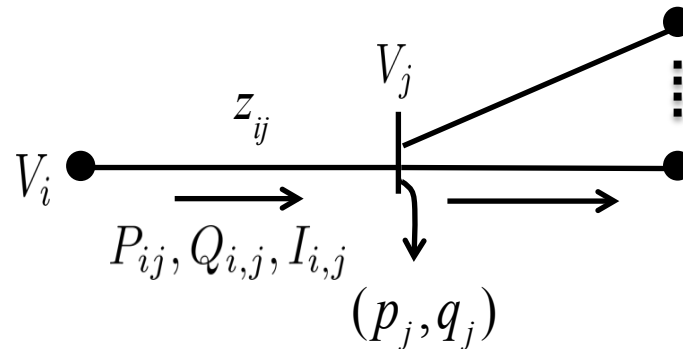
s. t. $x \in PFC(x)$ **nonconvex**

$$(v_i, p_i, q_i) \in OC(x)$$

Convex ?



Convexity relaxation



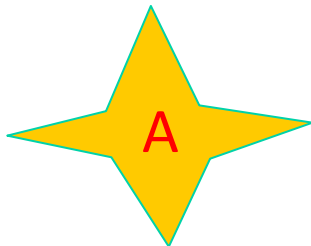
OPF:

$$\max \sum_i U_i(p_i) - C\left(\sum_{(0,j)} P_{0j}\right) - r_{i,j} |I_{i,j}|^2$$

over $x := (P, Q, I, V, p, q)$

s. t. $x \in PFC(x)$

$(v_i, p_i, q_i) \in OC(x)$



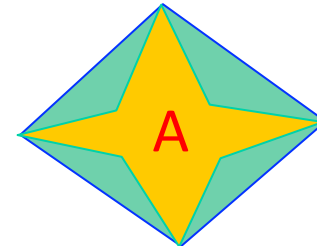
ROPF:

$$\max \sum_i U_i(p_i) - C\left(\sum_{(0,j)} P_{0j}\right) - r_{i,j} |I_{i,j}|^2$$

over $x := (P, Q, I, V, p, q)$

s. t. $x \in RPFC(x)$

$(v_i, p_i, q_i) \in OC(x)$




SOCP
relaxation

Exact relaxation

Theorem (Li-Chen-Low '12a): Convex relaxation is exact provided that for any i , $v_i^{\text{lin}}(p, q) < \bar{v}_i$ and for any link (k, l) in the network and (i, j) on the path from 0 to k ,

$$\text{If } \frac{r_{k,l}}{x_{k,l}} - \frac{X_k}{R_k} > 0, \text{ then } v_i + 2\underline{P}_i^{\text{lin}} \left(\frac{r_{k,l}}{x_{k,l}} X_k - R_{i,k} \right) + 2\underline{Q}_i^{\text{lin}} X_i > 0$$

$$\text{Otherwise, } v_i + 2\underline{Q}_i^{\text{lin}} \left(\frac{x_{k,l}}{r_{k,l}} R_k - X_{i,k} \right) + 2\underline{P}_i^{\text{lin}} R_i > 0$$

- ❑ if only load buses, relaxation is always exact
- ❑ relaxation is always exact for real systems where

$$v \sim 1, r, x \ll 1, P, Q < 1$$

- ❑ IEEE distribution test systems
 - ❑ Southern California Edison circuits
- ❑ many decomposition approaches (thus distributed algorithms) apply (Li-Chen-Low '12b, '12c)

