Connections II

Fundamentals of network science

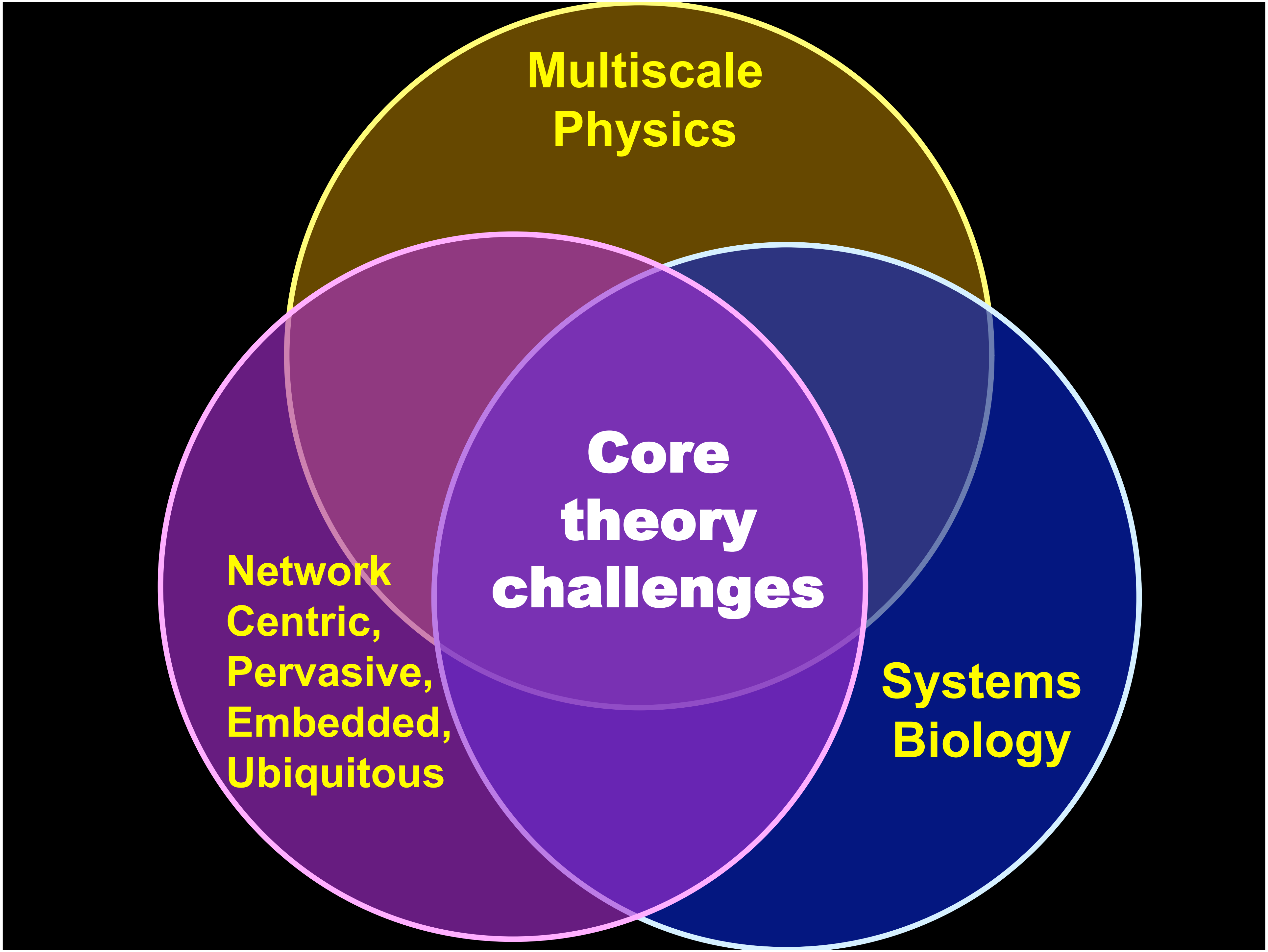
John Doyle

John G Braun Professor

Control and Dynamical Systems, BioEng, and ElecEng

Caltech

www.cds.caltech.edu/~doyle


Multiscale
Physics
Core
theory
challenges
Network
Centric,
Pervasive,
Embedded,
Ubiquitous
Systems
Biology

Core theory challenges

- Hard limits
- Short proofs
- Small models

- Architecture

Architecture?

- "*The bacterial cell and the Internet have*
  - *architectures*
  - *that are robust and evolvable*"

- What does "architecture" mean?
- What does it mean for an "architecture" to be robust and evolvable?
- Robust yet fragile?

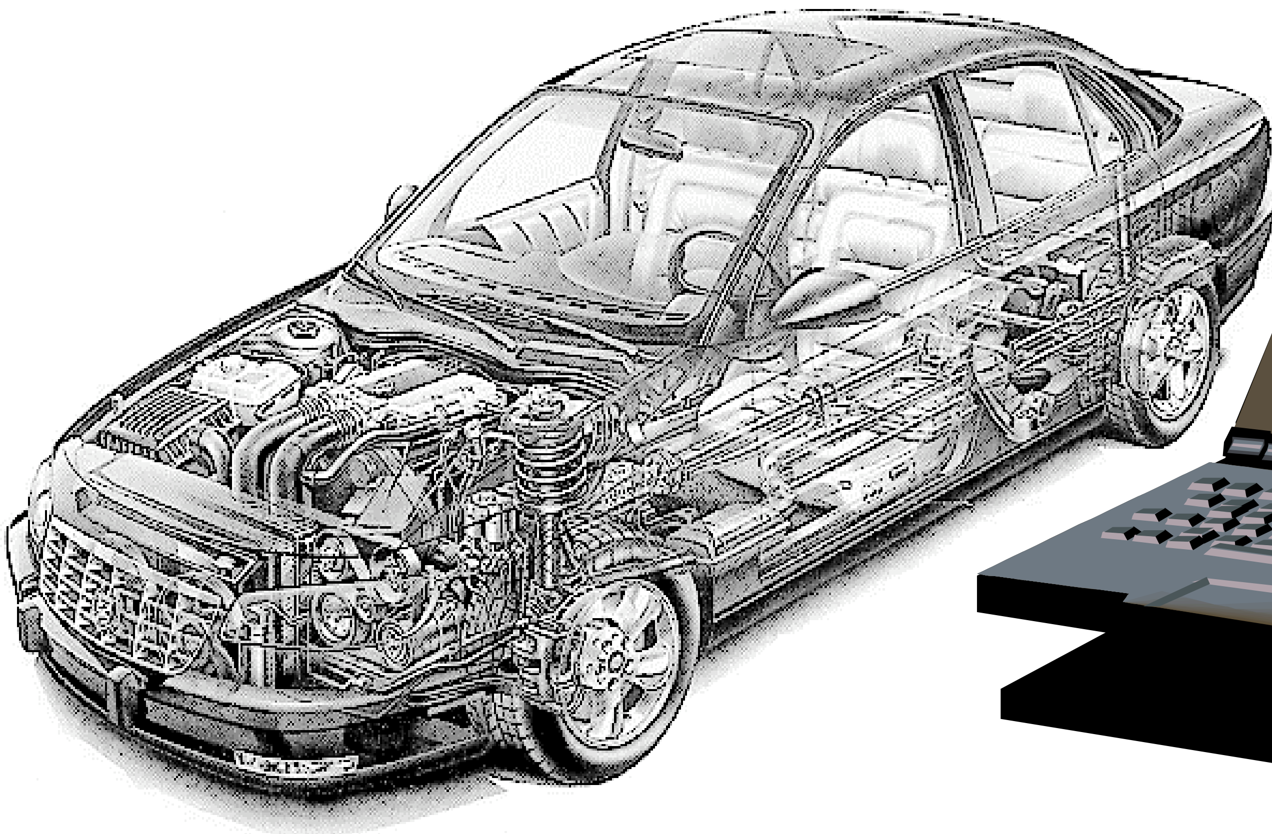
Robust

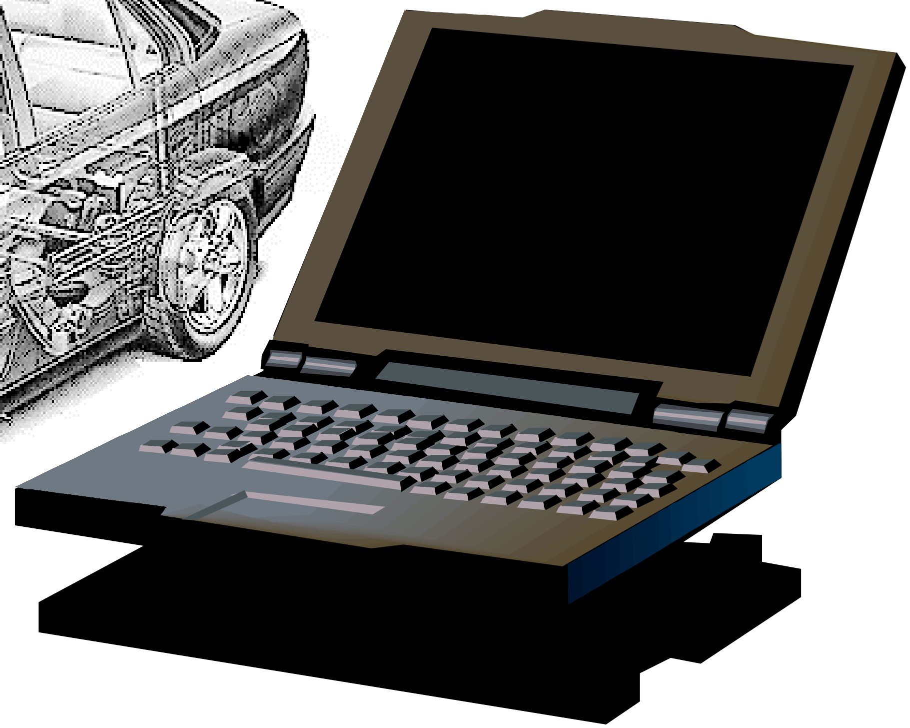
1. Efficient, flexible metabolism
2. Complex development
3. Immune systems
4. Regeneration & renewal
5. Complex societies

Yet Fragile

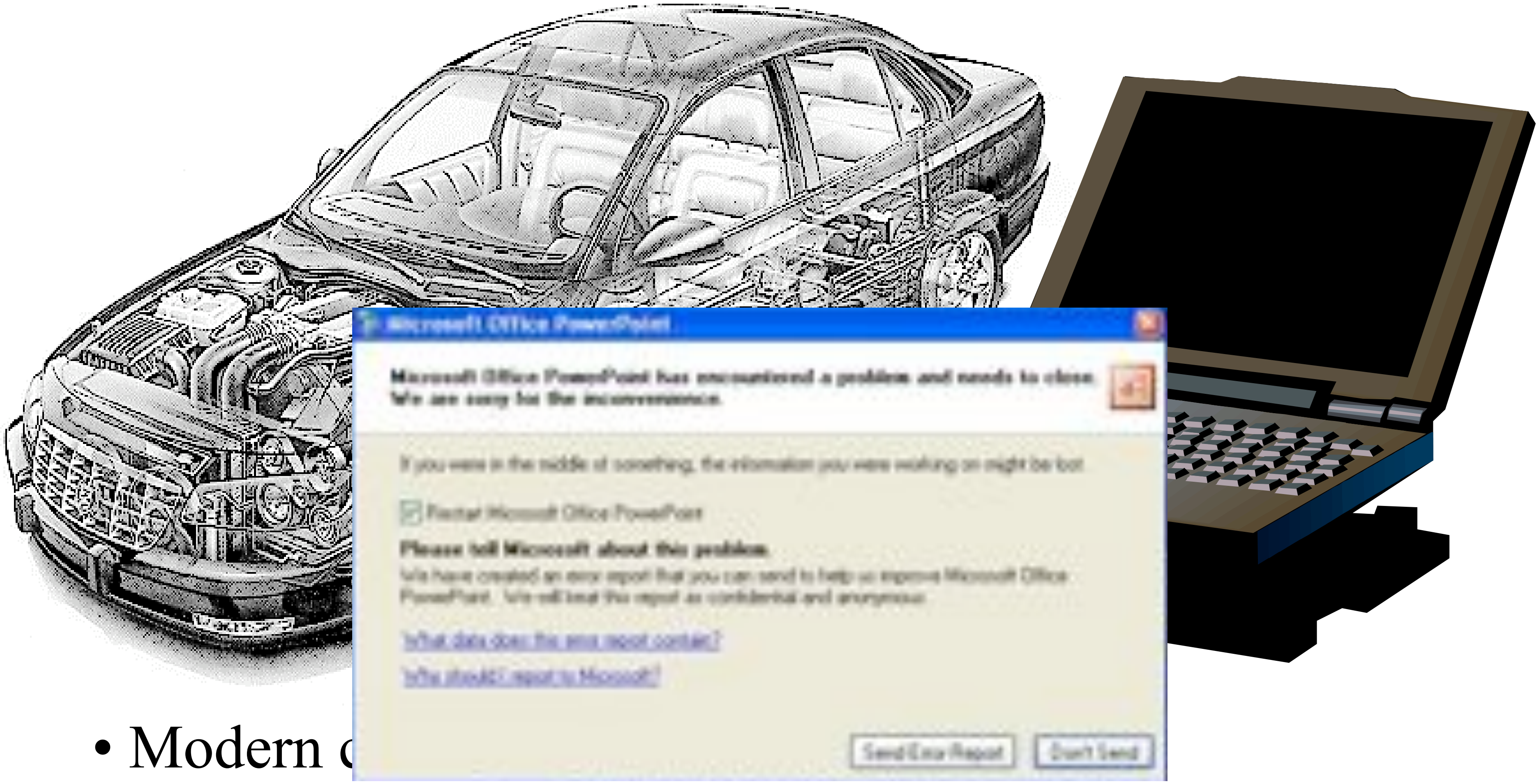
1. Obesity and diabetes
2. Rich parasite ecosystem
3. Auto-immune disease
4. Cancer
5. Epidemics, war, genocide, …

Human robustness and fragility

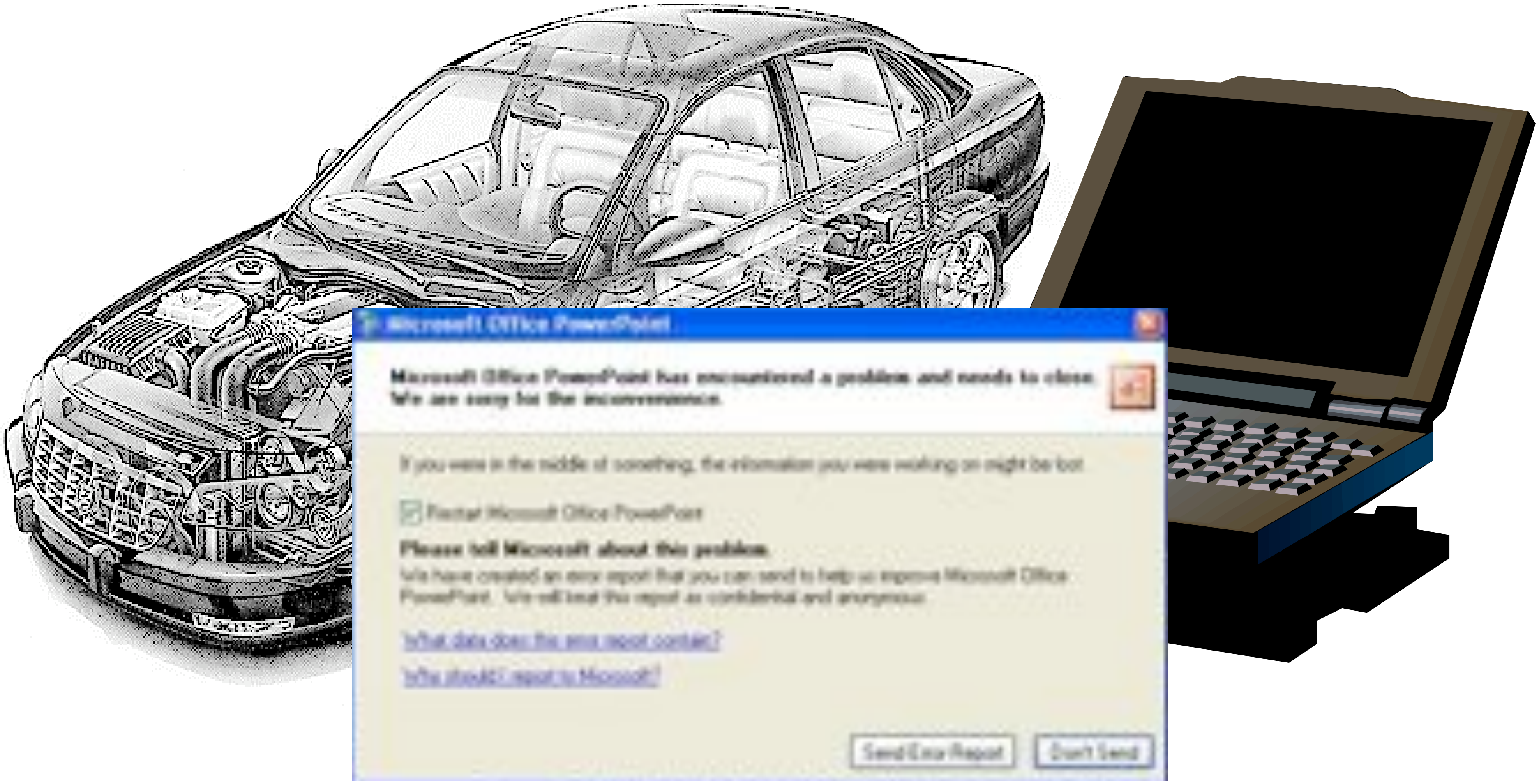




- Modern cars, planes, computers, etc have exploding internal complexity
- They are simpler to use and more robust.
- They tend to work perfectly or not at all.



- Modern c
exploding internal complexity
- They are simpler to use and more robust.
- They tend to work perfectly or not at all.



Robust yet Fragile

Nightmare?

Biology: We might accumulate more complete parts lists but never “understand” how it all works.

Technology: We might build increasingly complex and incomprehensible systems which will eventually fail completely yet cryptically.

Nothing in the orthodox views of complexity says this won’t happen (apparently).

HOPE?

Interesting systems are **robust yet fragile**.

Identify the fragility, evaluate and protect it.

The rest (robust) is **“easy”.**

Nothing in the orthodox views of complexity
says this can’t happen (apparently).

To pursue the hope

- Hard limits
- Short proofs
- Small models

- Architecture

- **Robustness**

Hard limits and tradeoffs

On systems and their components

- Thermodynamics (Carnot)
- Communications (Shannon)
- Control (Bode)
- Computation (Turing/Gödel)

Assume ***different*** architectures a priori.

- Fragmented and incompatible
- We need a more integrated view and have the beginnings

The nature of simplicity

Simple questions:
- Simple models
- Elegant theorems
- Elegant experiments

Simple answers:
- Predictable results
- Short proofs
- Simple outcomes

Reductionist science: Reduce the ***apparent complexity*** of the world to an underlying simplicity.

Physics has for centuries epitomized the success of this approach.

1930s: The end of certainty

Simple questions:

- Simple models
- Elegant theorems
- Elegant experiments

Simple answers:

- ~~Predictable results~~
- ~~Short proofs~~
- Simple outcomes

- Godel: Incompleteness
- Turing: Undecidability
- Profoundly effected mathematics and computation.
- Little impact on science.

1960s-Present: “Emergent complexity”

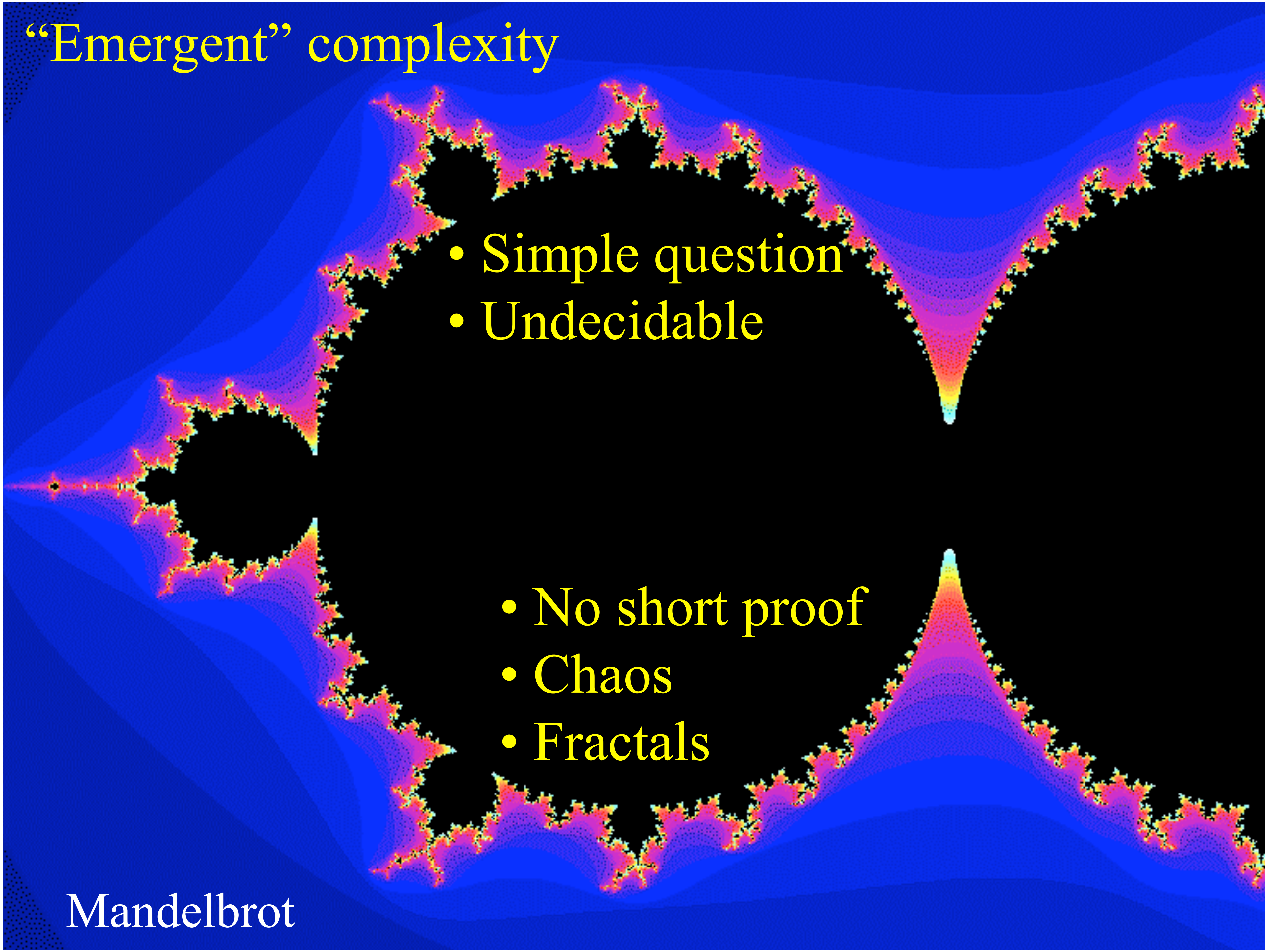
Simple questions:

- Simple models
- Elegant theorems
- Elegant experiments

Dominates scientific thinking today

Complexity:

- Unpredictabity
- Chaos, fractals
- Critical phase transitions
- Self-similarity
- Universality
- Pattern formation
- Edge-of-chaos
- Order for free
- Self-organized criticality
- Scale-free networks


"Emergent" complexity
• Simple question
• Undecidable
• No short proof
• Chaos
• Fractals
Mandelbrot

The “New Science of Complexity”

	Simple question
Predictable	Simplicity
Unpredictable	“Emergence”

Even simple systems with little uncertainty can yield completely unpredictable behavior.