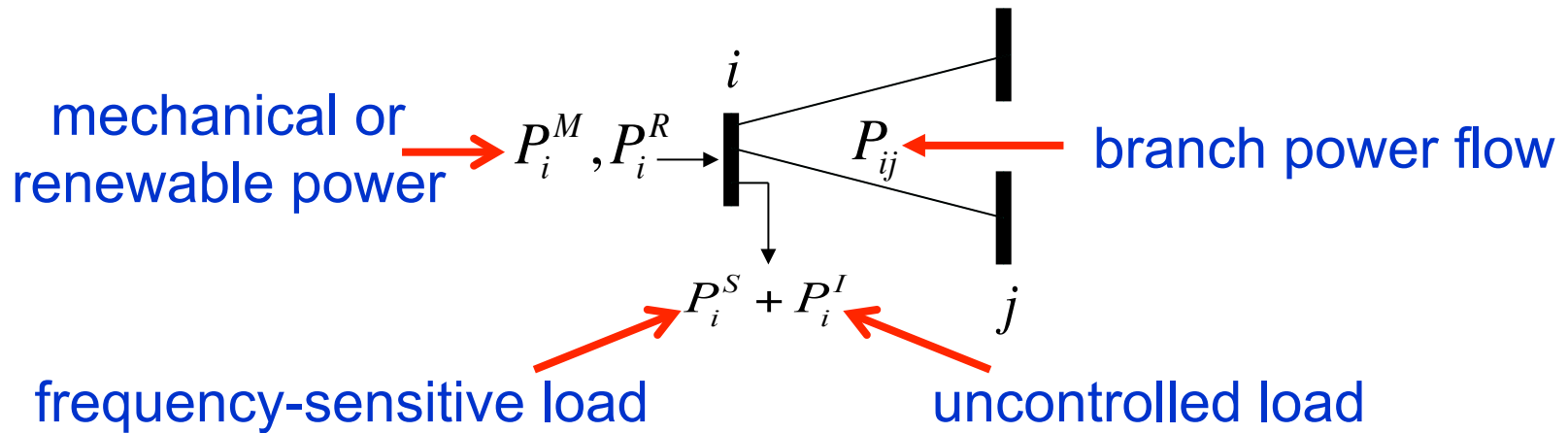
Convexification of OPF

- ❑ Exact convex relaxation
 - ❑ tremendous progress since Lavaei and Low '11; see survey Low '14
 - ❑ effectiveness depends on graph properties of underlying physical and/or communication networks
 - ❑ not always possible, and conditions may violate operation constraints
- ❑ Convex approximation?
 - ❑ geometry of power flow and its dependence on operation constraints and graph properties
 - ❑ systematic approach to construct convex approximation, to trade off tractability and optimality

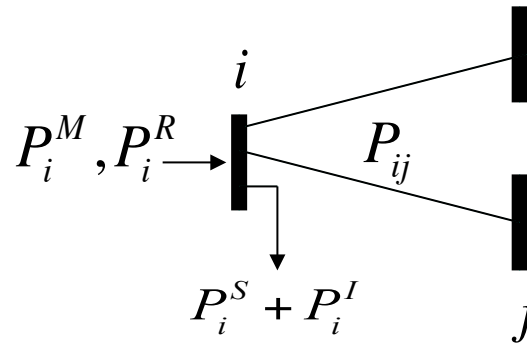
Distributed decomposition under dynamics constraints

- ❑ Power network is a physical system
 - ❑ cannot be “re-set” arbitrarily, but has to evolve from one state to another
 - ❑ algorithms must be “consistent” with system dynamics
- ❑ Reverse engineering
 - ❑ what can we learning from current system?
 - ❑ can we bridge existing local control with system-wide property?
- ❑ Forward engineering
 - ❑ **engineer the model** from reverse engineering to guide systematic design of new algorithms

Frequency control



Frequency control



focus on primary
control for insight

- Synchronous generator: $P_i^M = F_i(\omega_i)$
 - decreasing; e.g., $F_i(\omega_i) = -S_i\omega_i$
- Renewable generator: $P_i^R = H_i(\omega_i)$
 - decreasing
- Frequency sensitive load: $P_i^S = G_i(\omega_i)$
 - increasing; e.g., $G_i(\omega_i) = D_i\omega_i$

Dynamics

Synchronous generator bus:

$$M_i \dot{\omega}_i = F_i(\omega_i) - G_i(\omega_i) - P_i^I - \sum_{j:i \sim j} P_{ij}$$

Renewable generator bus:

$$0 = H_i(\omega_i) - G_i(\omega_i) - P_i^I - \sum_{j:i \sim j} P_{ij}$$

Load bus (no generator):

$$0 = G_i(\omega_i) + P_i^I + \sum_{j:i \sim j} P_{ij}$$

Real branch power flow:

$$\dot{P}_{ij} = b_{ij} (\omega_i - \omega_j)$$

system state

$(\omega(t), P(t))$

Cost/disutility functions

Control functions defines relations between equilibrium frequency and equilibrium generation and load

□ synchronous generator: $C_i^M(P_i^M) = -\int_0^{P_i^M} F_i^{-1}(P) dP$

□ renewable generator: $C_i^P(P_i^R) = -\int_0^{P_i^M} H_i^{-1}(P) dP$

□ frequency sensitive load: $C_i^S(P_i^S) = \int_0^{P_i^M} G_i^{-1}(P) dP$

The equivalence of control and decision problem

$$P_i^M = \arg \min_P C_i^M(P) + P\omega_i$$

depend only on the control function but is independent of how the feedback signal is updated

Reverse engineering

Theorem (You-Chen '14a): Power system dynamics is a distributed primal-dual gradient algorithm to solve

$$\begin{aligned} \min \quad & \sum_{i \in N^M} C_i^M(P_i^M) + \sum_{i \in N^R} C_i^R(P_i^R) + \sum_{i \in N} C_i^S(P_i^S) \\ \text{s.t.} \quad & P_i^S + P_i^I + \sum_j P_{ij} = P_i^M, \quad i \in N^M \\ & P_i^S + P_i^I + \sum_j P_{ij} = P_i^R, \quad i \in N^R \\ & P_i^S + P_i^I + \sum_j P_{ij} = 0, \quad i \in N^L \end{aligned} \quad \begin{array}{l} \text{DC OPF} \\ \text{problem} \end{array}$$

and the dual variables are frequencies and equal.

network dynamics as optimization algorithms

Network dynamics as optimization algorithms

- ❑ A new perspective to understand collective behavior arising from interaction between local controls
 - ❑ structural properties of the equilibrium point
 - ❑ efficiency and tradeoffs, etc
- ❑ Suggests a Lyapunov function for global stability or convergence analysis
 - ❑ important both theoretically and practically
- ❑ Suggests a principled way to systematically design new algorithms and control schemes

Forward engineering

Suggests a principled way to systematically design new algorithms and control schemes

- ❑ new design goals (e.g., frequency recovery, fairness, and economic efficiency) incorporated by engineering the global objective function and the constraints
- ❑ new control schemes with different dynamical properties based on various optimization algorithms
- ❑ insights from reverse engineering can guide particular way to engineer the model and derive the algorithm

Nominal frequency recovery

- Key observation: $\omega = 0$ can be ensured if $\sum_{i \in N} G_i(\omega) = 0$ at equilibrium



$$\sum_{i \in N} P_i^I = \sum_{i \in N^M} P_i^M + \sum_{i \in N^R} P_i^R$$

do not have the decoupling structure

- Impose the above indirectly by imposing decoupling constraints

$$P_i^I + \sum_j Q_{ij} = P_i^M, \quad i \in N^M$$

$$P_i^I + \sum_j Q_{ij} = P_i^R, \quad i \in N^R$$

$$P_i^I + \sum_j Q_{ij} = 0, \quad i \in N^L$$

$$Q_{ij} = Q_{ji}$$

Nominal frequency recovery

A new optimization problem (You-Chen '14a):

$$\max \sum_{i \in N^M} C_i^M(P_i^M) + \sum_{i \in N^R} C_i^R(P_i^R) + \sum_{i \in N} C_i^S(P_i^S)$$

$$s.t. \quad P_i^S + P_i^I + \sum_j P_{ij} = P_i^M, \quad i \in N^M$$

$$P_i^S + P_i^I + \sum_j P_{ij} = P_i^R, \quad i \in N^R$$

$$P_i^S + P_i^I + \sum_j P_{ij} = 0, \quad i \in N^L$$

all are
physical
variables

$$P_i^I + \sum_j Q_{ij} = P_i^M, \quad i \in N^M$$

$$P_i^I + \sum_j Q_{ij} = P_i^R, \quad i \in N^R$$

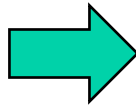
$$P_i^I + \sum_j Q_{ij} = 0, \quad i \in N^L$$

Q is not
physical

Nominal frequency recovery

New control scheme (You-Chen '14a):

$$\begin{aligned} P_i^M &= F_i(\omega_i), i \in N^M \\ P_i^R &= H_i(\omega_i), i \in N^R \end{aligned}$$



$$P_i^M = F_i(2\omega_i - \nu_i), i \in N^M$$

$$P_i^R = H_i(\omega_i + \mu_i), i \in N^R$$

$$\dot{\nu}_i = -(G_i(\omega_i) + \sum_j (P_{ij} - Q_{ij})) / M_i, i \in N^M$$

$$\dot{\mu}_i = \xi_i (G_i(\omega_i) + \sum_j (P_{ij} - Q_{ij})), i \in N^R \cup N^L$$

$$\dot{Q}_{ij} = \varepsilon_{ij} (\mu_i - \mu_j)$$

distributed control

Economic dispatch

Theorem (You-Chen '14a): Power system dynamics with the new control scheme solves economic dispatch problem

$$\begin{aligned} \min \quad & \sum_{i \in N^M} C_i^M(P_i^M) + \sum_{i \in N^R} C_i^R(P_i^R) + \sum_{i \in N} C_i^S(P_i^S) \\ \text{s.t.} \quad & P_i^I + \sum_j P_{ij} = P_i^M, \quad i \in N^M \\ & P_i^I + \sum_j P_{ij} = P_i^R, \quad i \in N^R \\ & P_i^I + \sum_j P_{ij} = 0, \quad i \in N^L \end{aligned}$$