1900s: The triumph (and horror) of organization

~~Simple questions~~:

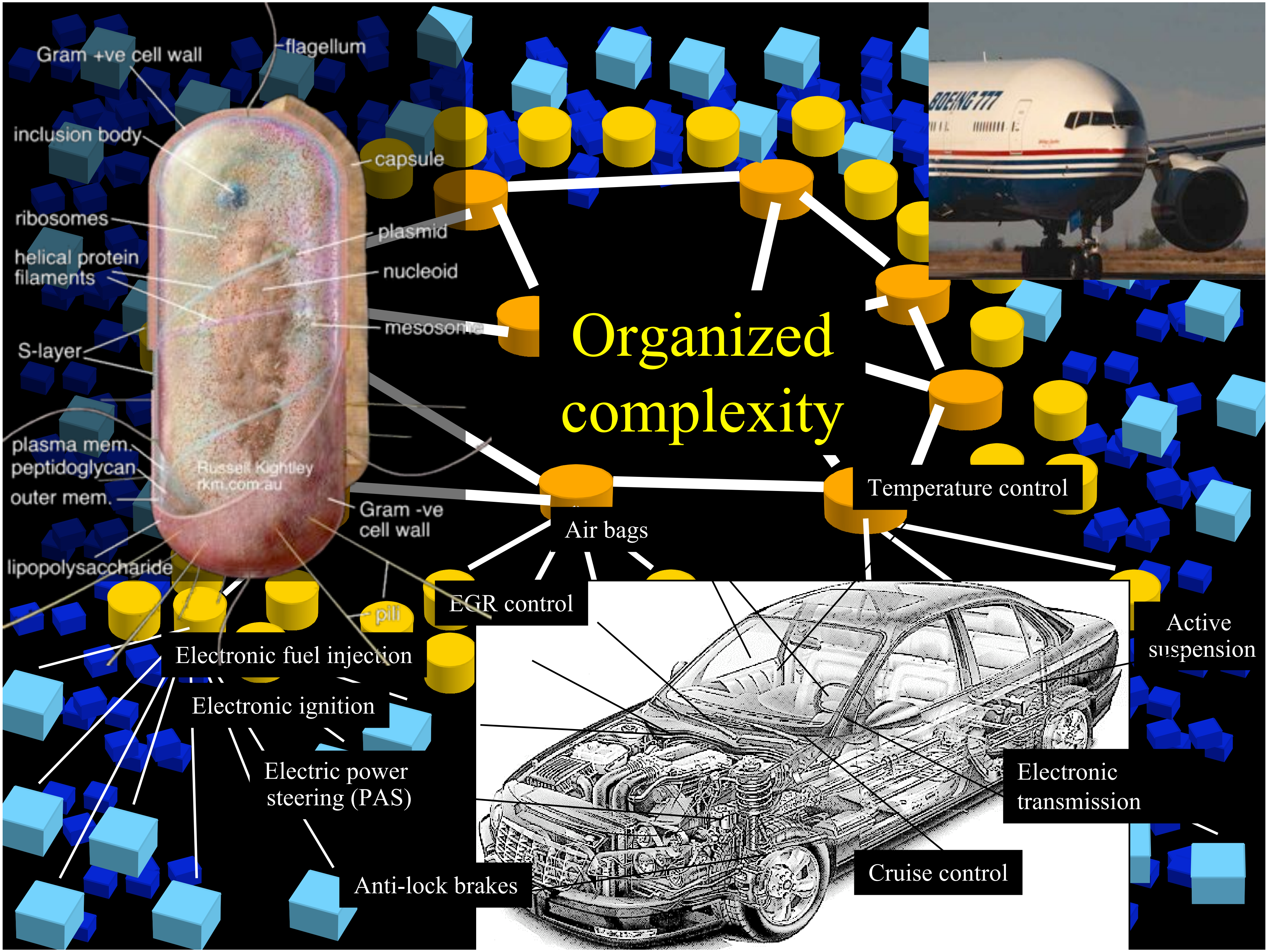
- ~~Simple models~~
- ~~Elegant theorems~~
- ~~Elegant experiments~~

Simple answers:

- Predictable results
- Short proofs
- Simple outcomes

- Complex, uncertain, hostile environments
- Unreliable, uncertain, changing components
- Limited testing and experimentation

- Yet predictable, robust, reliable, adaptable, evolvable systems


Gram +ve cell wall
flagellum
inclusion body
capsule
ribosomes
plasmid
helical protein filaments
nucleoid
mesosome
S-layer
plasma mem.
peptidoglycan
outer mem.
Gram -ve cell wall
lipopolysaccharide
pili
Russell Kightley rkm.com.au
BOEING 777
Organized complexity
Temperature control
Air bags
EGR control
Active suspension
Electronic fuel injection
Electronic ignition
Electric power steering (PAS)
Electronic transmission
Cruise control
Anti-lock brakes

Organized complexity

- Requires highly organized interactions, by design or evolution
- Completely different theory and technology from emergence

Simple answers:

- Predictable results
- Short proofs
- Simple outcomes

- Complex, uncertain, hostile environments
- Unreliable, uncertain, changing components
- Limited testing and experimentation

- Yet predictable, robust, reliable, adaptable, evolvable systems

Mathematics and technology

Question / Answer	Simple	Complex
Predictable	Simplicity	**Organization**
Unpredictable	Emergence	

Emergence and organization are opposites, but can be viewed in a unified framework.

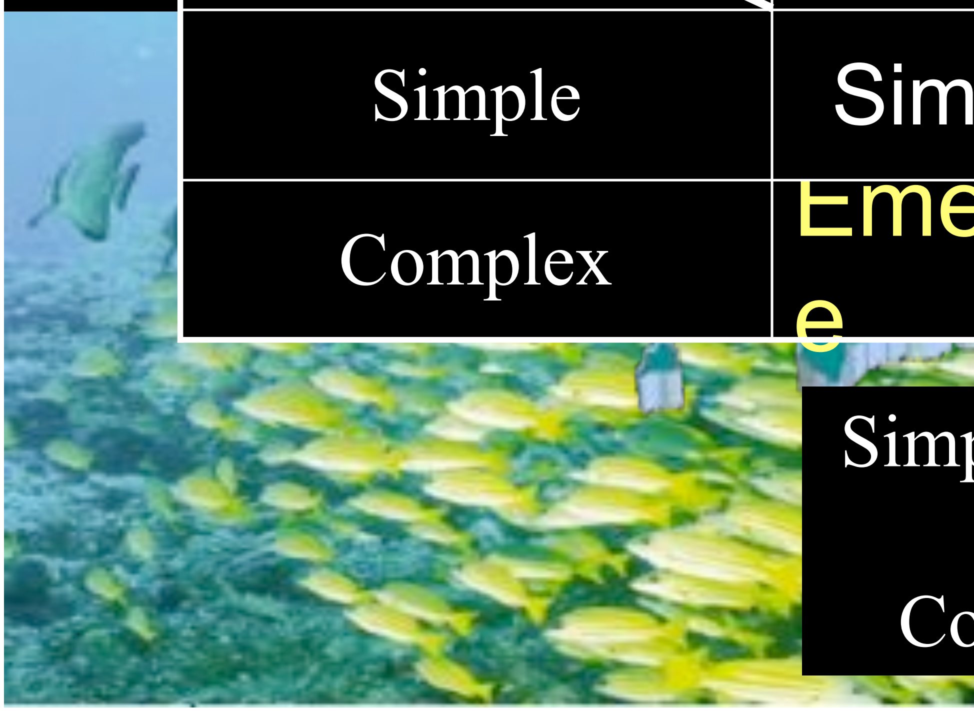
Irreducible complexity?

Question / Answer	Simple	Complex
Predictable	Simplicity	**Organization**
Unpredictable	Emergence	?

Complexity and unpredictability are the key to successful cryptography and other security technologies.

The complete picture

Question / Answer	Simple	Complex
Simple	Simplicity	**Organization**
Complex	Emergence	**Irreducibility**

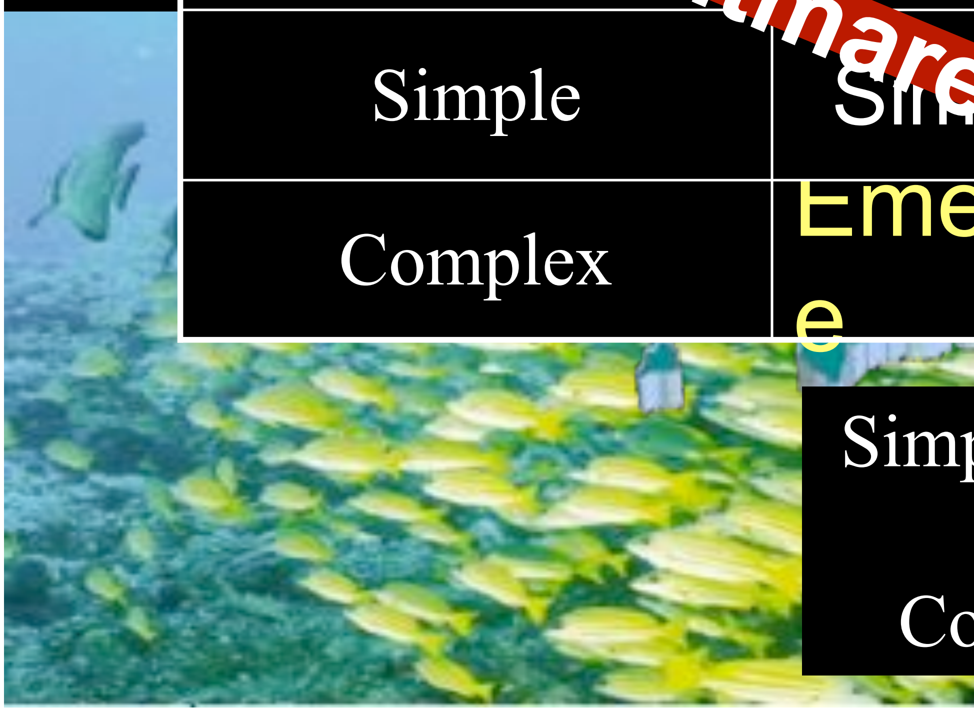


Simple, predictable, reliable, robust
versus
Complex, unpredictable, fragile

The complete picture

Question / Answer	Simple	Complex
Simple	Simplicity	Organization
Complex	Emergence	Irreducibility

Nightmare



Simple, predictable, reliable, robust
versus
Complex, unpredictable, fragile

The complete picture

Question / Answer	Simple	Complex
Simple	Simplicity	Organization
Complex	Emergence	Irreducibility

The challenge

Models / Proofs	Small	Large
Short		
Long		

How can we treat complex networks and systems with small models and short proofs?

The complete picture

Models / Proofs	Small	Large
Short	Simplicity	Organization
Long	Emergence	Irreducibility

Breaking hard problems

- SOSTOOLS proof theory and software (Parrilo, Prajna, Papachristodoulou, …)
- Nested family of (dual) proof algorithms
- Each family is polynomial time
- Recovers many "gold standard" algorithms as special cases, and immediately improves
- Nonlinear, hybrid, stochastic, …
- No a priori polynomial bound on depth (otherwise P=NP=coNP)
- Conjecture: Complexity implies fragility

Architecture?

- "*The bacterial cell and the Internet have*
  - *architectures*
  - *that are robust and evolvable* ***(yet fragile)*** "

- What does "architecture" mean?
- What does it mean for an "architecture" to be robust and evolvable?
- Robust yet fragile?
- Rigorous and coherent theory?

A look back and forward

- The Internet architecture was designed without a “theory.”
- Many academic theorists told the engineers it would never work.
- We now have a nascent theory that confirms that the engineers were right (Kelly, Low, Vinnicombe, Paganini, Papachristodoulou, …)

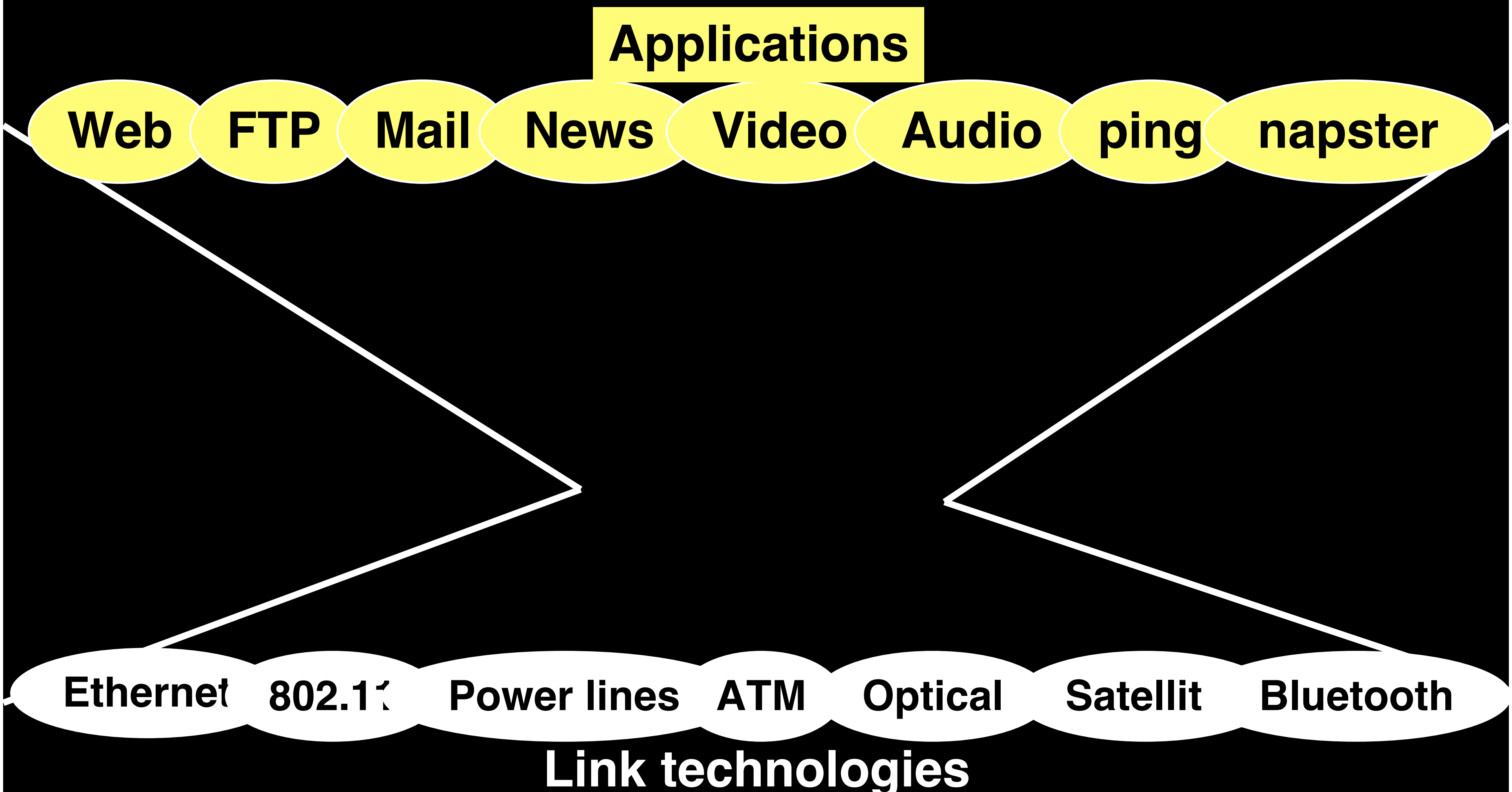
- Parallel stories exist in “theoretical biology.”
- For future networks, “systems of systems,” and other new technologies, as well as ***systems biology of the cell, organism and brain***, …
- …let’s hope we can avoid a repeat of this history. (Looks like we have a good start…)

Architecture?

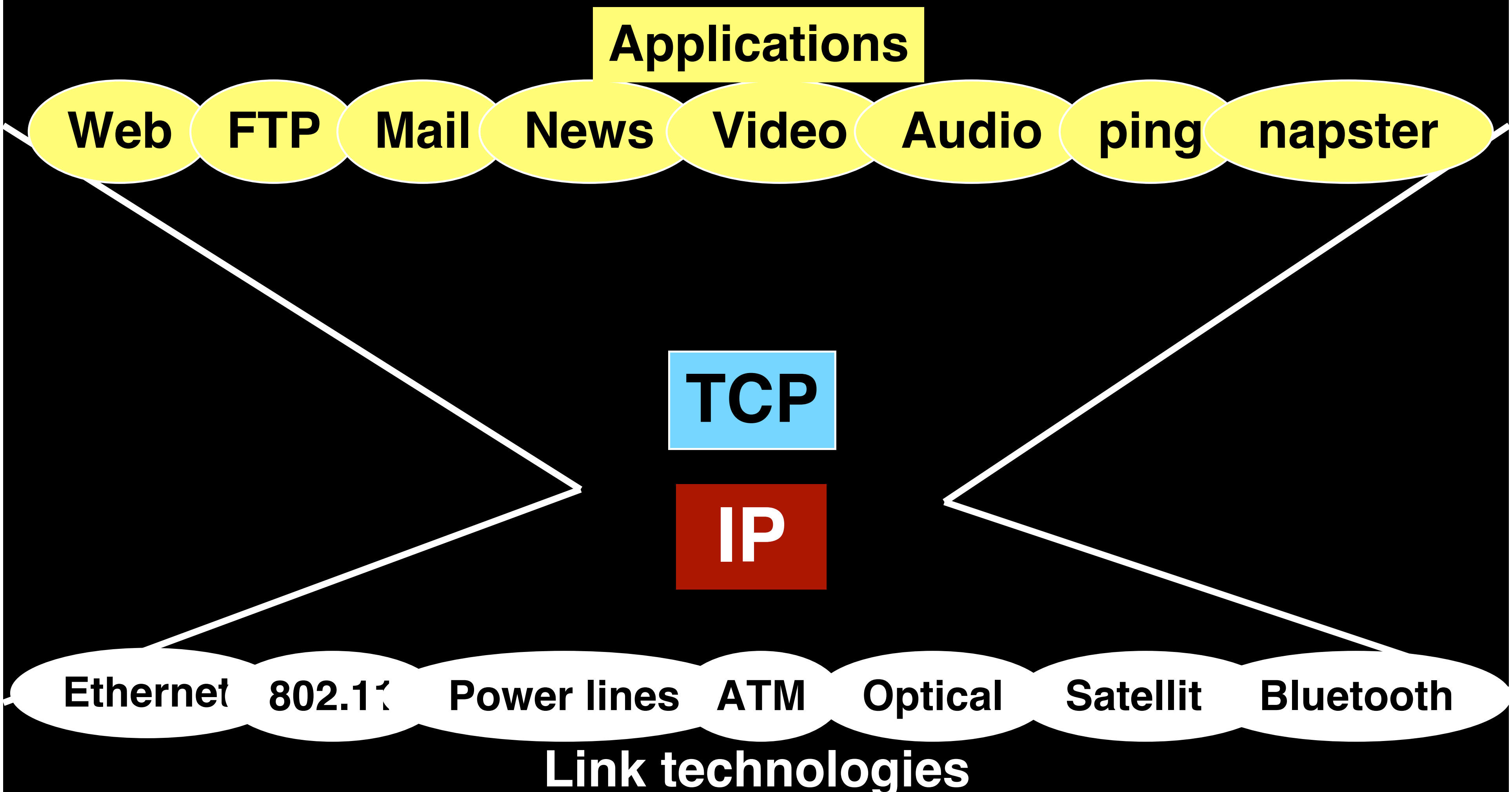
- "*The bacterial cell and the Internet have*
  - *architectures*
  - *that are robust and evolvable*"

- For the Internet, we know how all the parts work, and we can ask the architects

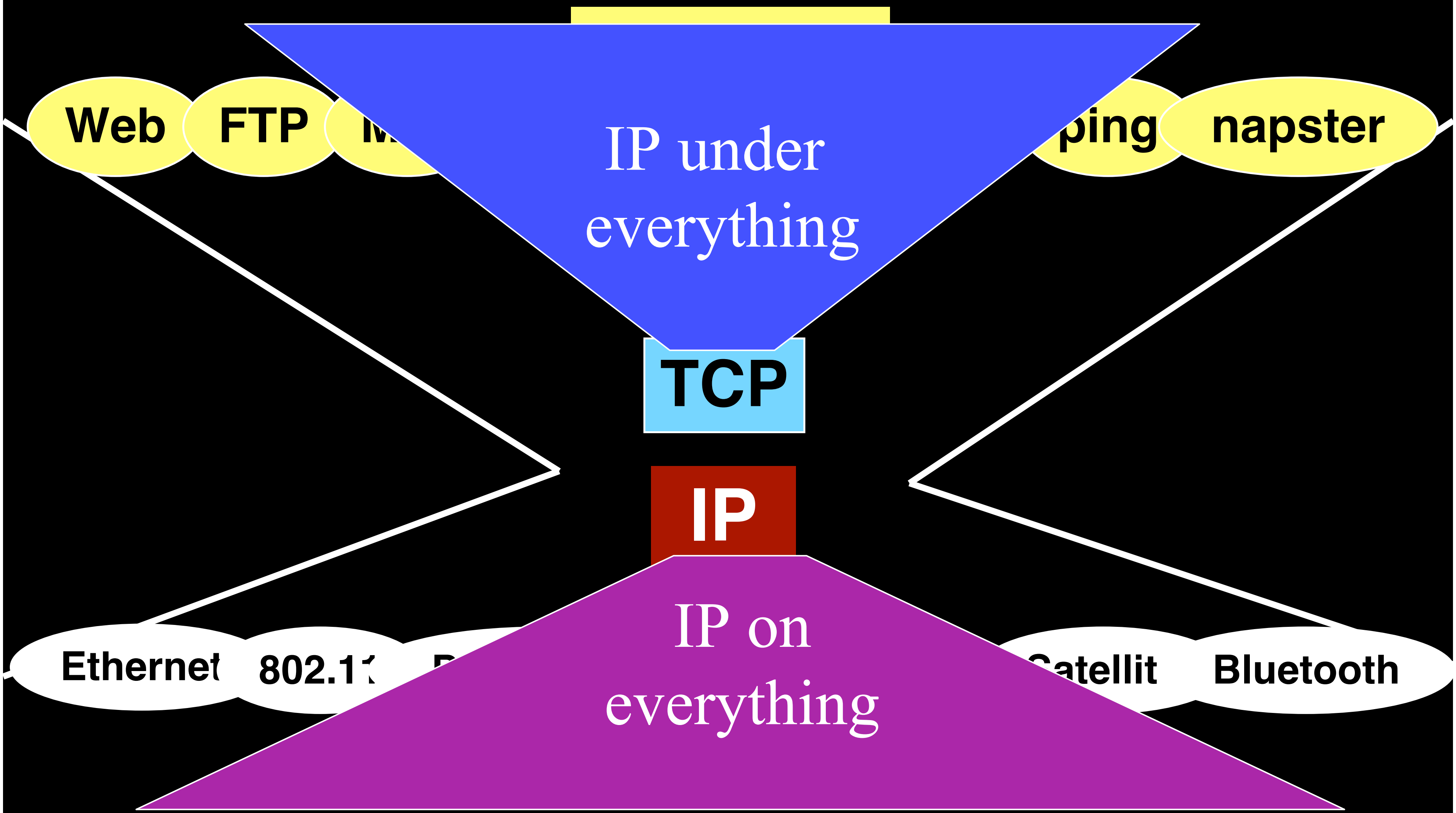
The Internet hourglass

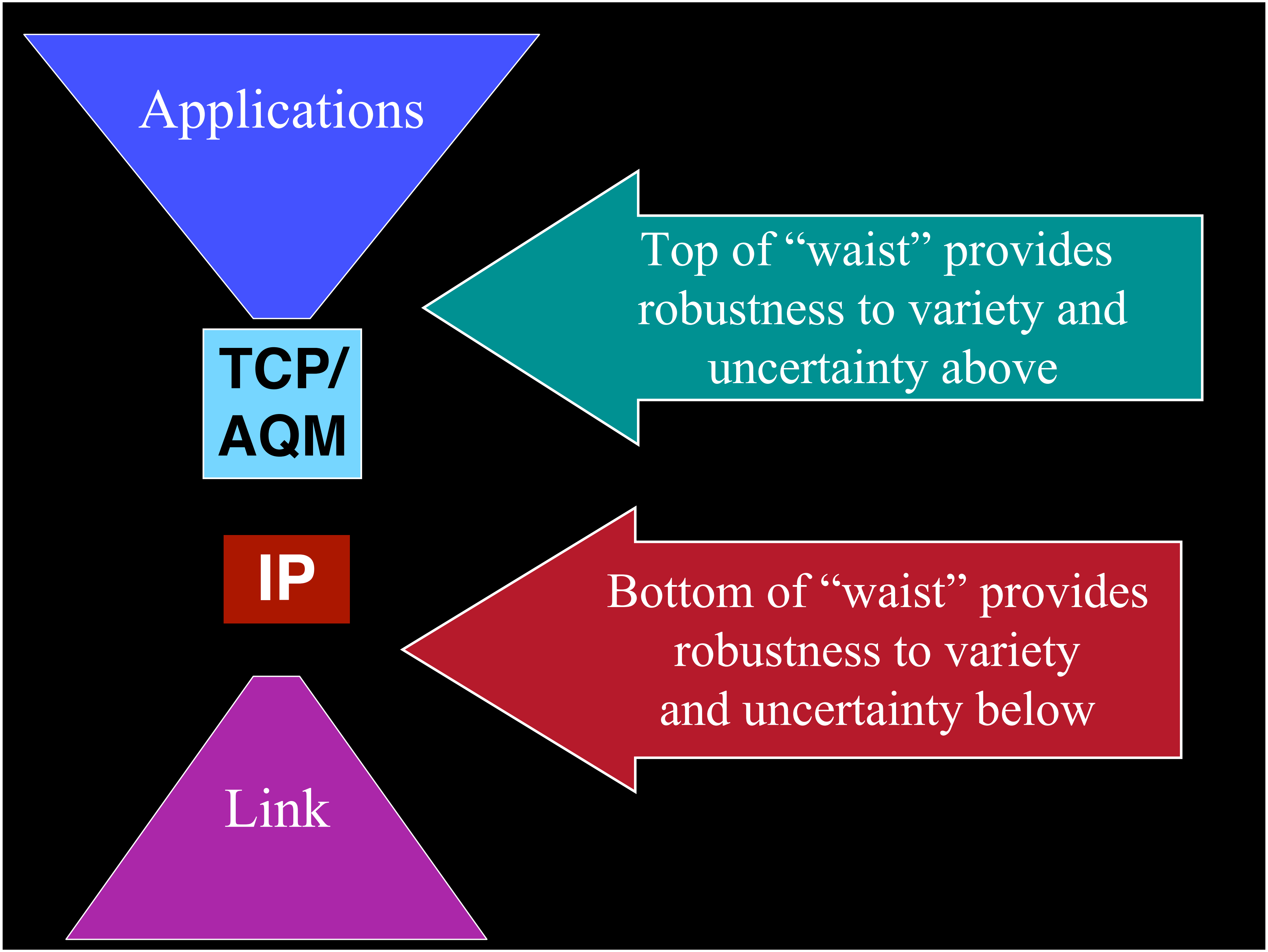


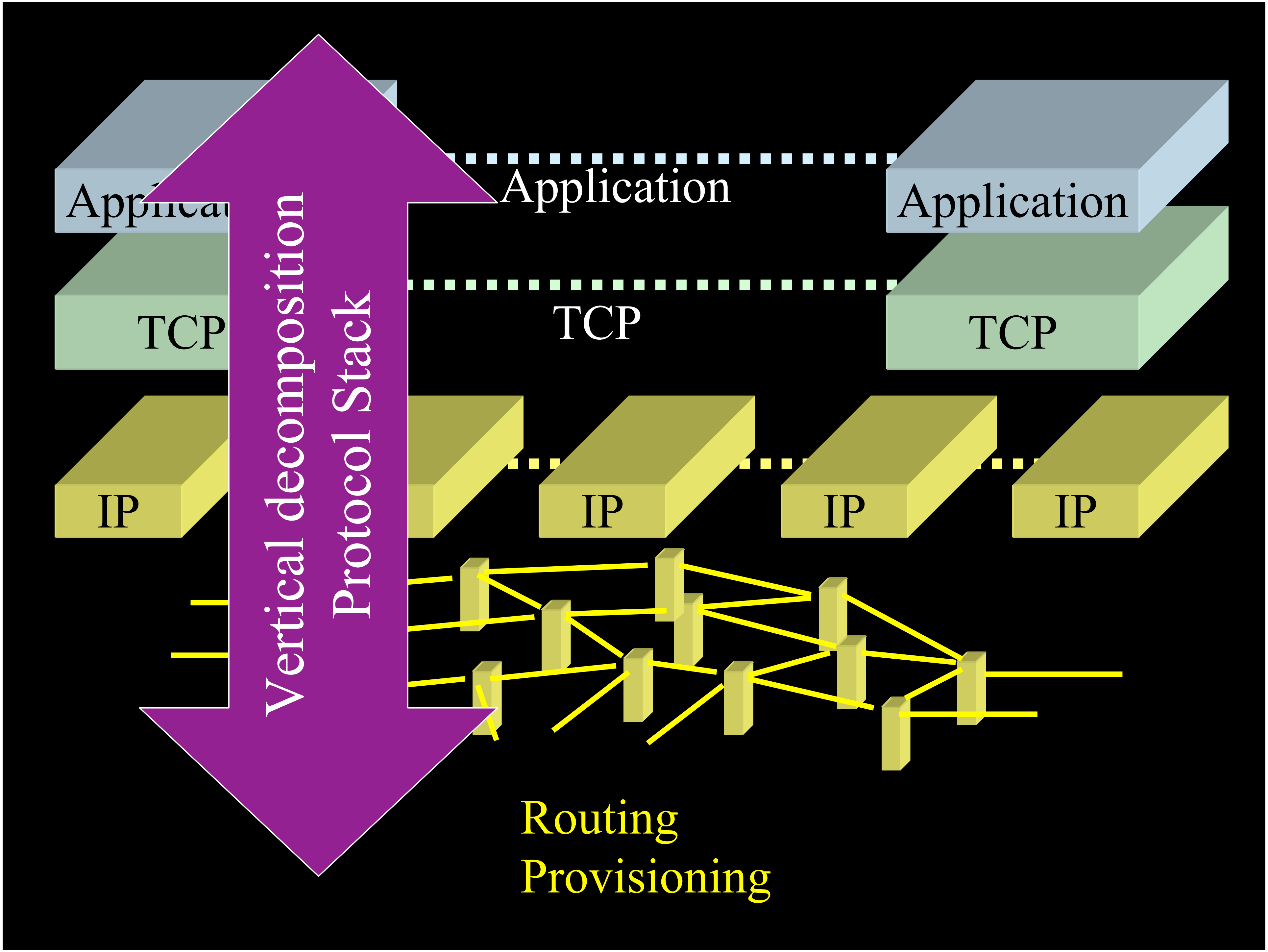
The Internet hourglass



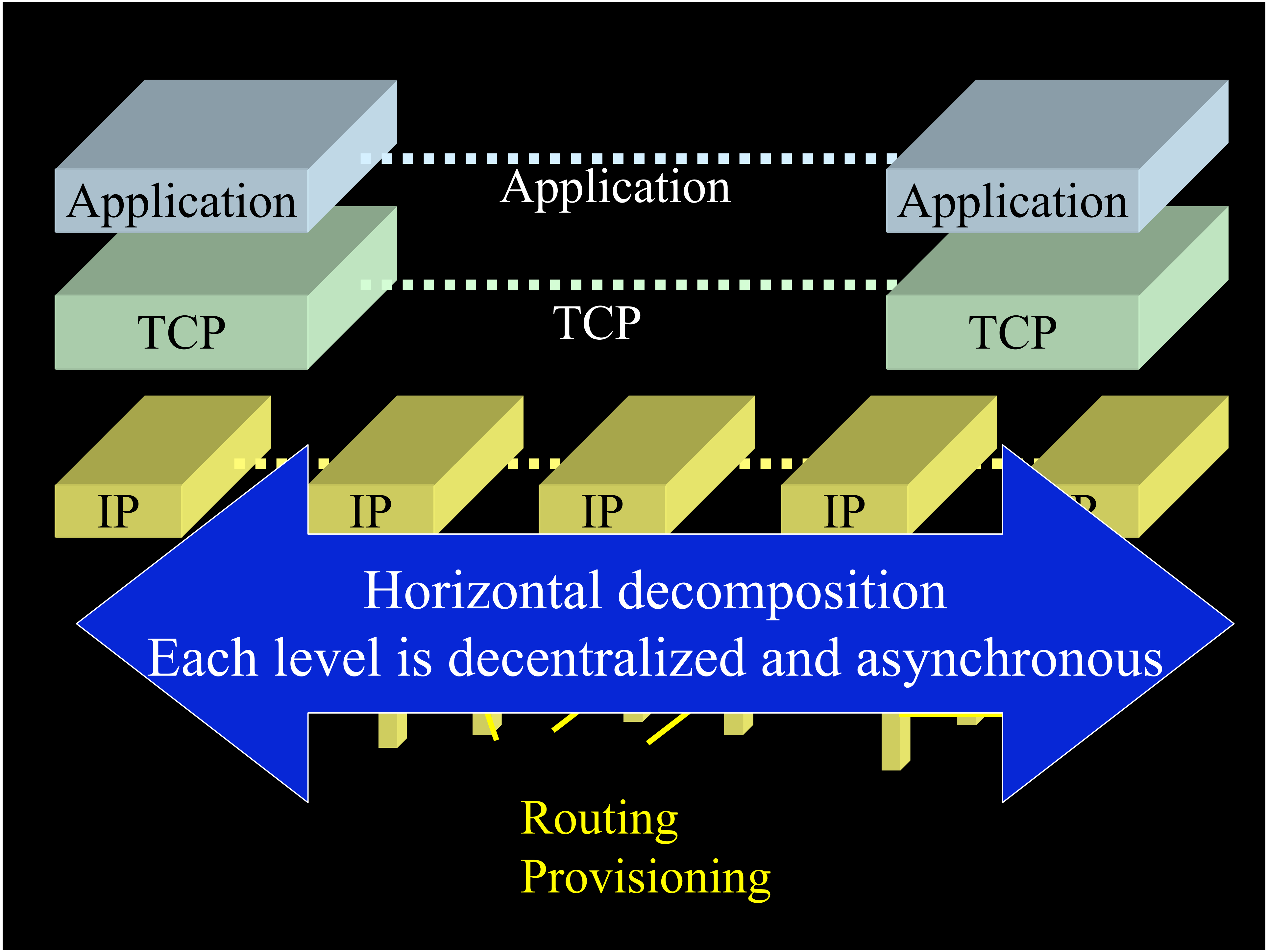
The Internet hourglass

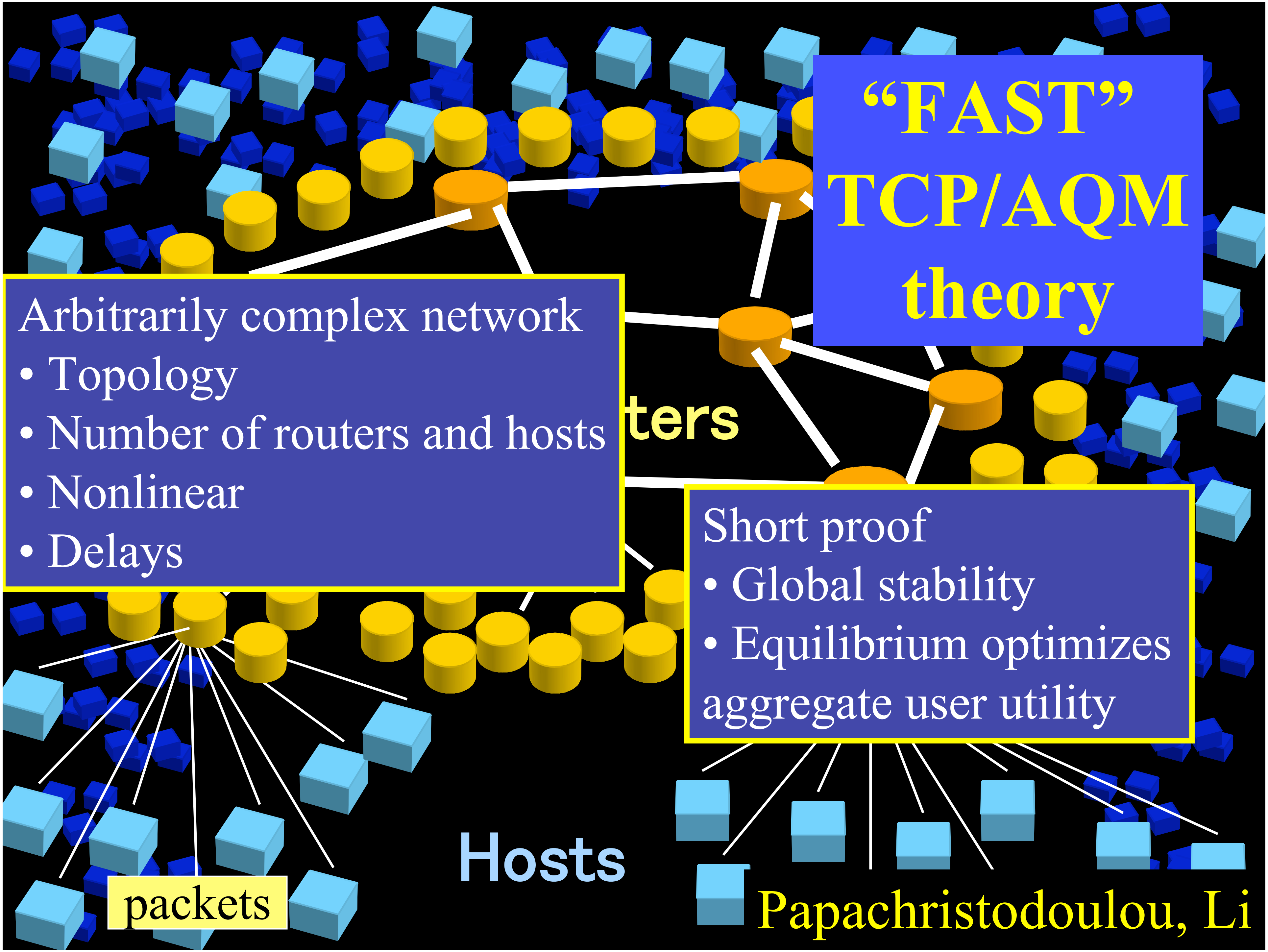



Applications
TCP/
AQM
IP
Link
Top of "waist" provides
robustness to variety and
uncertainty above
Bottom of "waist" provides
robustness to variety
and uncertainty below



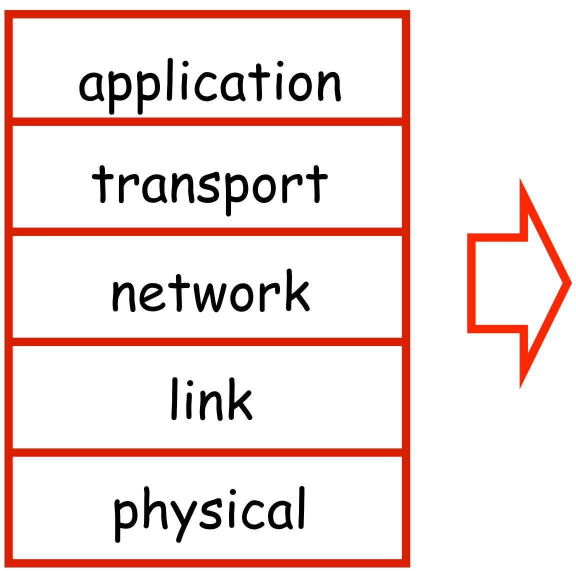
Application
Application
TCP
TCP
TCP
IP
IP
IP
IP
IP
Vertical decomposition
Protocol Stack
Routing
Provisioning


Application
Application
Application
TCP
TCP
TCP
IP
IP
IP
IP
Horizontal decomposition
Each level is decentralized and asynchronous
Routing
Provisioning


"FAST" TCP/AQM theory
Arbitrarily complex network
• Topology
• Number of routers and hosts
• Nonlinear
• Delays
Short proof
• Global stability
• Equilibrium optimizes aggregate user utility
Hosts
packets
Papachristodoulou, Li

Layering as optimization decomposition

- Each layer is abstracted as an optimization problem
- Operation of a layer is a distributed solution
- Results of one problem (layer) are parameters of others
- Operate at different timescales



Application: utility

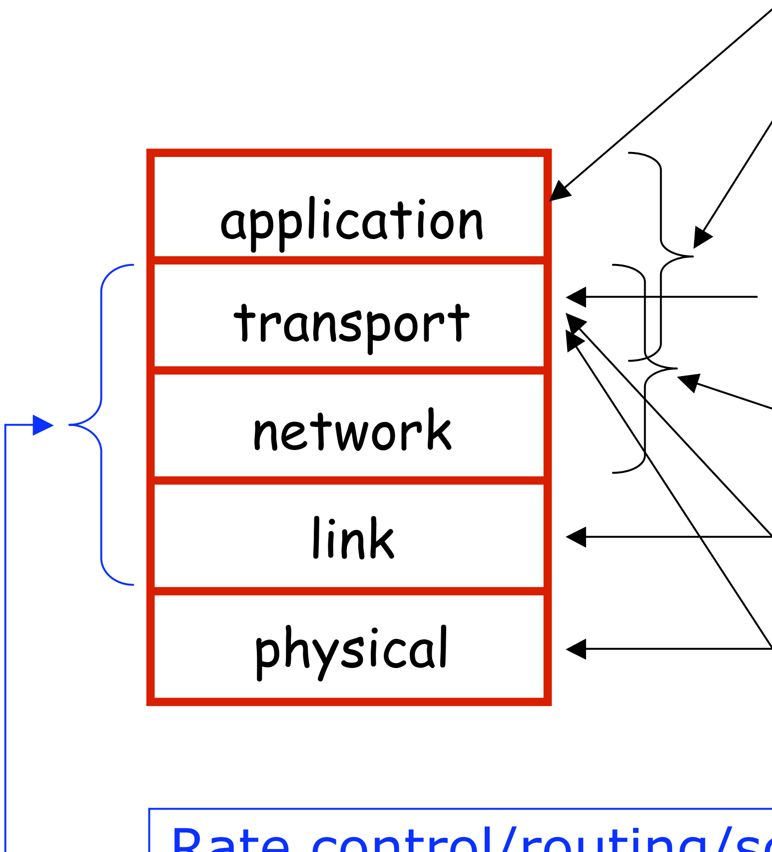
Phy: power

$$
\begin{aligned}
\max_{x \geq 0} \quad & \sum_i U_i(x_i) \\
\text{subj to} \quad & Rx \leq c(p) \\
& x \in X
\end{aligned}
$$

IP: routing

Link: scheduling

Examples



Optimal web layer: Zhu, Yu, Doyle ’01

HTTP/TCP: Chang, Liu ’04

TCP: Kelly, Maulloo, Tan ’98, ……

TCP/IP: Wang et al ’05, ……

TCP/MAC: Chen et al ’05, ……

TCP/power control: Xiao et al ’01, Chiang ’04, ……

Rate control/routing/scheduling: Eryilmax et al ’05, Lin et al ’05, Neely, et al ’05, Stolyar ’05, this paper

detailed survey in Proc. of IEEE, 2006

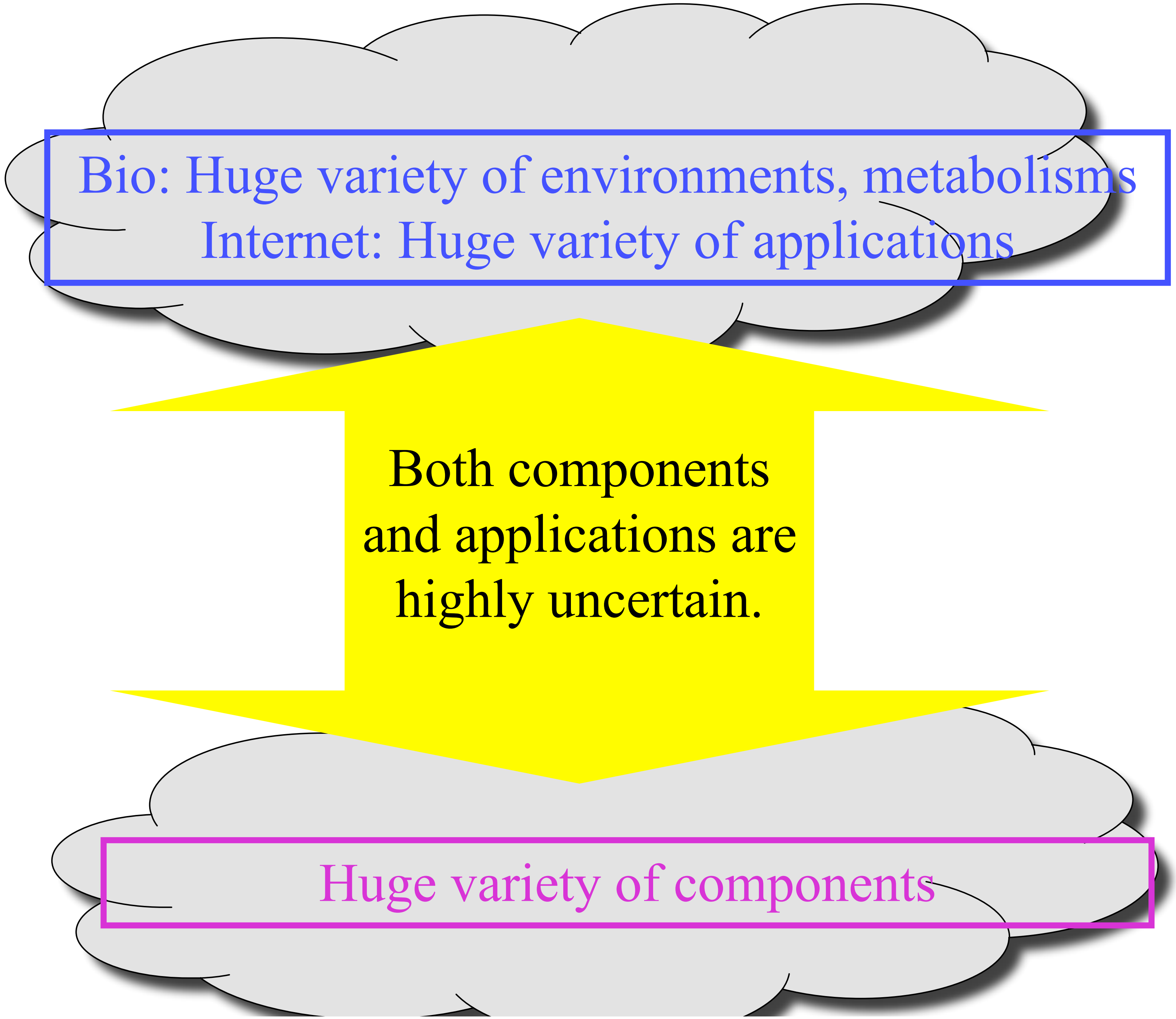
Architecture?