- ❑ real-time frequency control recovering frequency and achieving economic efficiency at the same time
- ❑ different from current approach achieving these objectives at different timescales and with centralized control
- ❑ needed for future smart grid to cope with rapid/large fluctuations and manage a huge number of control points

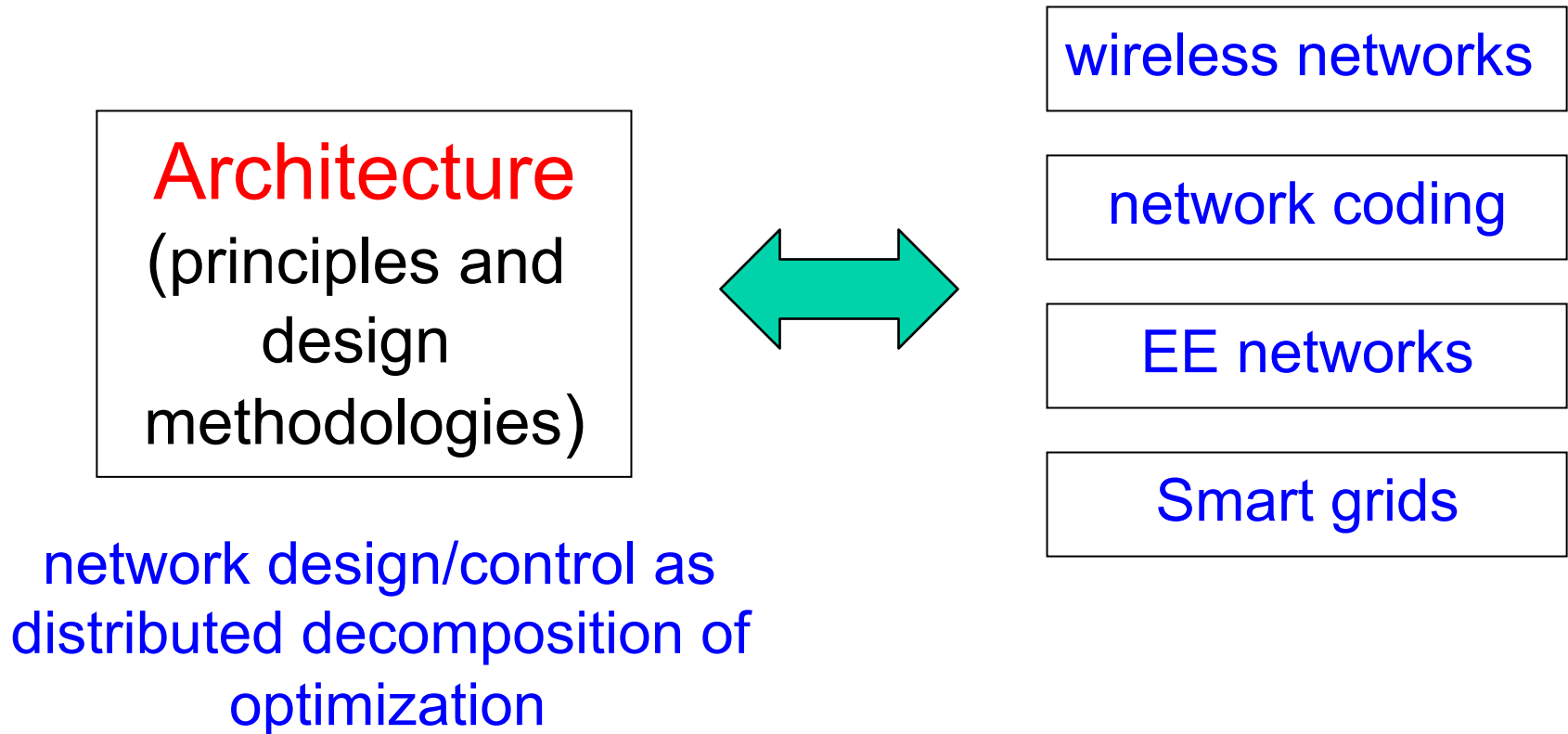
Network dynamics as optimization algorithms