- “*The bacterial cell and the Internet have*
  - *architectures*
  - *that are robust and evolvable*”

- For the Internet, we know how all the parts work, and we can ask the architects
- For biology, we know how some parts work, and evolution is the “architect” (another source of confusion)

Bio: Huge variety of environments, metabolisms
Internet: Huge variety of applications

Huge variety of components


Bio: Huge variety of environments, metabolisms
Internet: Huge variety of applications
Both components and applications are highly uncertain.
Huge variety of components

Bio: Huge variety of environments, metabolisms
Internet: Huge variety of applications

Huge variety of genomes
Huge variety of physical networks

Huge variety of components

Hourglass architectures

Bio: Huge variety of environments, metabolisms
Internet: Huge variety of applications

Feedback control

Identical control architecture

Huge variety of genomes
Huge variety of physical networks

Huge variety of components

TCP/IP

Metabolism/biochem

5:Application/function: variable supply/demand

4:TCP

3:IP

Feedback Control

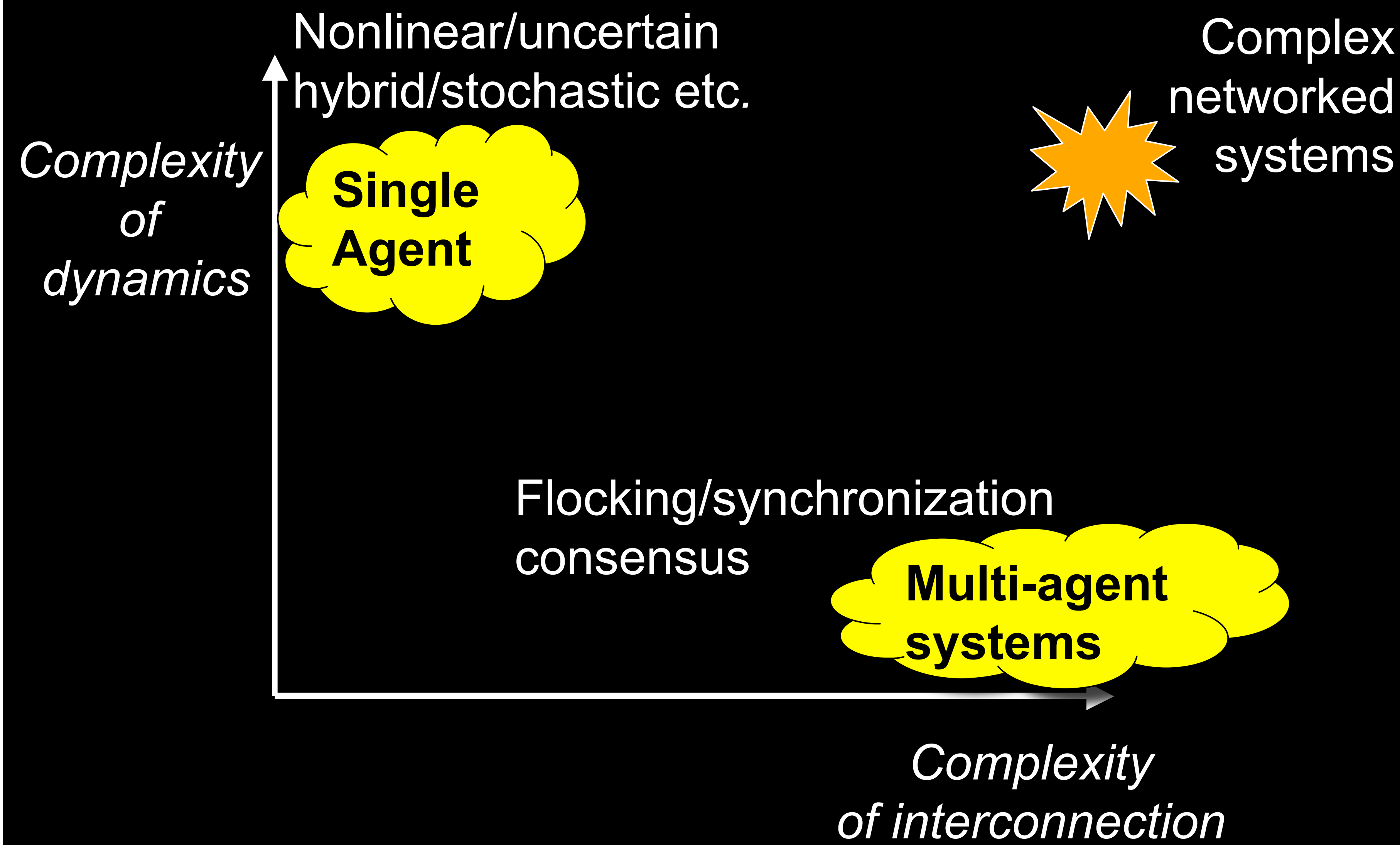
4:Allosteric

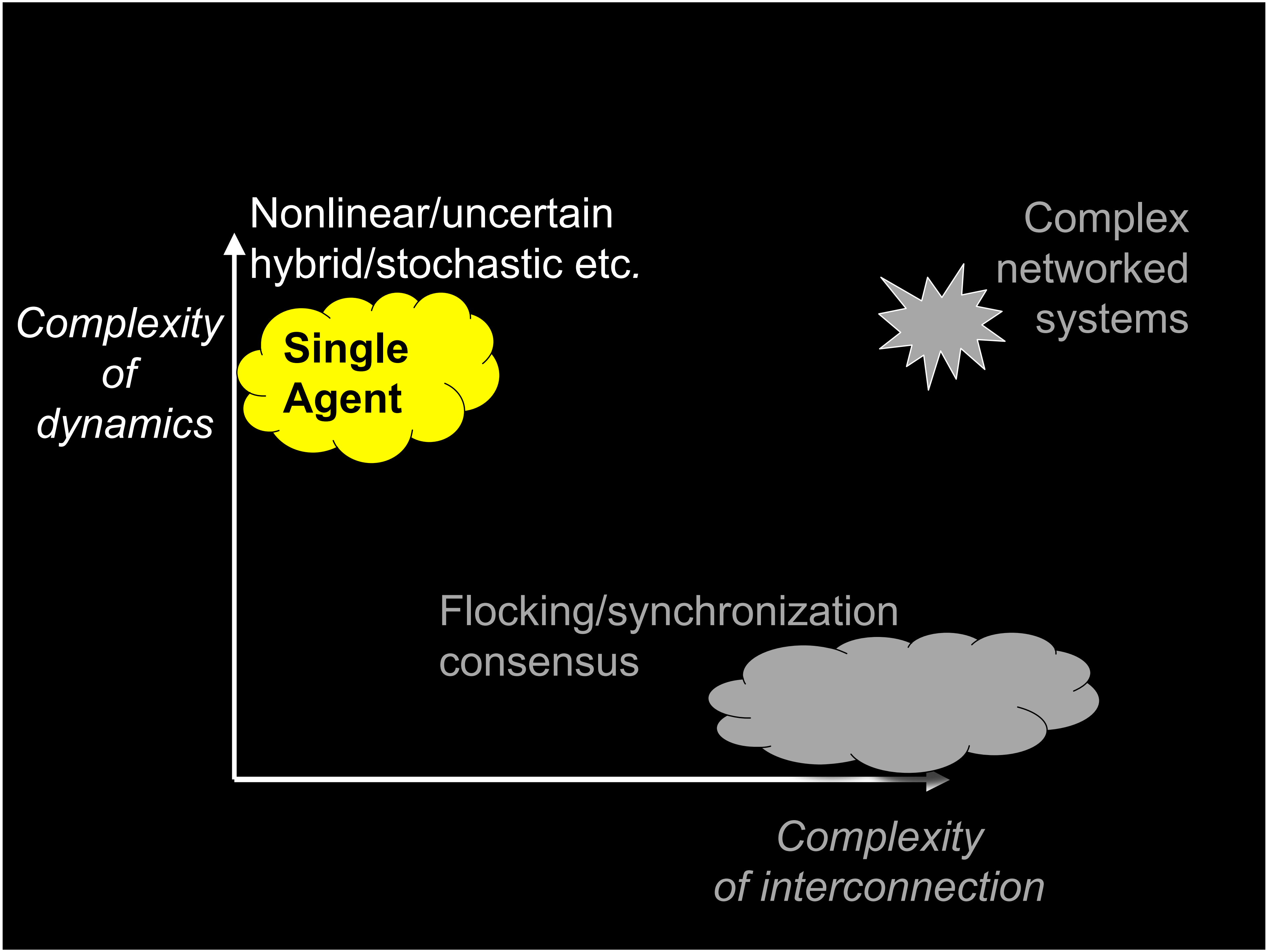
3:Transcriptional

2:Potential physical network

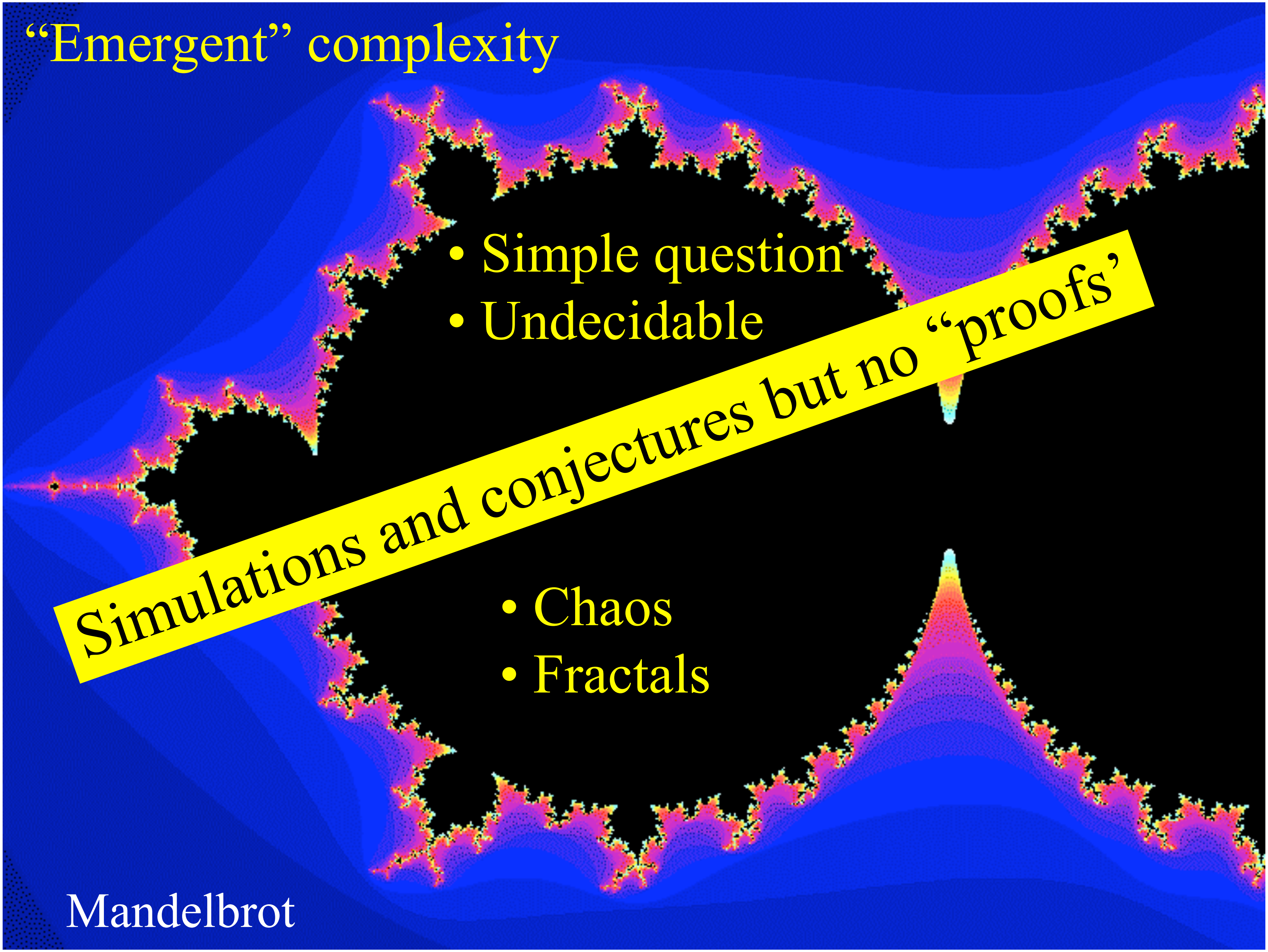
1:Hardware components

Networked dynamical systems

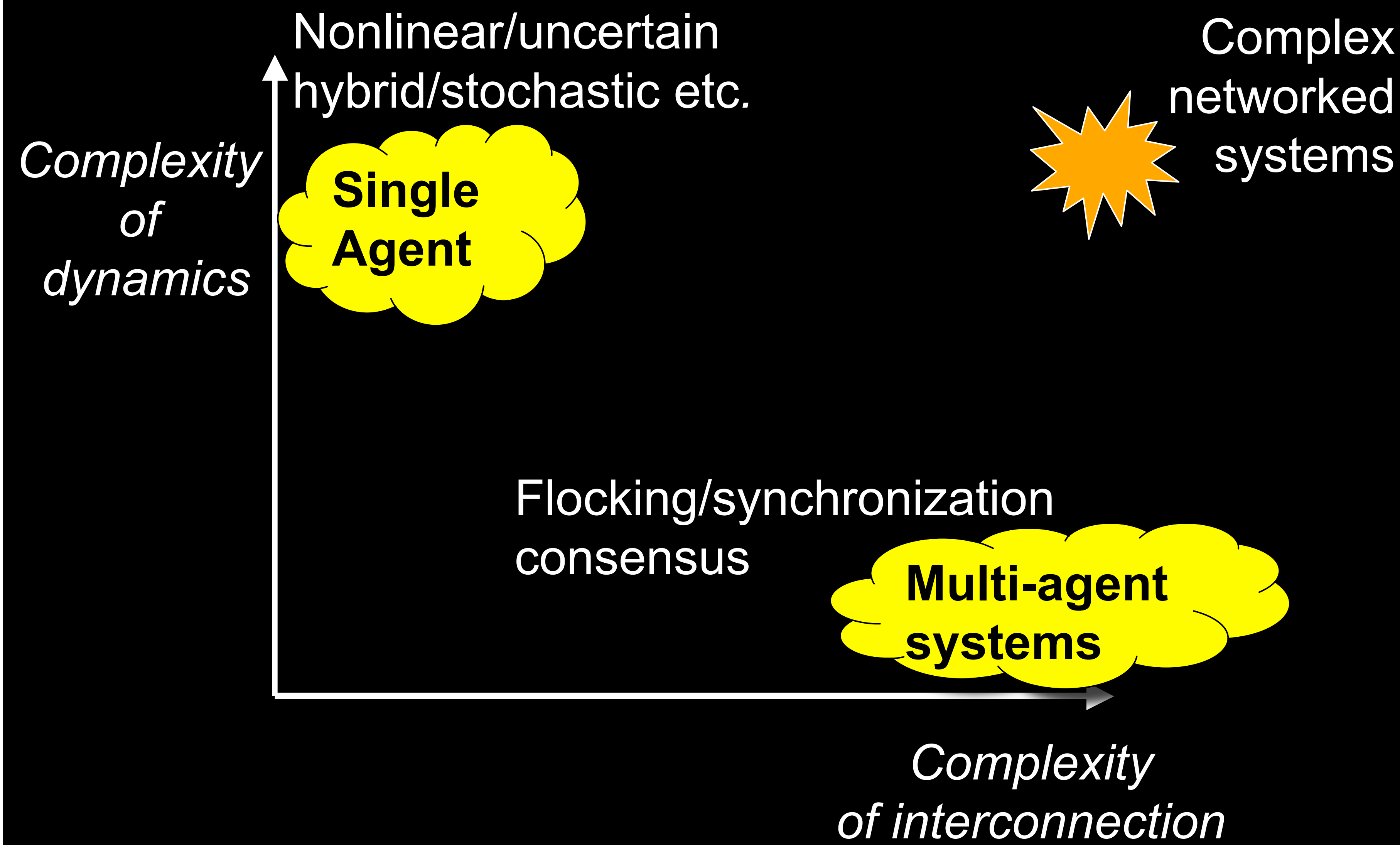


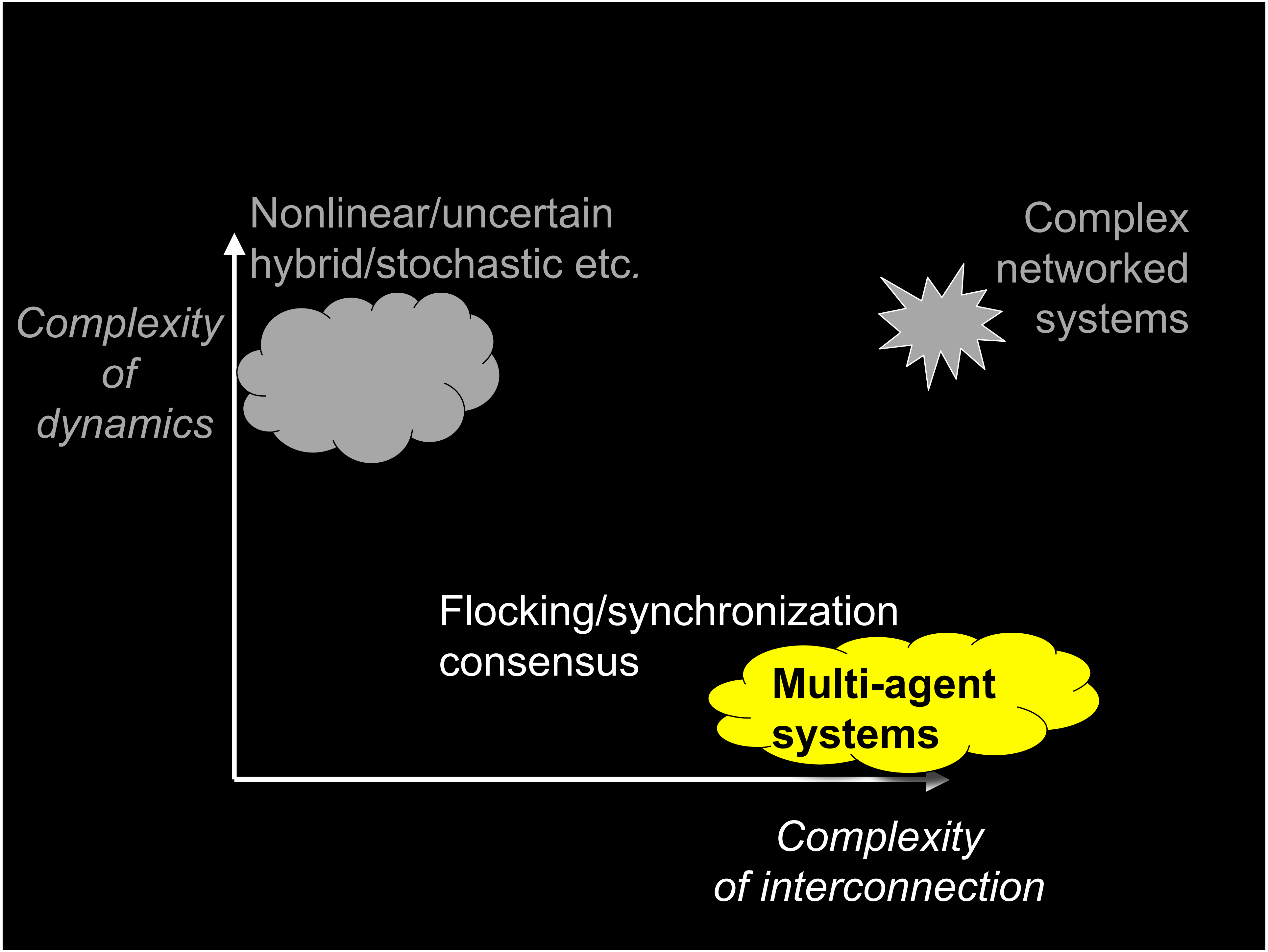


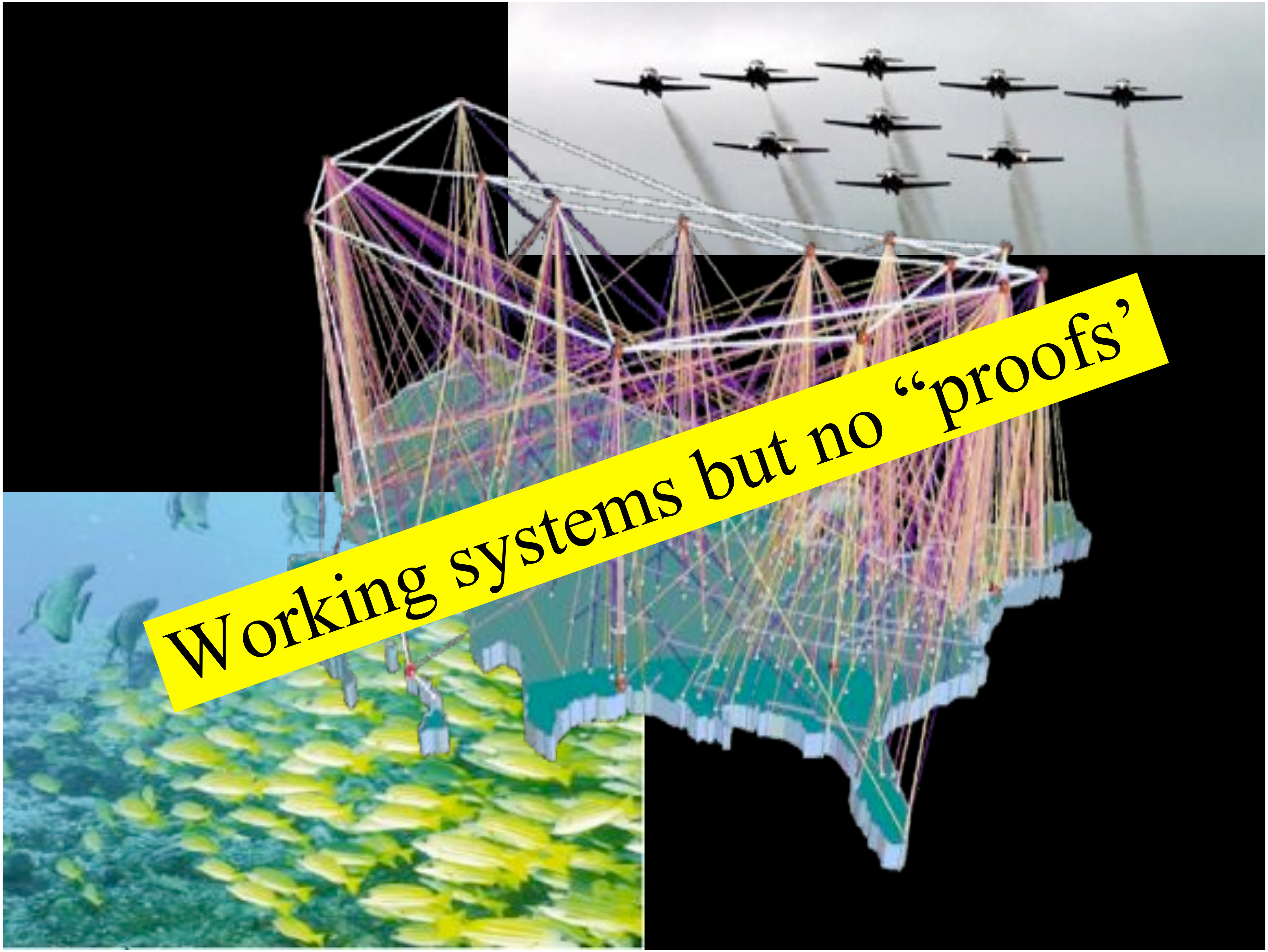
Nonlinear/uncertain
hybrid/stochastic etc.
Complex
networked
systems
Complexity
of
dynamics
Single
Agent
Flocking/synchronization
consensus
Complexity
of interconnection


"Emergent" complexity
• Simple question
• Undecidable
Simulations and conjectures but no "proofs'
• Chaos
• Fractals
Mandelbrot

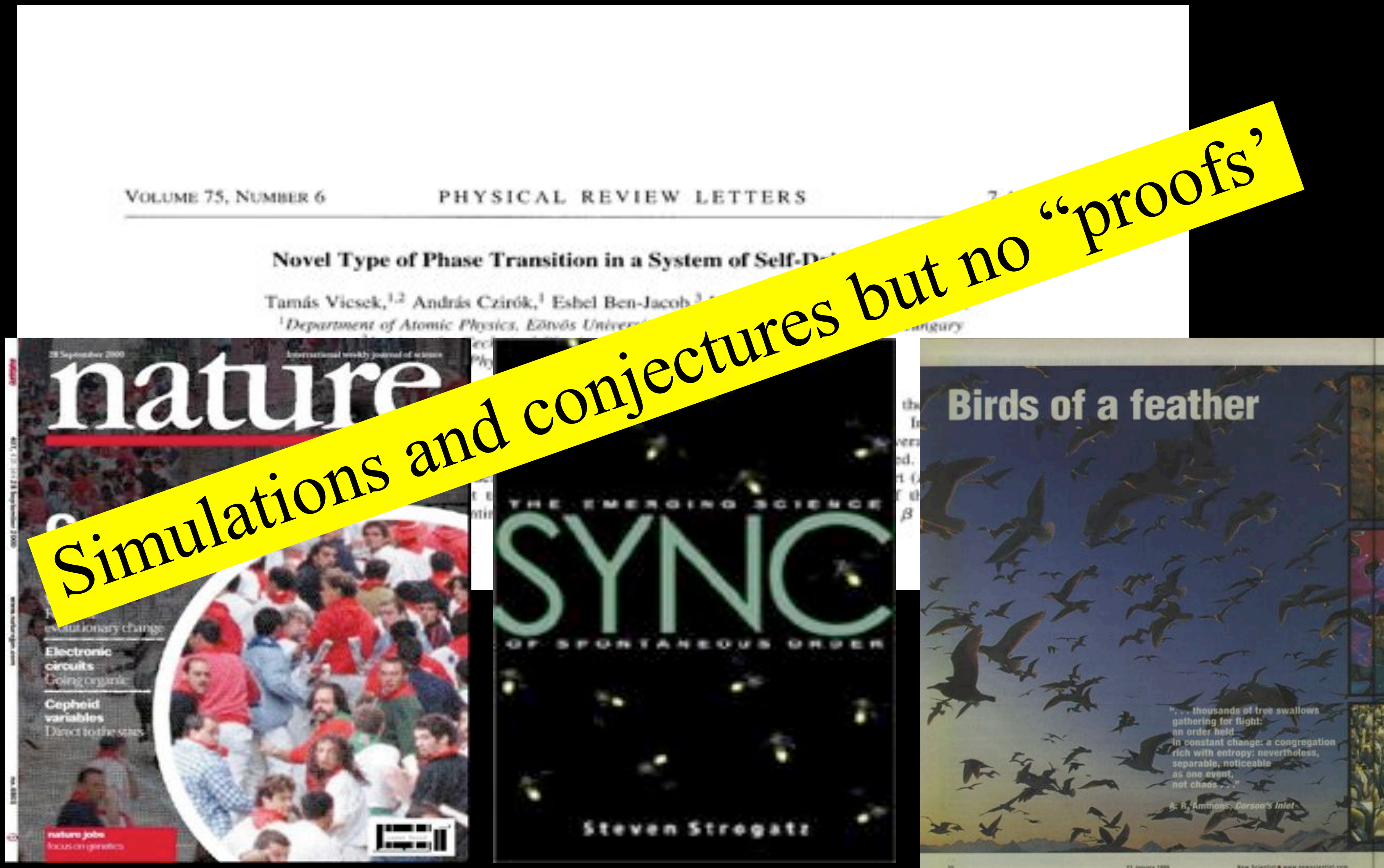
Networked dynamical systems




Nonlinear/uncertain
hybrid/stochastic etc.
Complex
networked
systems
Complexity
of
dynamics
Flocking/synchronization
consensus
Multi-agent
systems
Complexity
of interconnection

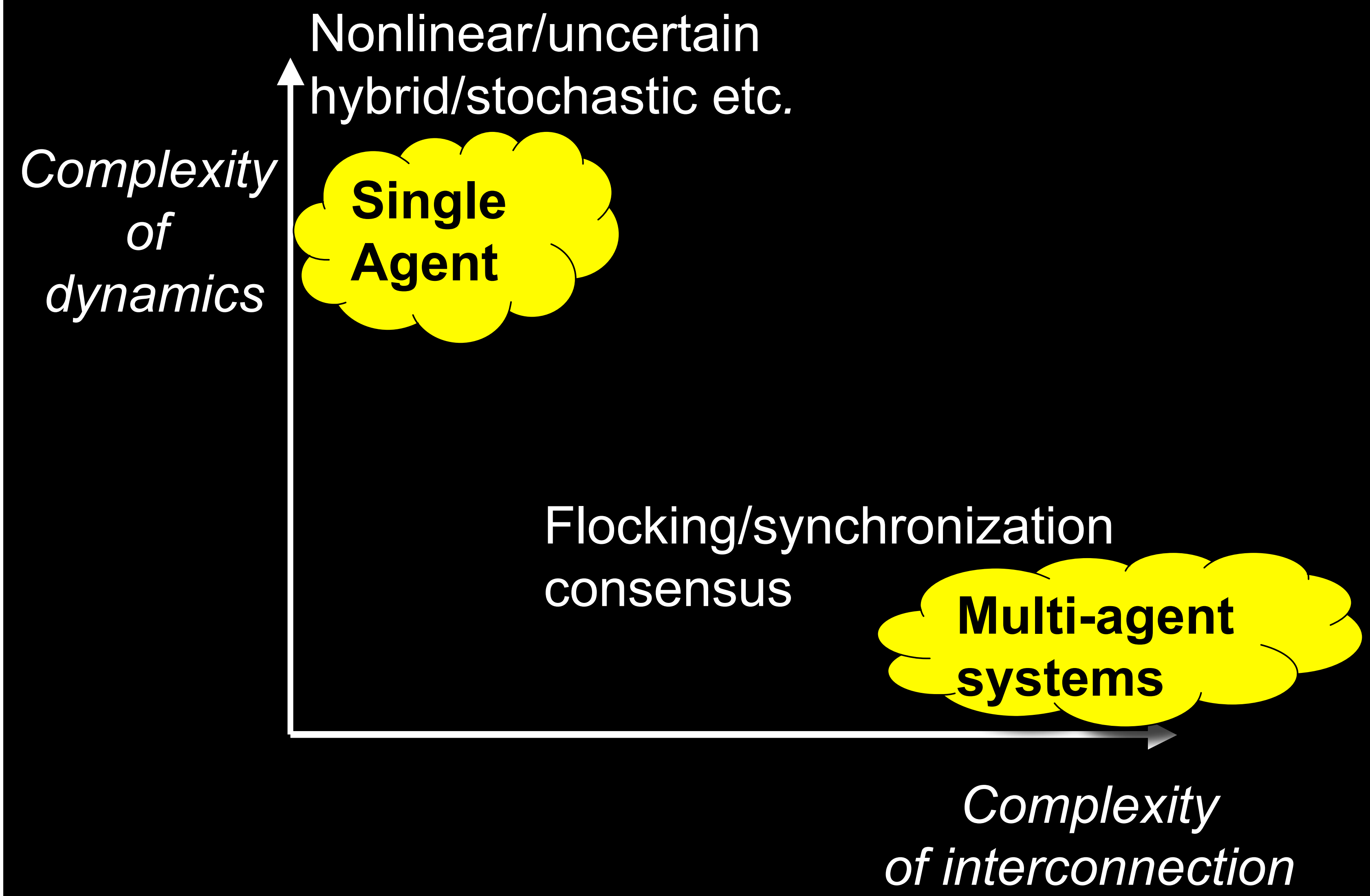

Working systems but no “proofs’

Statistical Physics and emergence of collective behavior

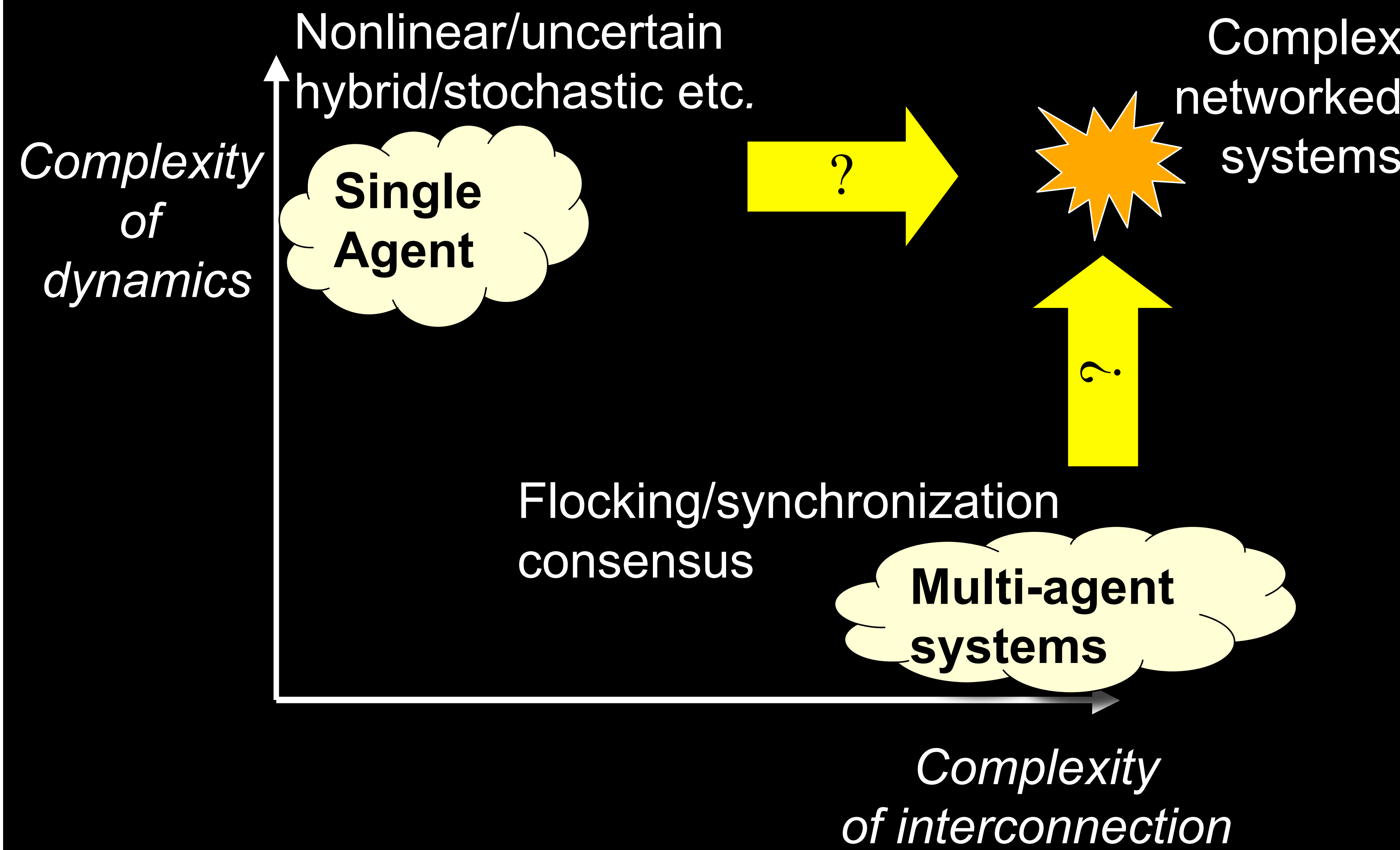


Simulations and conjectures but no “proofs’

Spectacular progress



Open questions



Pursuing the hope

- Hard limits (Tues)
- Short proofs (Wed)
- Small models (Thu)

- Architecture

Pursuing the hope

- **Hard limits (Tues)**
- Short proofs (Wed)
- Small models (Thu)

- Architecture

Pursuing the hope

- Hard limits
- Short proofs
- Small models

- Architecture

- **Robustness**

Robustness **=**
Invariance of a [property] of a [system] to a [set of perturbations]

Fragility **=**
Variability of a [property] of a [system] due to a [set of perturbations]

- Both can be quantitative or qualitative
- Robustness and fragility coexist in the same system
- Are there conservation laws?

Robustness = Invariance of a [property] of a [system] to a [set of perturbations]

Fragility = Variability of a [property] of a [system] due to a [set of perturbations]

Qualitative

Robustness/fragility = (In)variance of a [property] of a [system] to a [set of perturbations]

A [property] of a [system] can be ***robust*** to one [set of perturbations] ***yet fragile*** to another [set of perturbations]

Given a [system] and a [set of perturbations]: One [property] can be ***robust while another*** [property] ***is fragile***.

Hard limits and tradeoffs

On systems and their components

- Thermodynamics (Carnot)
- Communications (Shannon)
- Control (Bode)
- Computation (Turing/Gödel)

- Fragmented and incompatible
- We need a more integrated view and have the beginnings

Hard limits and tradeoffs

On systems and their components

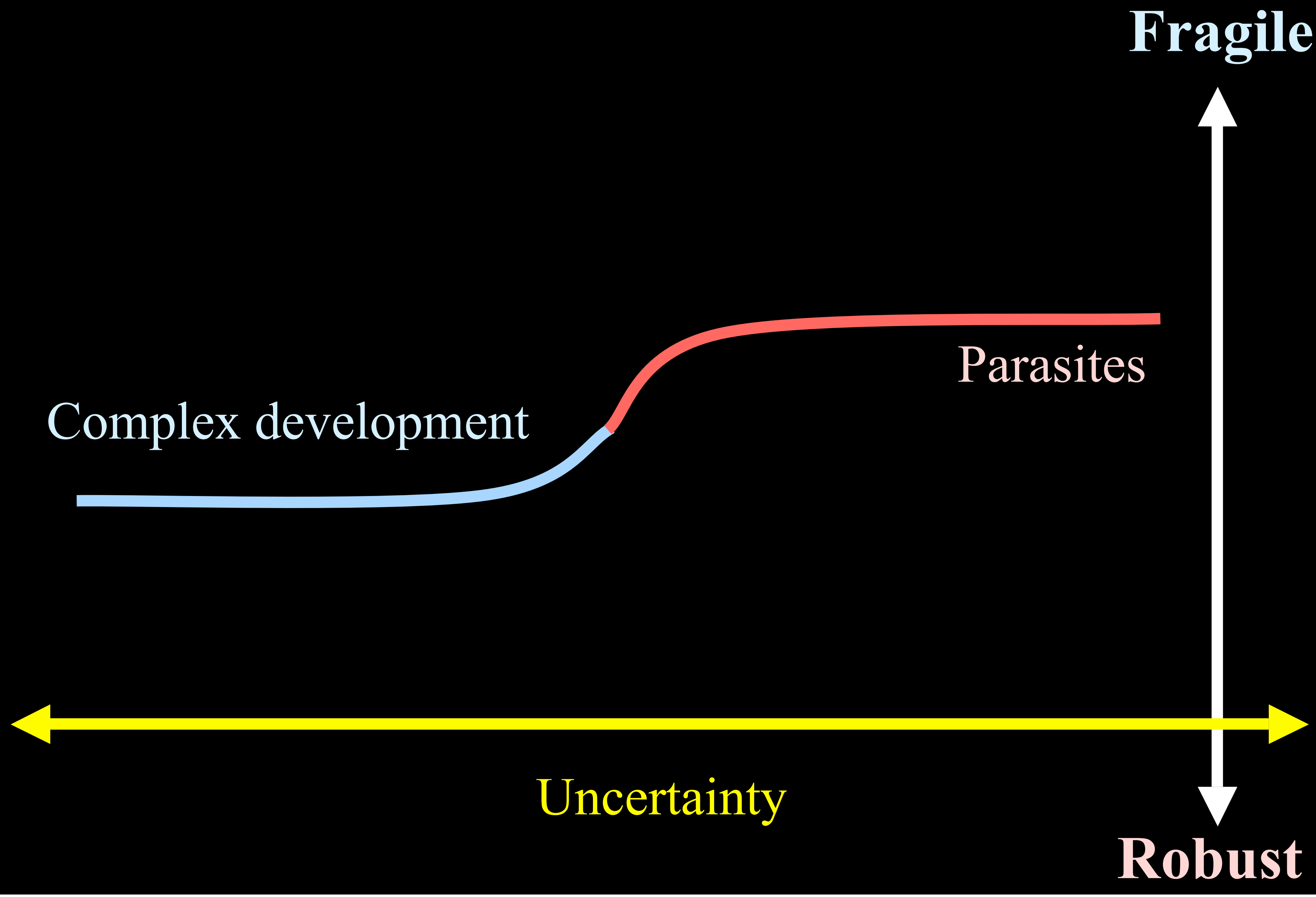
- Thermodynamics (Carnot)
- Communications (Shannon)
- **Control (Bode)**
- Computation (Turing/Gödel)

- Fragmented and incompatible
- We need a more integrated view and have the beginnings

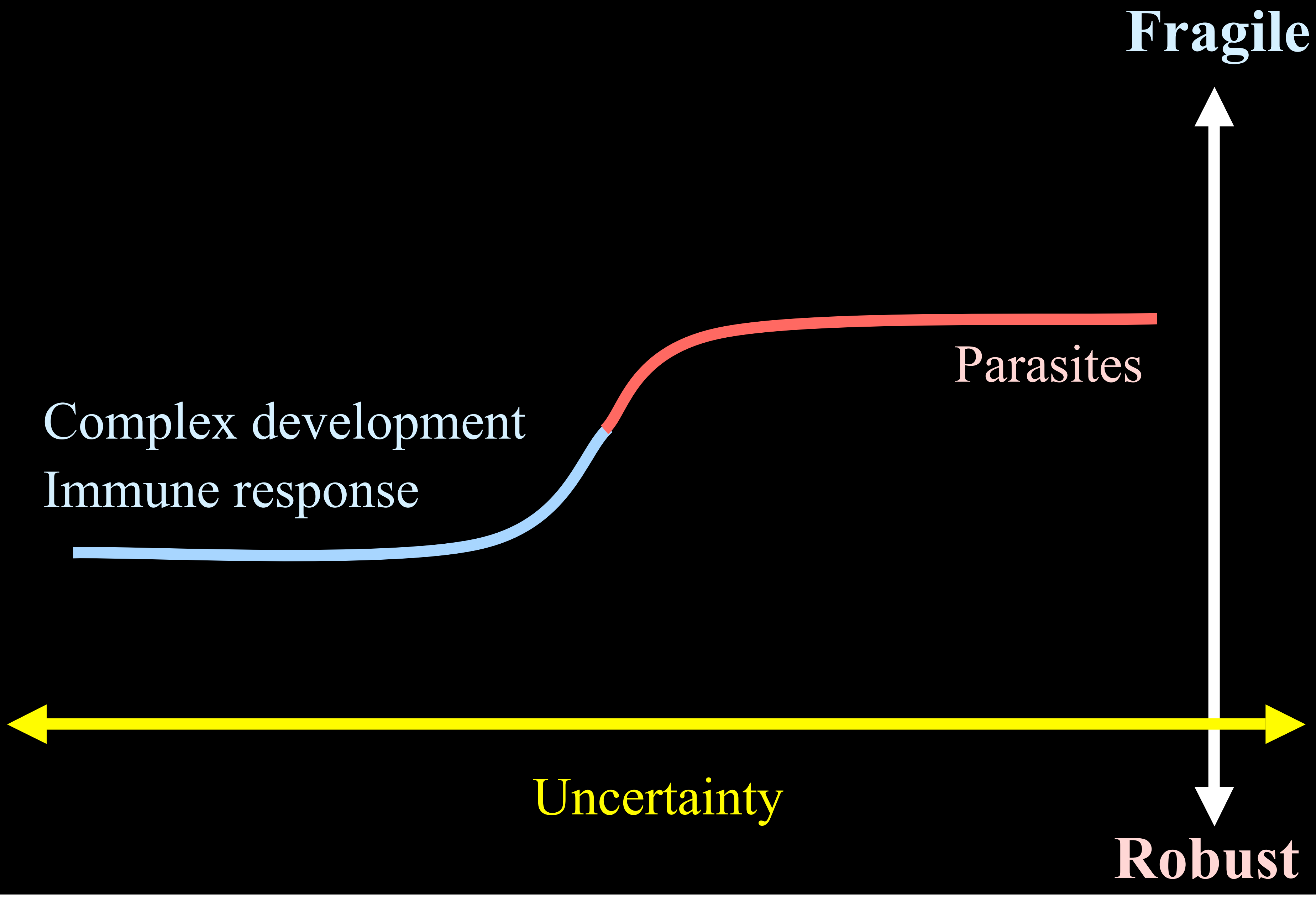
Robust	Yet Fragile
1. Efficient, flexible metabolism	1. Obesity and diabetes
2. Complex development	2. Rich parasite ecosystem
3. Immune systems	3. Auto-immune disease
4. Regeneration & renewal	4. Cancer
5. Complex societies	5. Epidemics, war, genocide, …