- ❑ Natural system dynamics exploited for simplicity, scalability, and robustness
 - ❑ desired for distributed real-time control
- ❑ Lots of progress
 - ❑ automatic generation control (Li-Chen-Zhao-Low '14); local volt/var control (Farivar-Chen-Low '13); load side frequency control (Zhao *et al* '12, '14, Mallada *et al* '14); distributed frequency control in microgrids (Dorfler *et al* '14)
- ❑ More work needed
 - ❑ remove approximations
 - ❑ integrate frequency and voltage control
 - ❑ distributed decomposition of full-fledged AC OPF problem

Outline

- ❑ Layering and constrained optimization
- ❑ Network dynamics as optimization algorithms
- ❑ Look into the future

Theory-based network design



Theory-based network design

network design/control as distributed
decomposition of optimization

The most important feature

- ❑ not the specific algorithms proposed and analyzed
- ❑ but that we can derive
 - the layering structure and modularity of the various mechanisms
 - the interfaces between these mechanisms
 - the control/signaling information that must cross these interfaces to achieve a certain performance and robustness

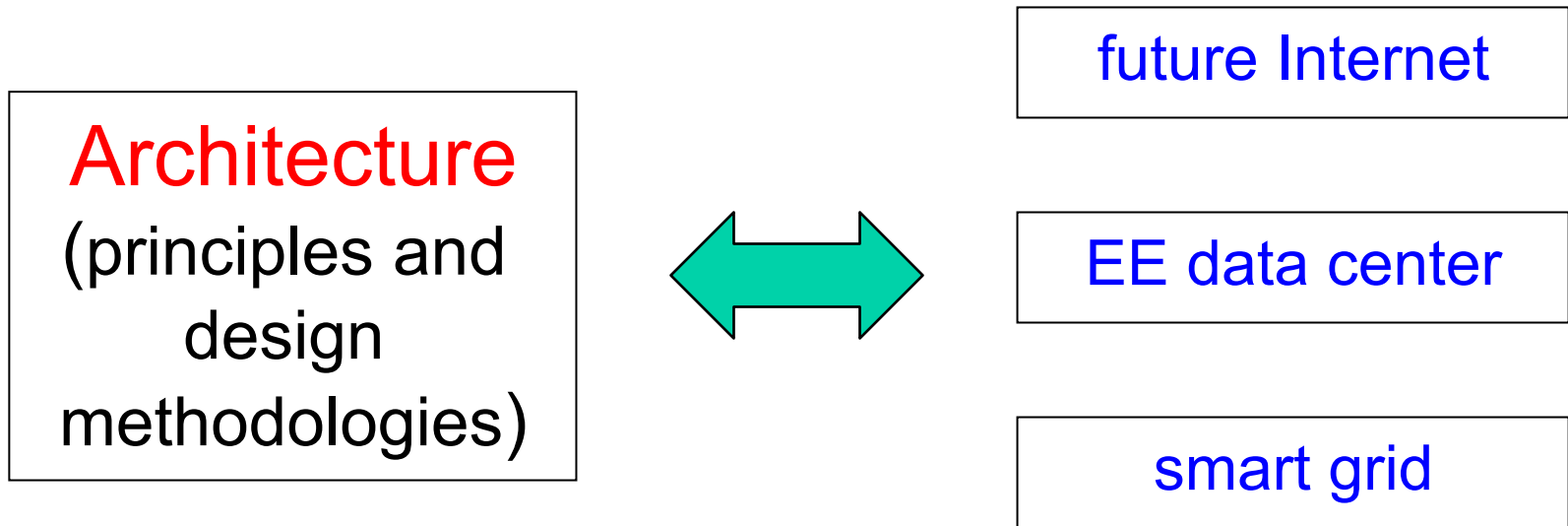
Foundations for a theory of architecture

Architecture
(principles and
design
methodologies)

network design/control as
distributed decomposition of
optimization and game

- A common analytical framework and language
 - handle and integrate computation, communication, control, and incentives
 - allow rigorous analysis and systematic design
- Two of the key components
 - explore architectural implication of complexities of network engineering
 - network compatible mechanism design