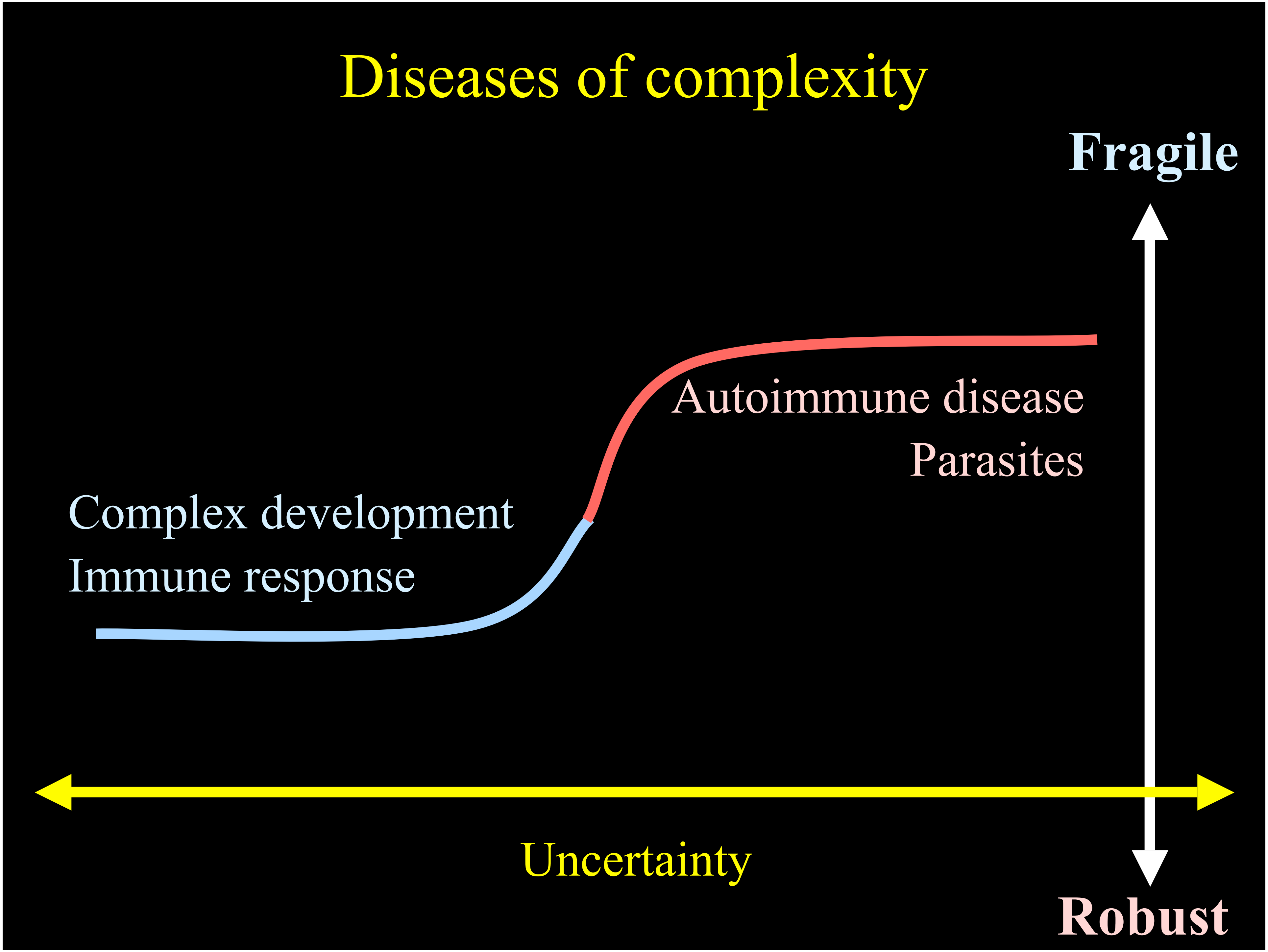
Human robustness and fragility

Diseases of complexity

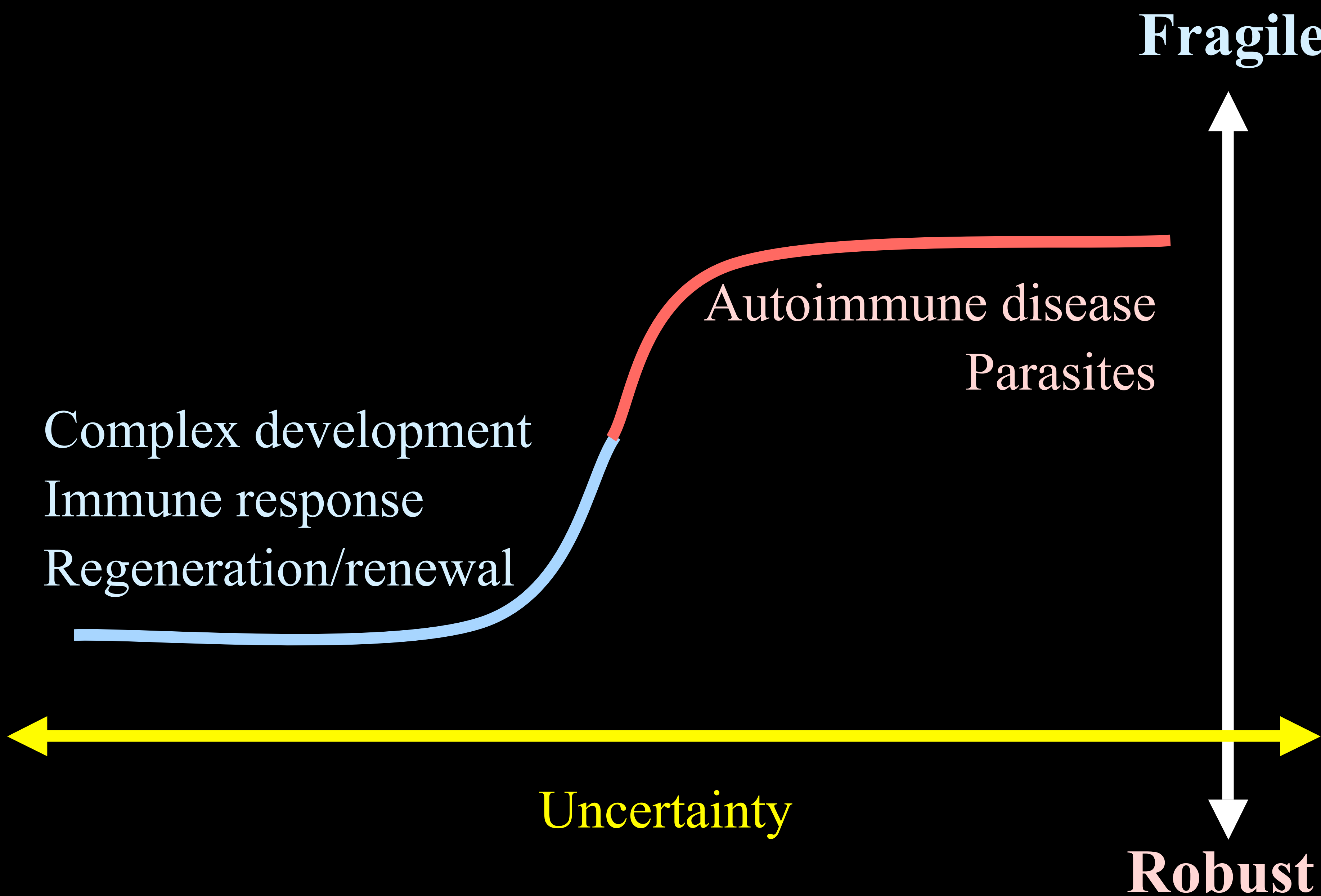


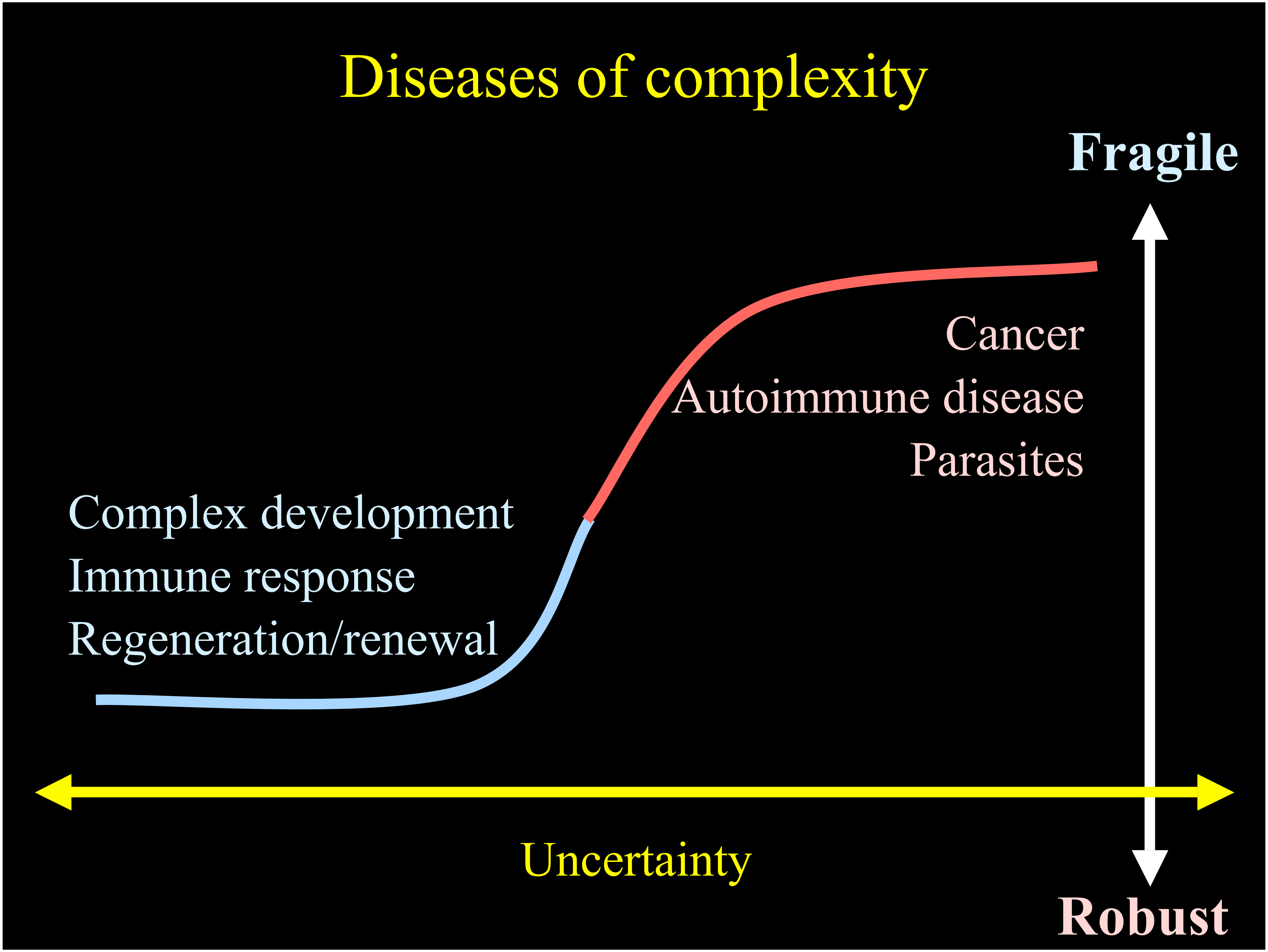
Diseases of complexity

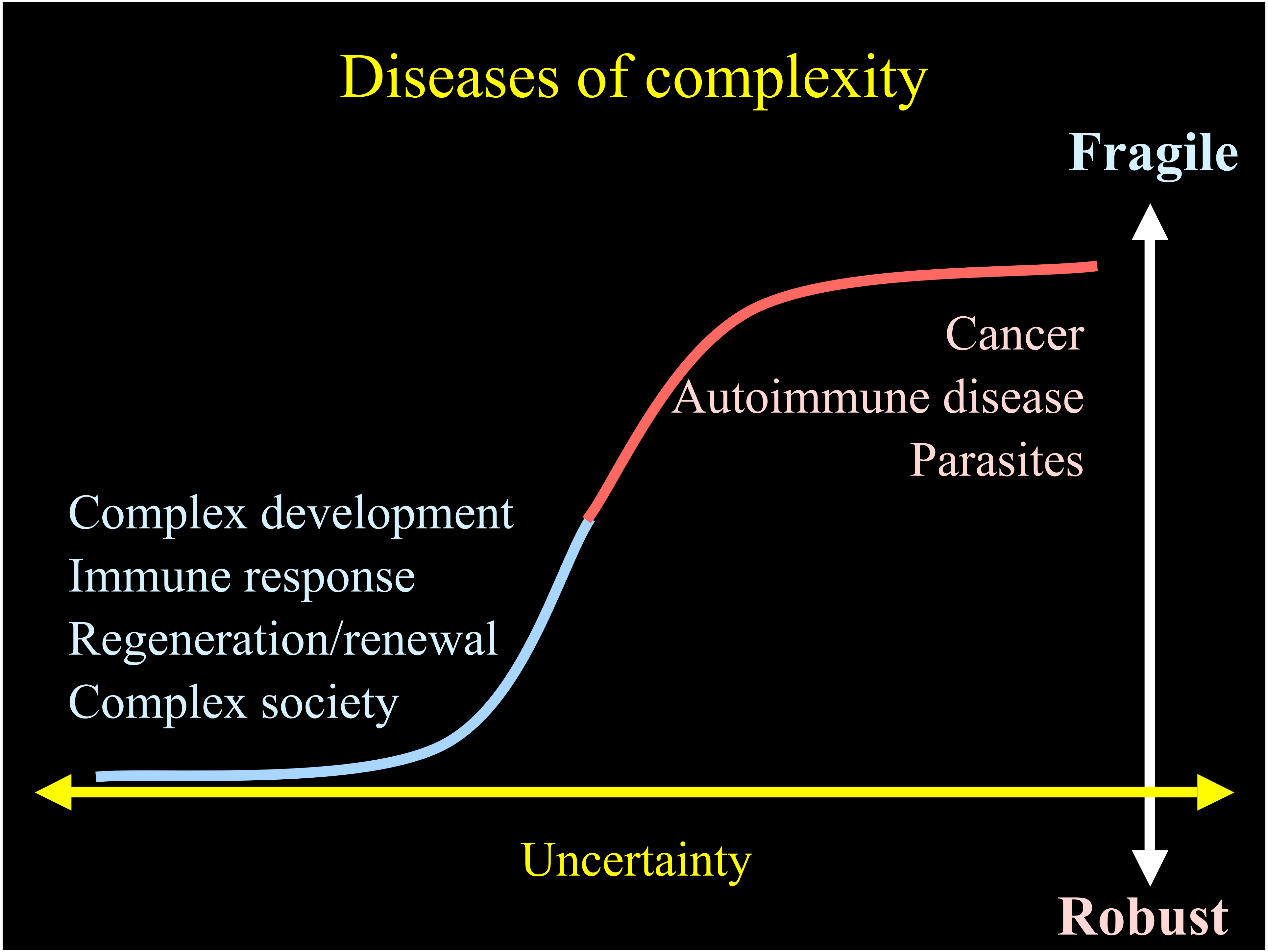


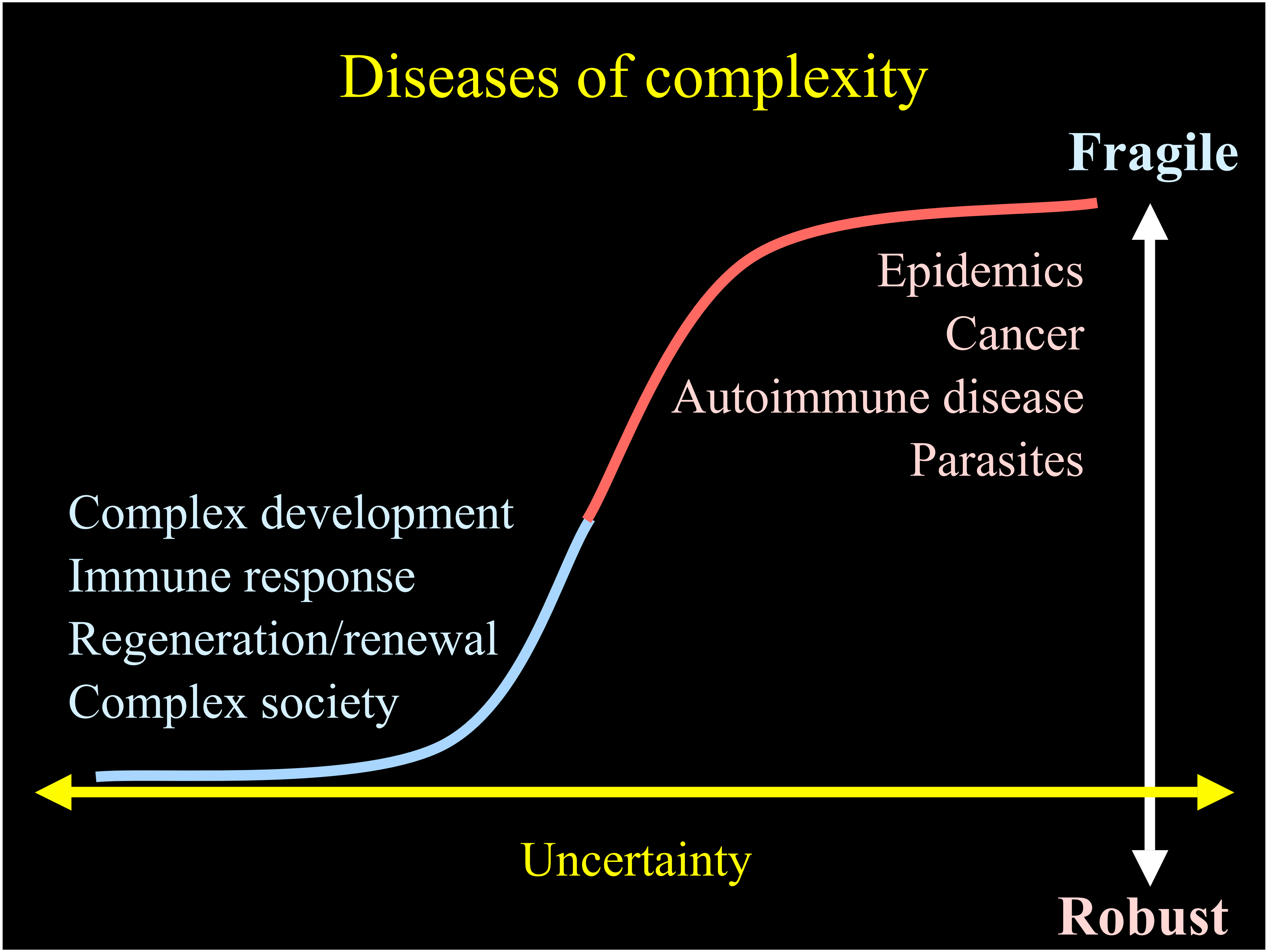

Diseases of complexity
Fragile
Autoimmune disease
Parasites
Complex development
Immune response
Uncertainty
Robust

Diseases of complexity

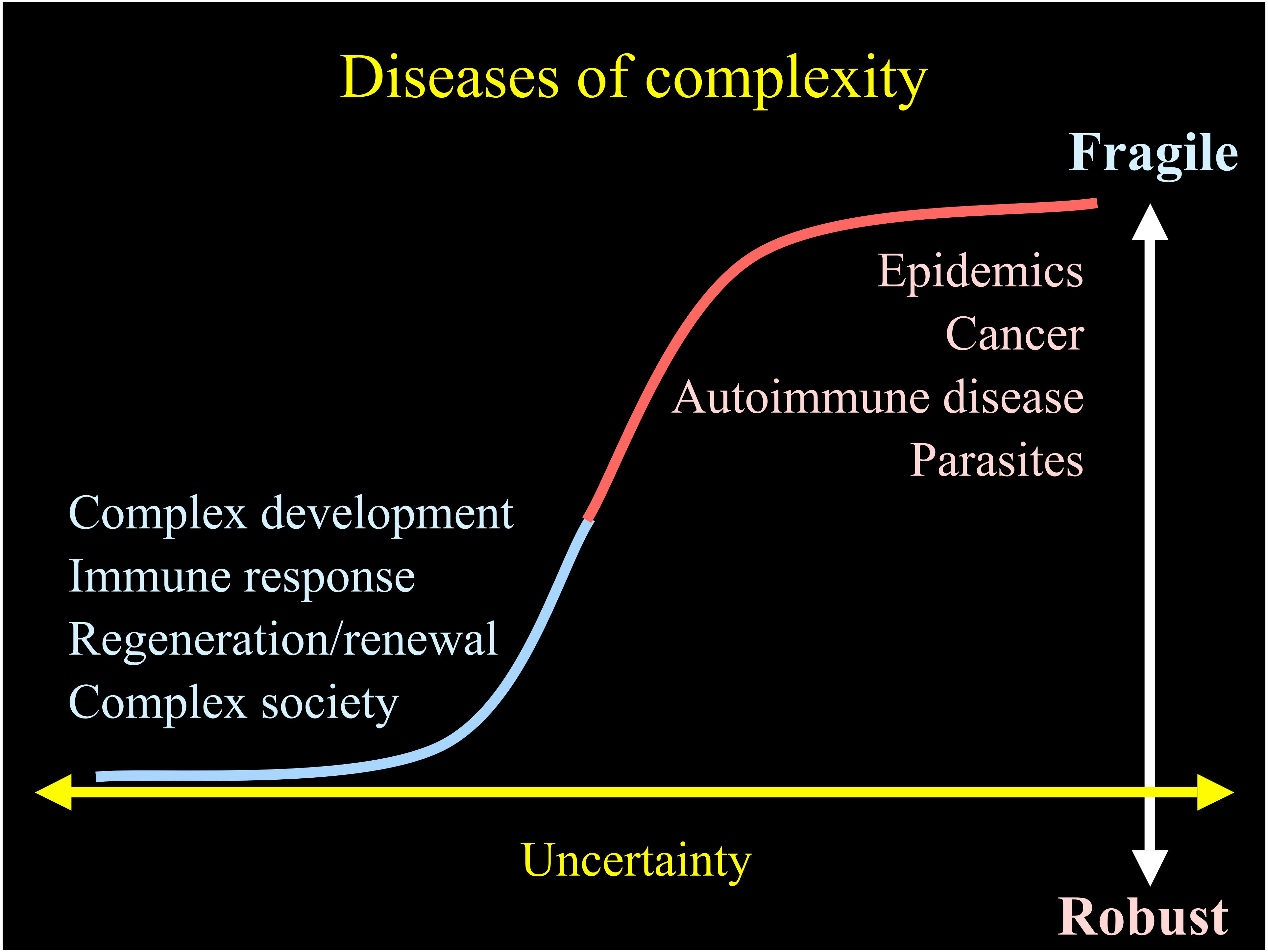


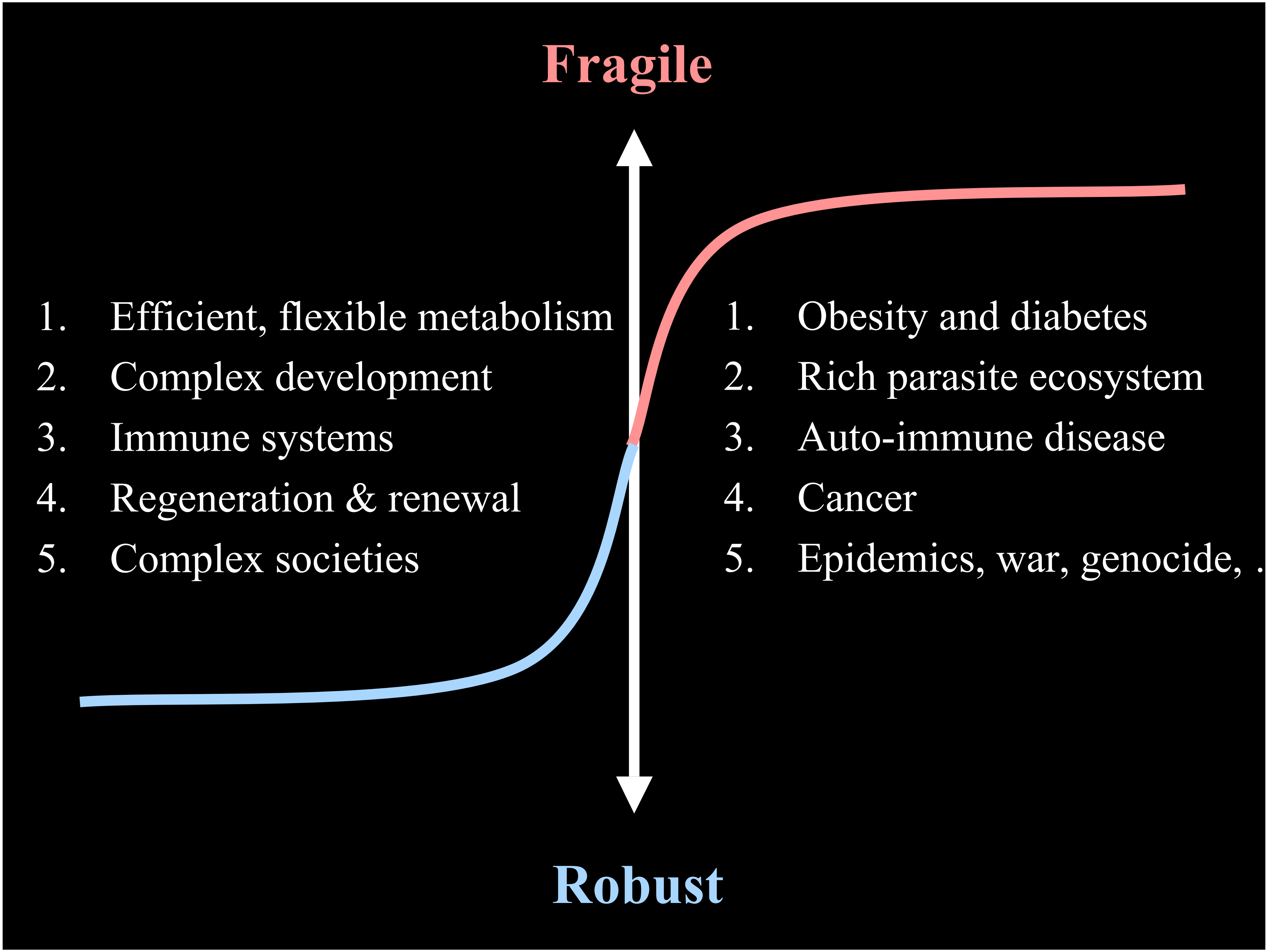

Diseases of complexity
Fragile
Cancer
Autoimmune disease
Parasites
Complex development
Immune response
Regeneration/renewal
Uncertainty
Robust

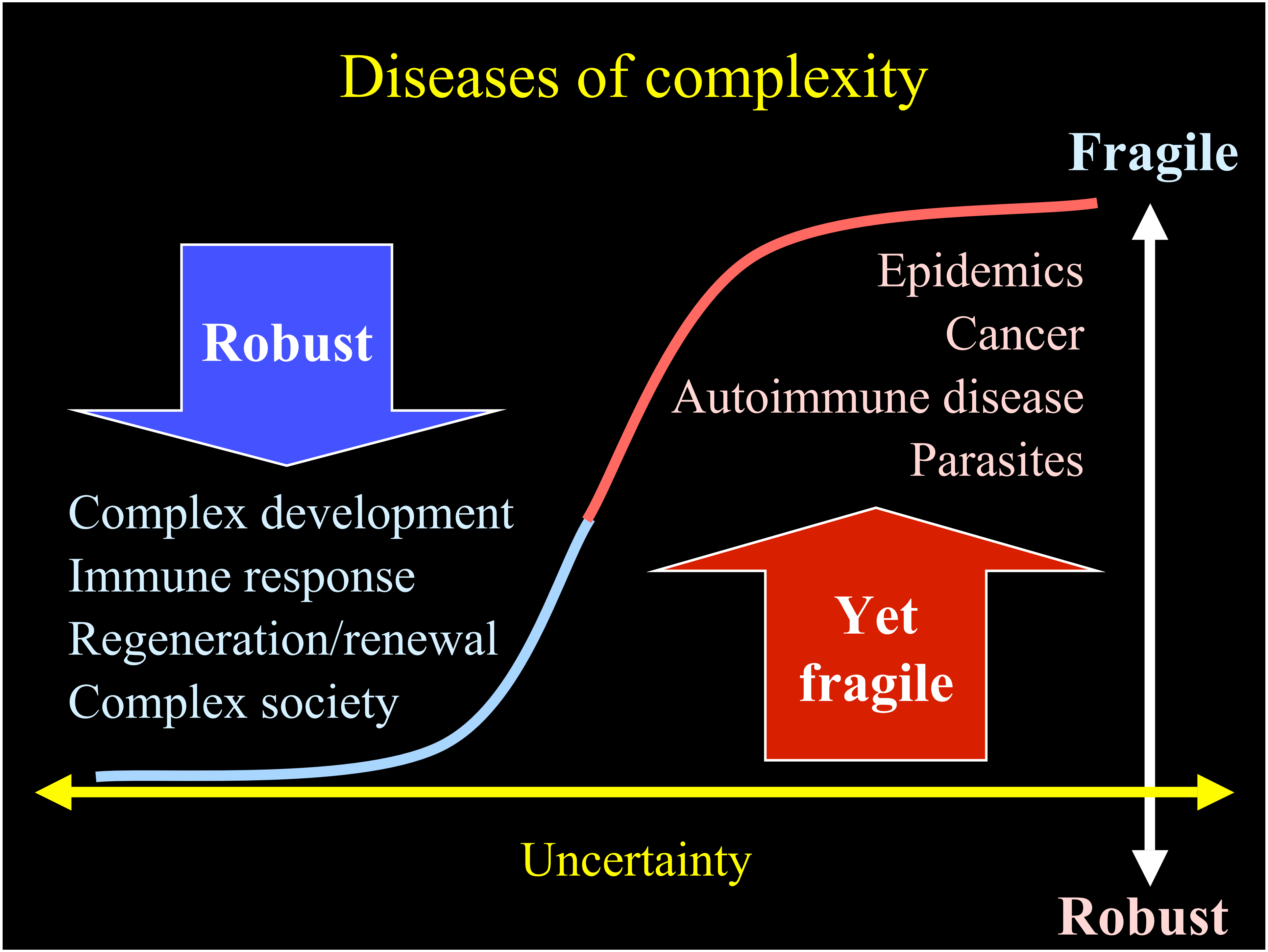

Diseases of complexity
Fragile
Cancer
Autoimmune disease
Parasites
Complex development
Immune response
Regeneration/renewal
Complex society
Uncertainty
Robust

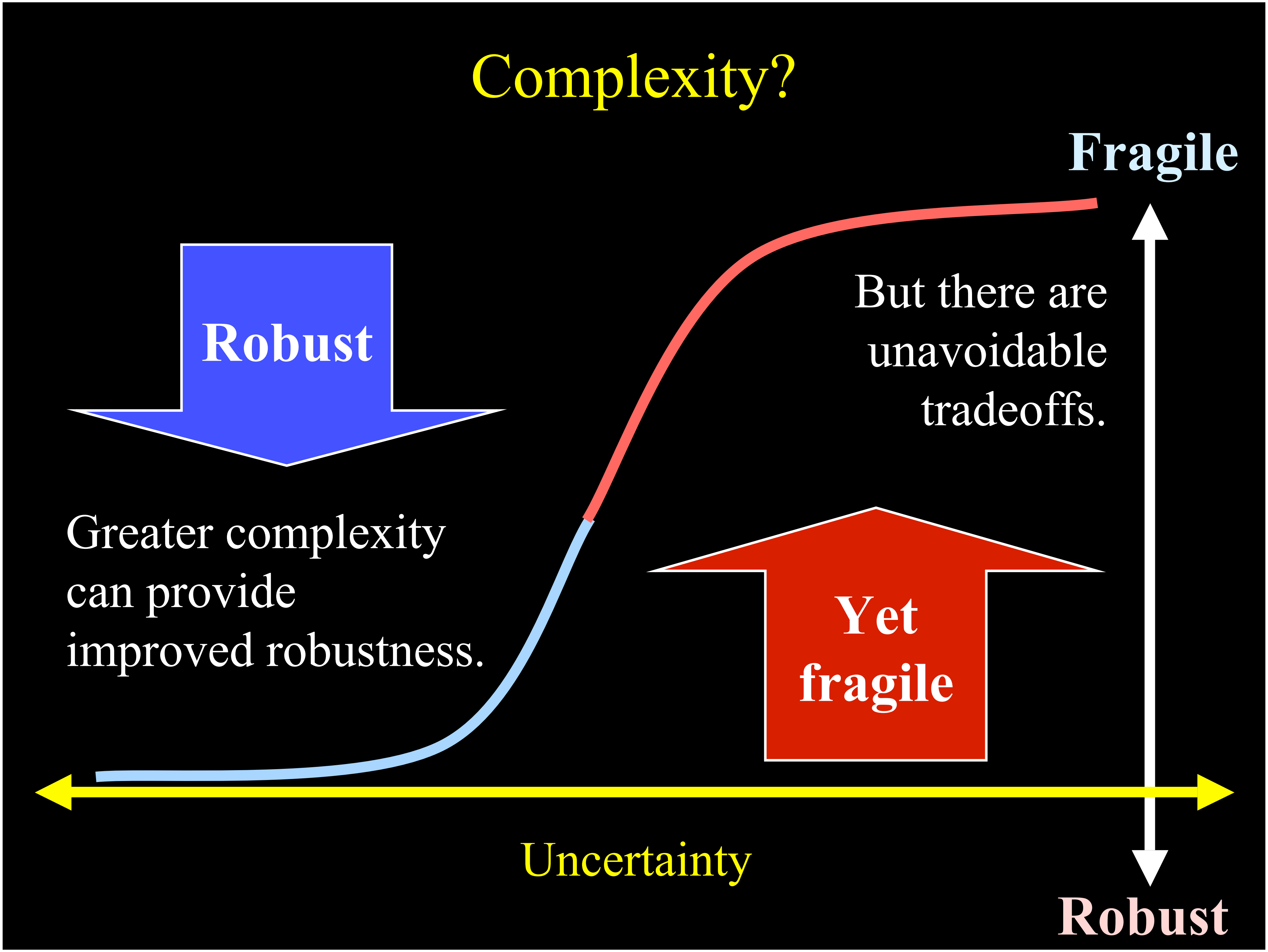


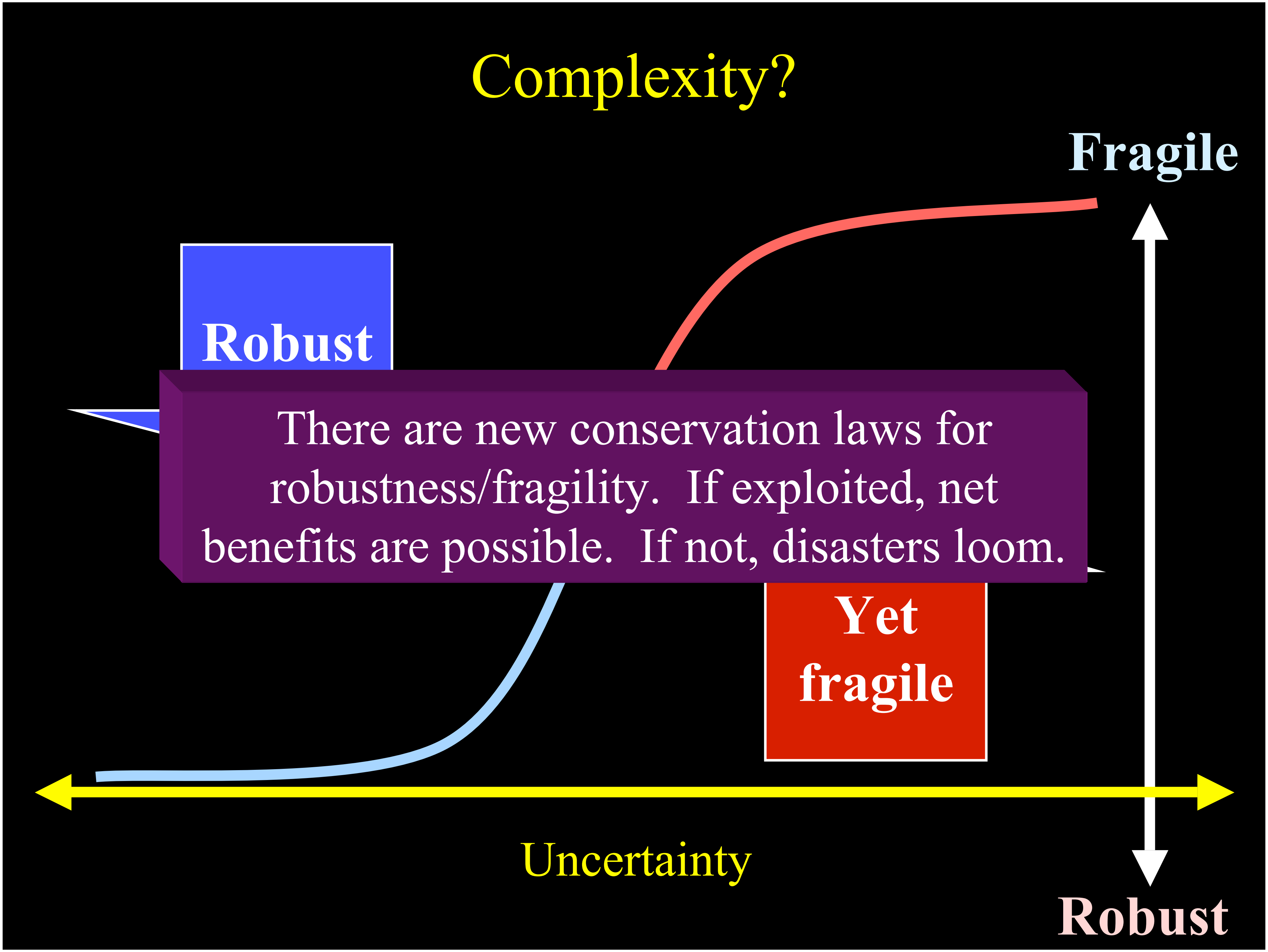
Diseases of complexity
Fragile
Epidemics
Cancer
Autoimmune disease
Parasites
Complex development
Immune response
Regeneration/renewal
Complex society
Uncertainty
Robust


Diseases of complexity
Fragile
Epidemics
Cancer
Autoimmune disease
Parasites
Complex development
Immune response
Regeneration/renewal
Complex society
Uncertainty
Robust

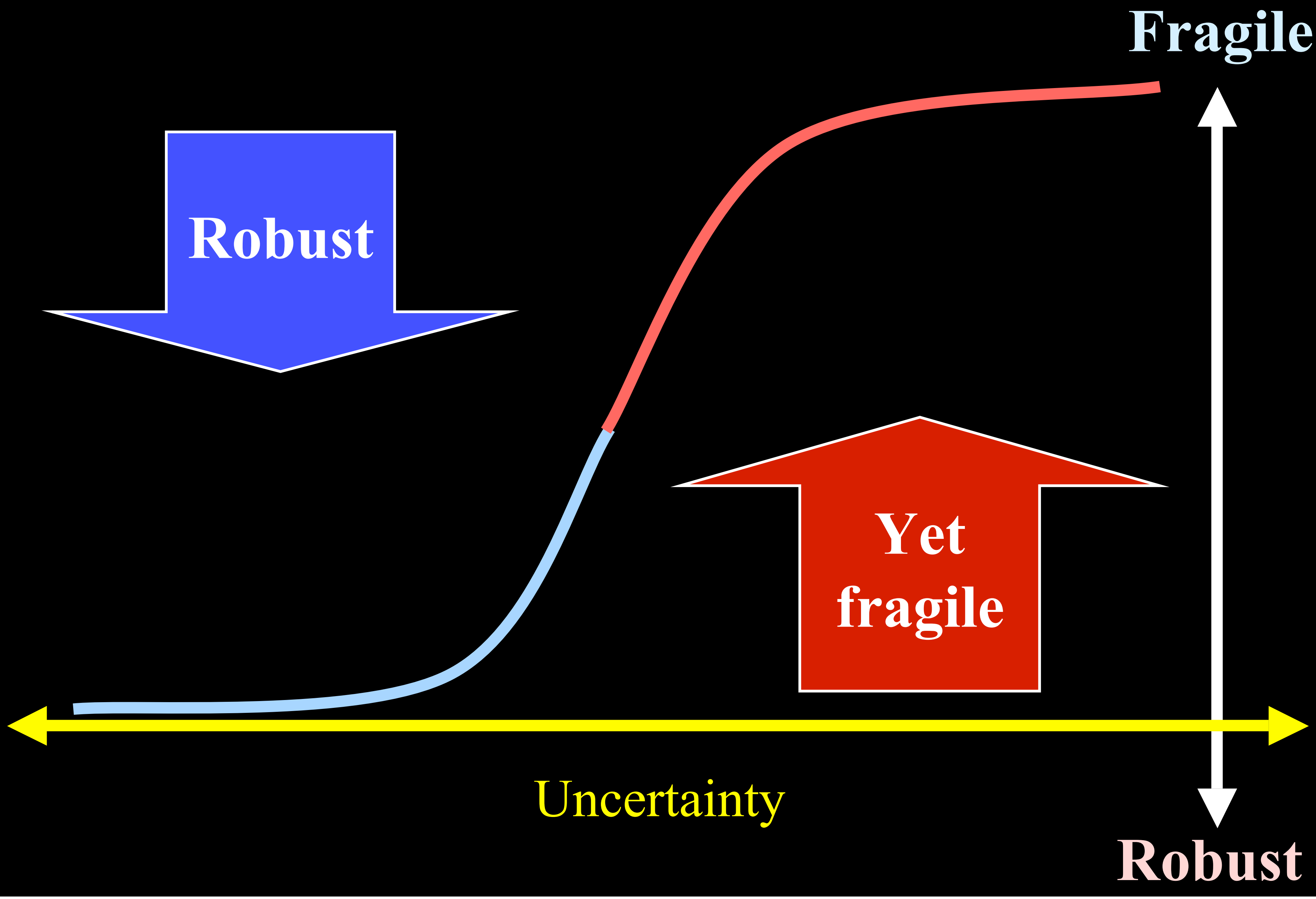

Fragile
1. Efficient, flexible metabolism
2. Complex development
3. Immune systems
4. Regeneration & renewal
5. Complex societies
1. Obesity and diabetes
2. Rich parasite ecosystem
3. Auto-immune disease
4. Cancer
5. Epidemics, war, genocide, .
Robust


Diseases of complexity
Fragile
Robust
Complex development
Immune response
Regeneration/renewal
Complex society
Epidemics
Cancer
Autoimmune disease
Parasites
Yet fragile
Uncertainty
Robust


Complexity?
Fragile
Robust
But there are unavoidable tradeoffs.
Greater complexity can provide improved robustness.
Yet fragile
Uncertainty
Robust


Complexity?
Fragile
Robust
There are new conservation laws for robustness/fragility. If exploited, net benefits are possible. If not, disasters loom.
Yet fragile
Uncertainty
Robust

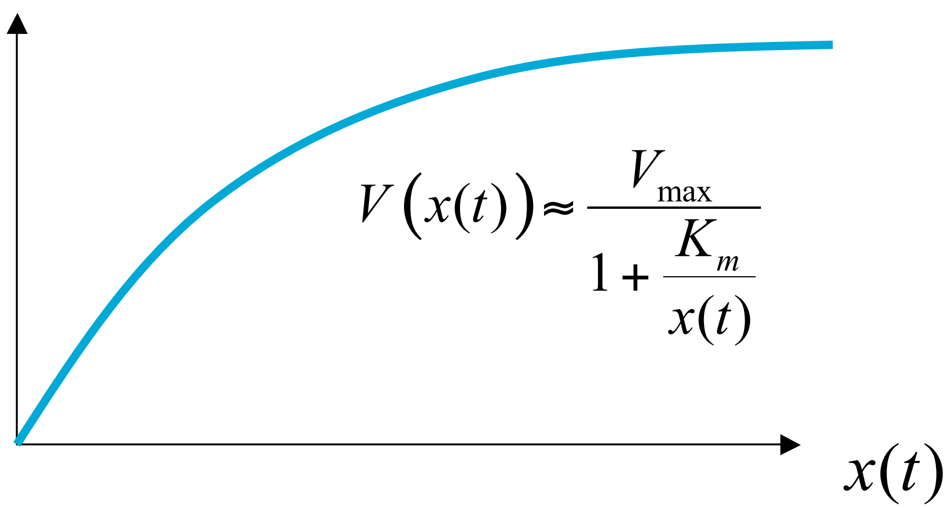
Quantitative?