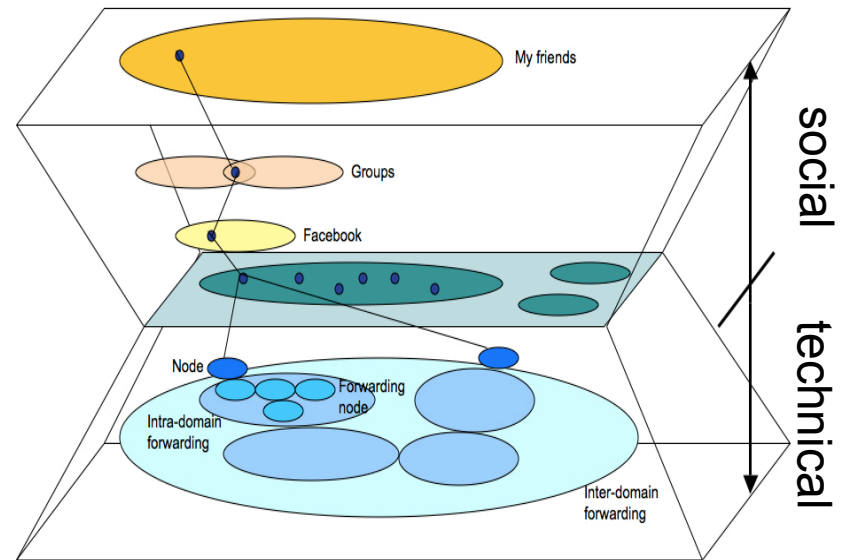
Approach



- ❑ must be both **abstract** and **concrete**
- ❑ must be both **foundational** and **practical**

Lower sub-architecture design

- ❑ A clean slate for the technology infrastructure
 - ❑ content (re)placement, content discovery, cache selection
 - ❑ multipath multicast routing
 - ❑ congestion control
 - ❑ inter-scope/domain issues
- ❑ Apply theory-based approach
 - ❑ how to layer various components (optimization)
 - ❑ inter-scope/domain design (game)



information-centric networks

Zegura, Wroclawski, *et al*, 2010

Energy-efficient data center

- ❑ A new branch of research with its own rich structures and unique challenges
- ❑ Aim to develop models and theories to guide the analysis and design of practical algorithms for energy efficient data centers
 - ❑ have taken initial step on some of these issues (Chen-Li-Low '10, Chen-Li '13, Chen-Andrew-Wierman '14)

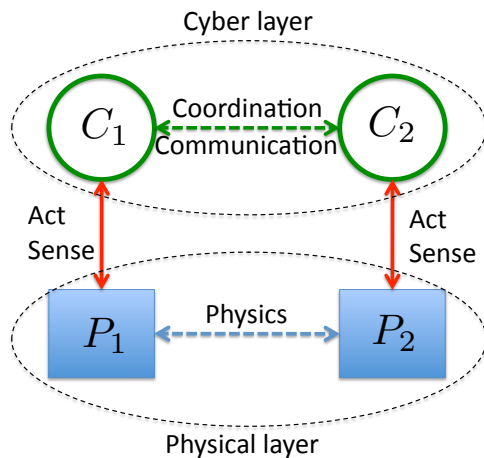


3-5% of total US energy use

Smart grid



- ❑ Nonconvexity of power flows
 - ❑ convex relaxation
 - ❑ convex approximation
- ❑ Network dynamics as optimization algorithms
 - ❑ reverse and forward engineering
 - ❑ distributed decomposition of full fledged AC OPF
- ❑ Integrating sensing, communication, control
 - ❑ fundamental limits on control performance under sensing and communication constraints (You-Chen '14b, Shihadeh-You-Chen '14)
 - ❑ communication/networking for distributed control

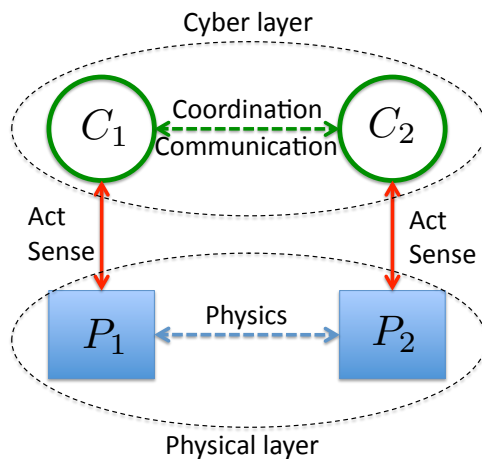


Smart grid



□ Architecture and layering

- mathematical underpinning of smart grid architecture
- systematic methods to develop and evaluate design choices and algorithms
- overarching framework to reason about architectural questions
 - design goals: to allow distributed management, to support different types of energy services, to allow plug-n-play microgrids, ...
 - design principles: layering, division of functionality, placement of intelligence, ...



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