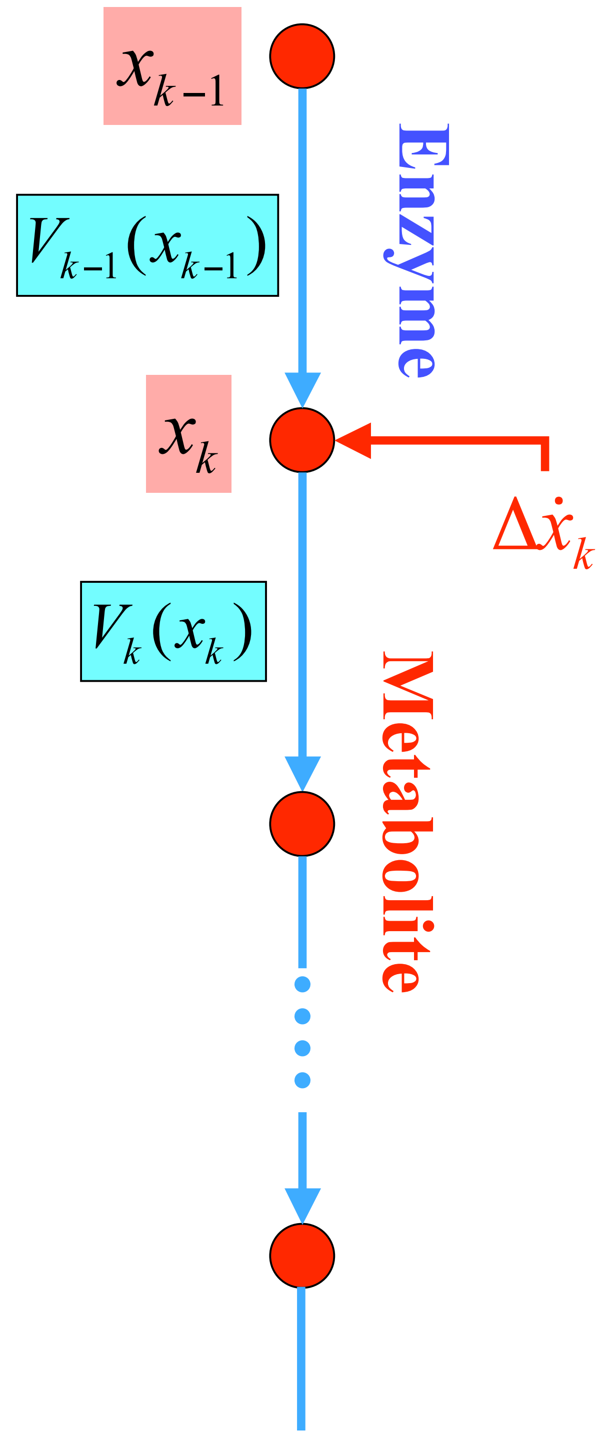


Thermodynamics and metabolism

$$\dot{x}_k = -V_k(x_k) + V_{k-1}(x_{k-1}) + \Delta\dot{x}_k$$

$$V\left(x(t)\right) \approx \frac{V_{\max}}{1 + \dfrac{K_m}{x(t)}}$$





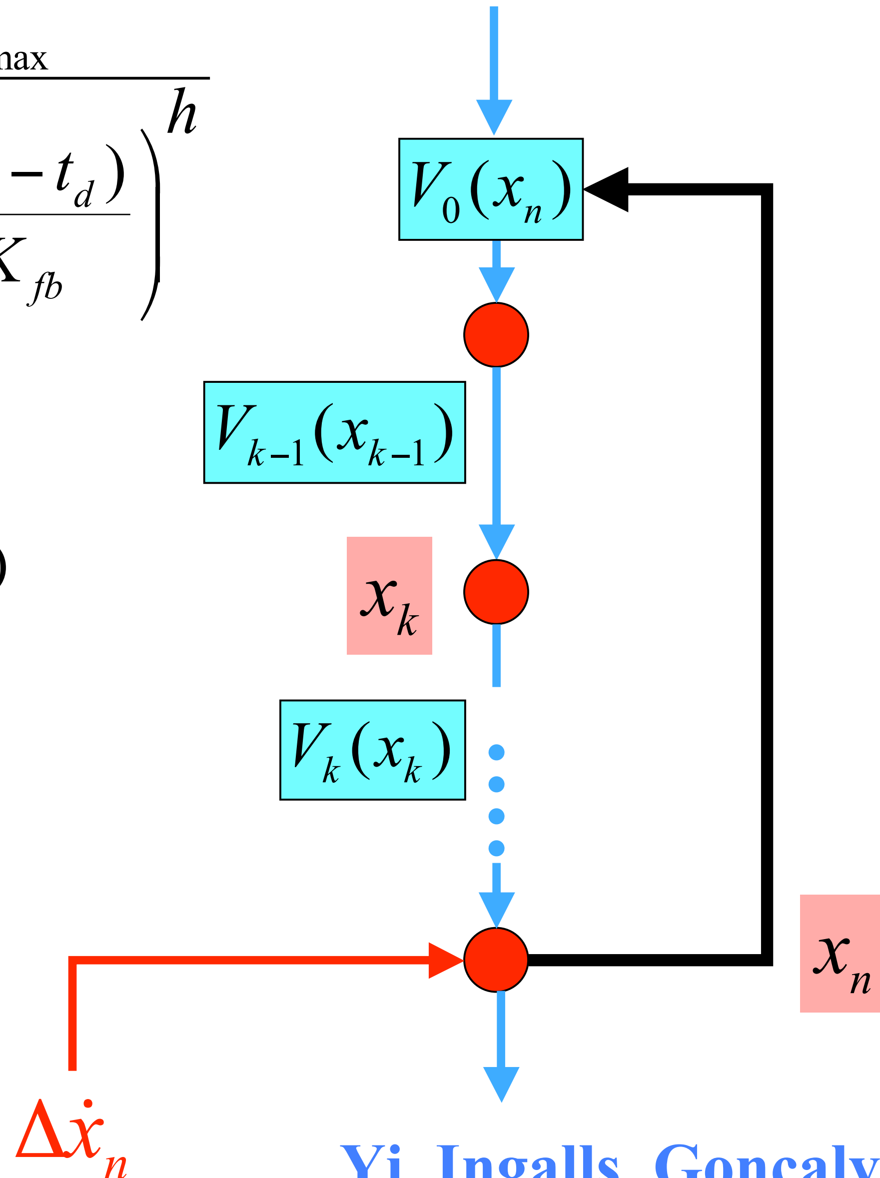
Product inhibition

$$\dot{x}_1 = V_0(x_n) - V_1(x_1)$$

$$V_0\big(x(t)\big) \approx \frac{V_{\max}}{1 + \left(\dfrac{x(t-t_d)}{K_{fb}}\right)^h}$$

$$\dot{x}_k = -V_k(x_k) + V_{k-1}(x_{k-1})$$

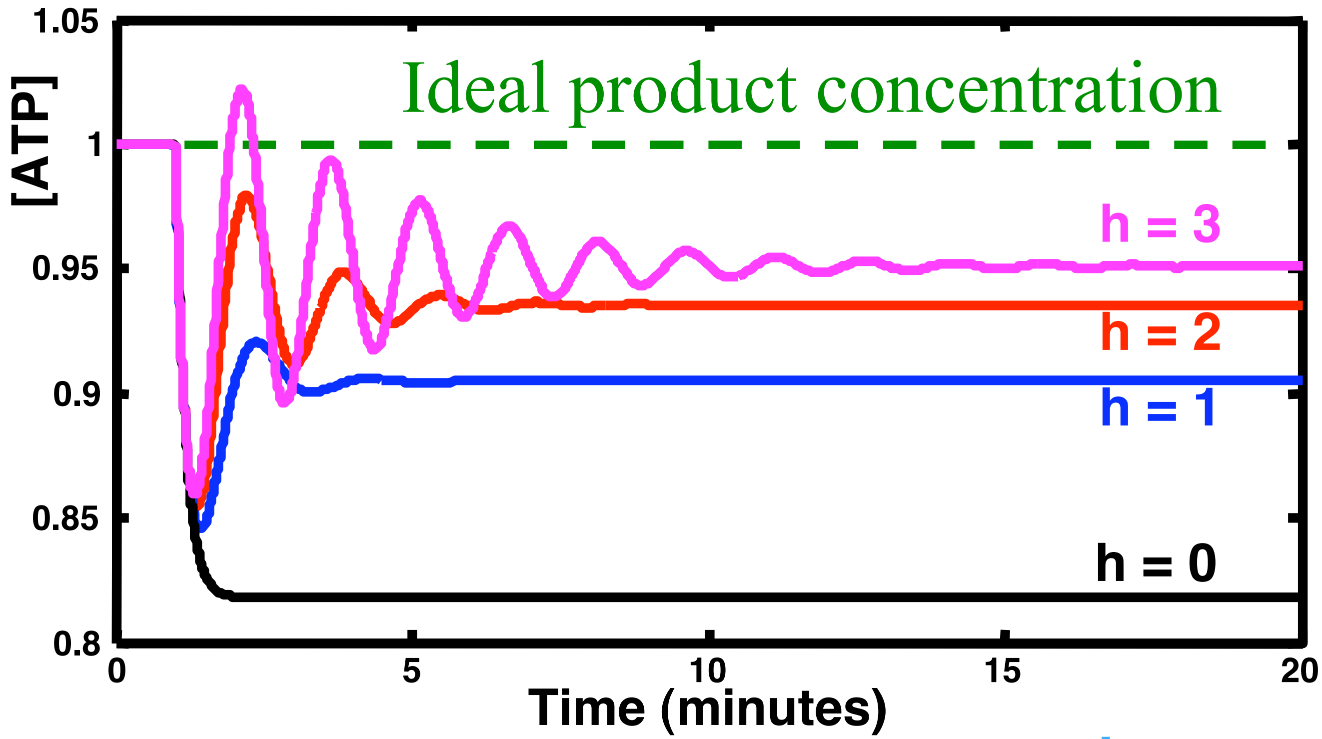
$$V\big(x(t)\big) \approx \frac{V_{\max}}{1 + \dfrac{K_m}{x(t)}}$$



perturbation

Yi, Ingalls, Goncalves, Sauro

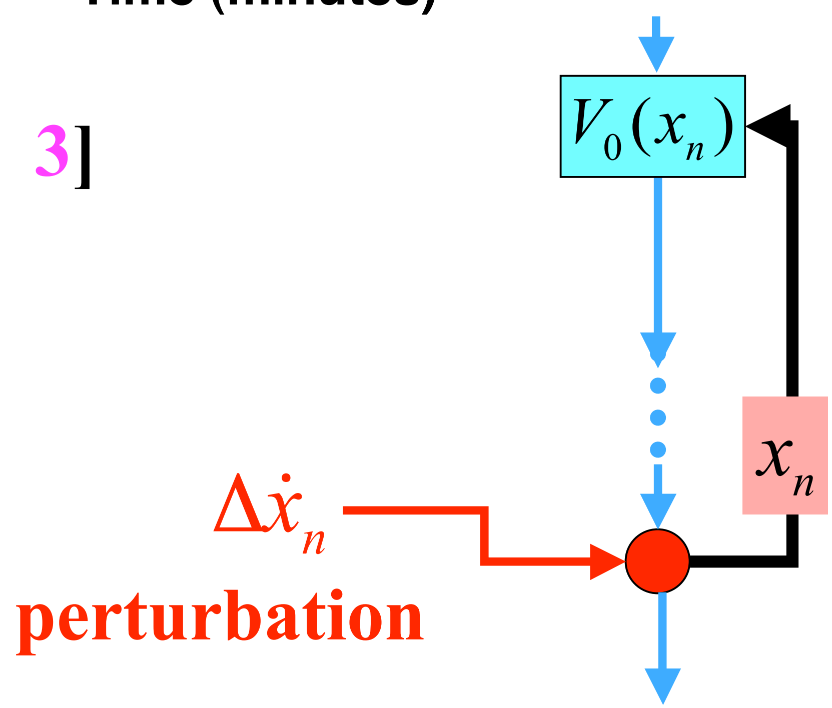
Step increase in demand for product or “ATP.”

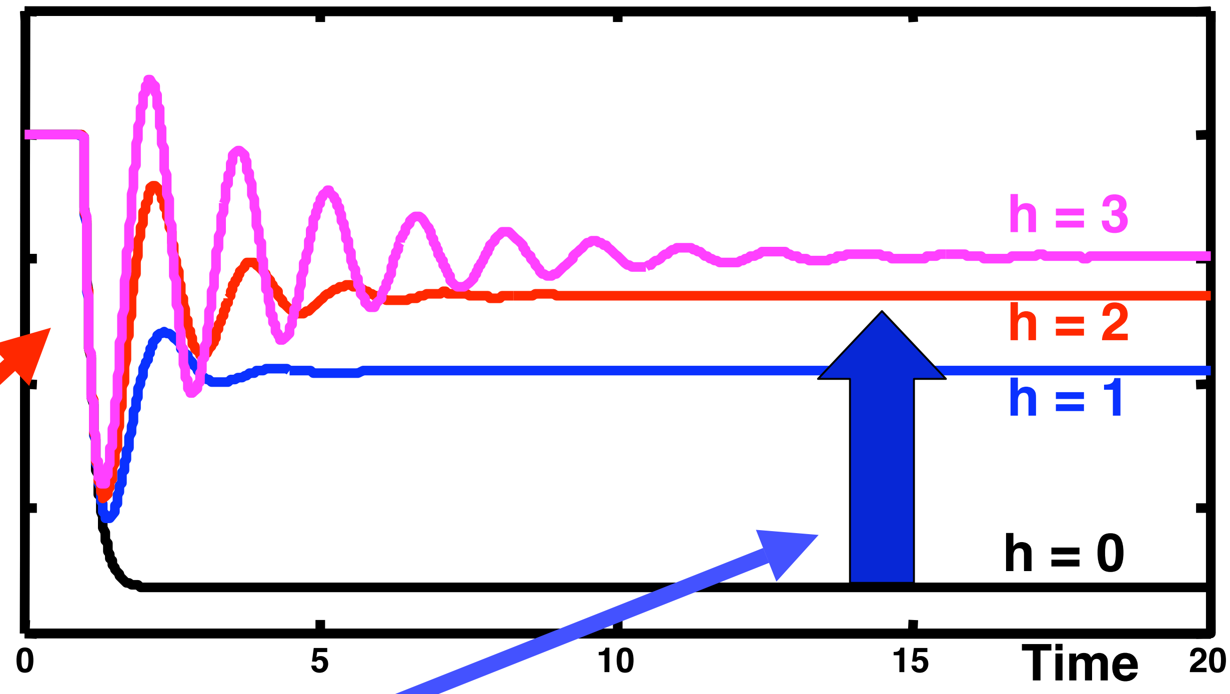


h = [0 1 2 3]

$$\dot{x}_1 = -V_1(x_1) + V_0(x_n)$$

$$V_0\big(x(t)\big) \approx \frac{V_{\max}}{1 + \left(\dfrac{x(t - t_d)}{K_{fb}}\right)^{h}}$$



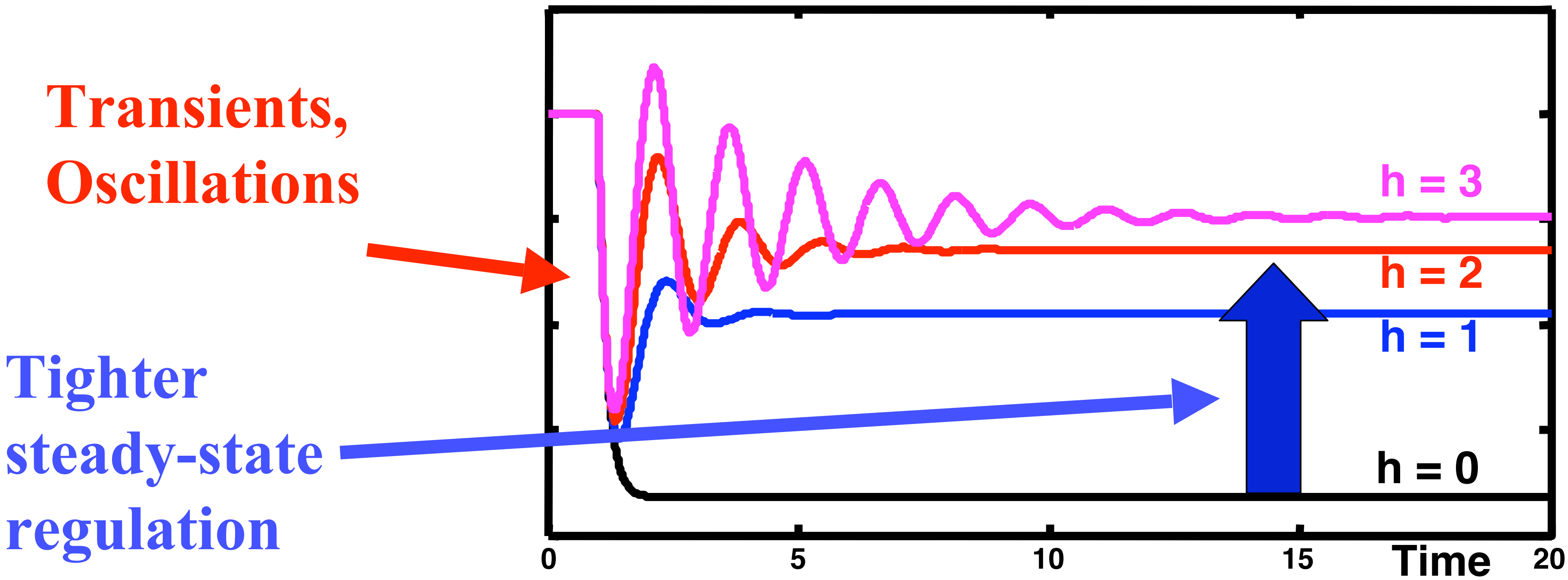


Transients, Oscillations

Tighter steady-state regulation

Higher feedback "gain"

It is well-known that many biological regulatory networks can oscillate, and presumably many more will be discovered

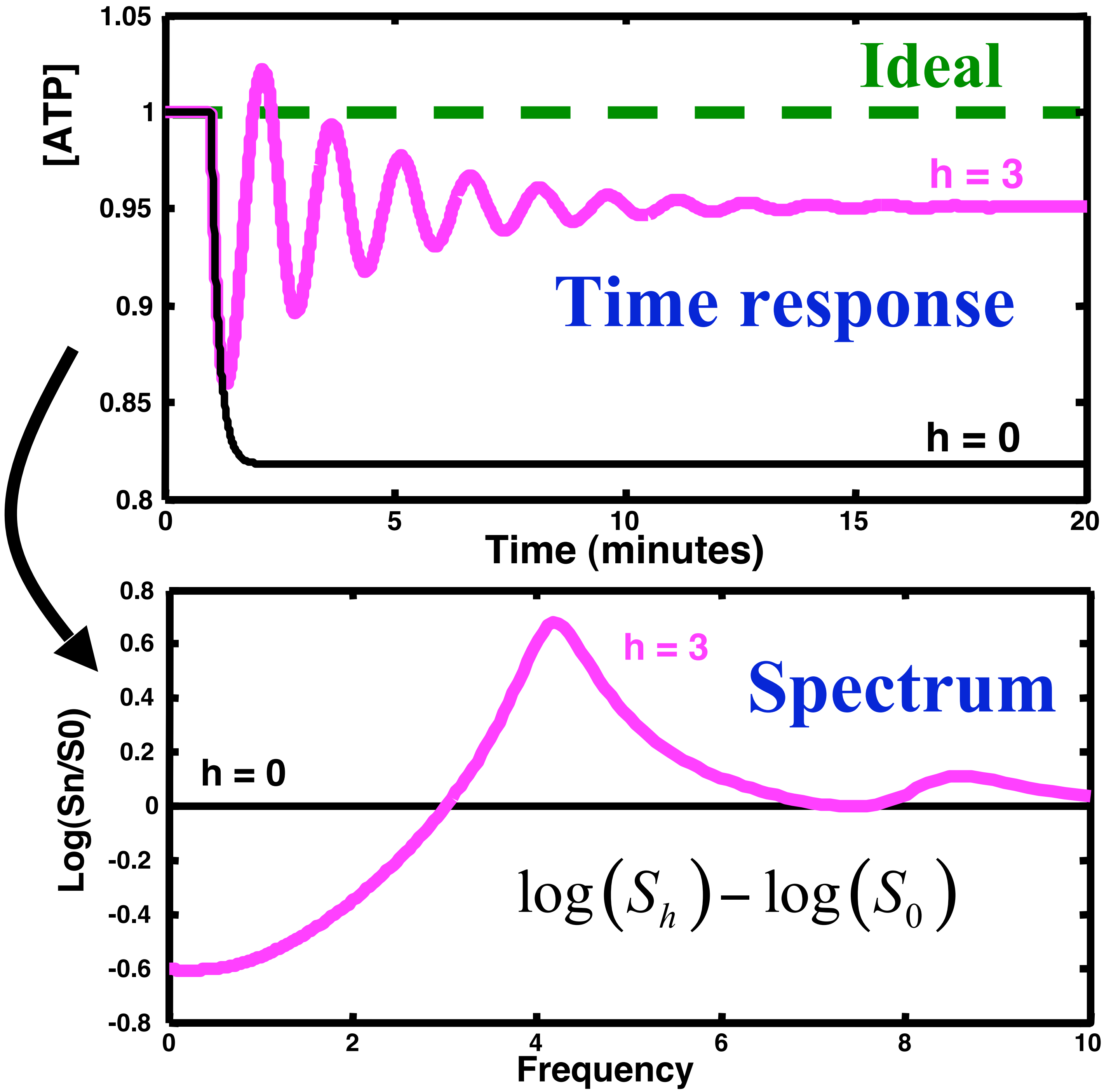


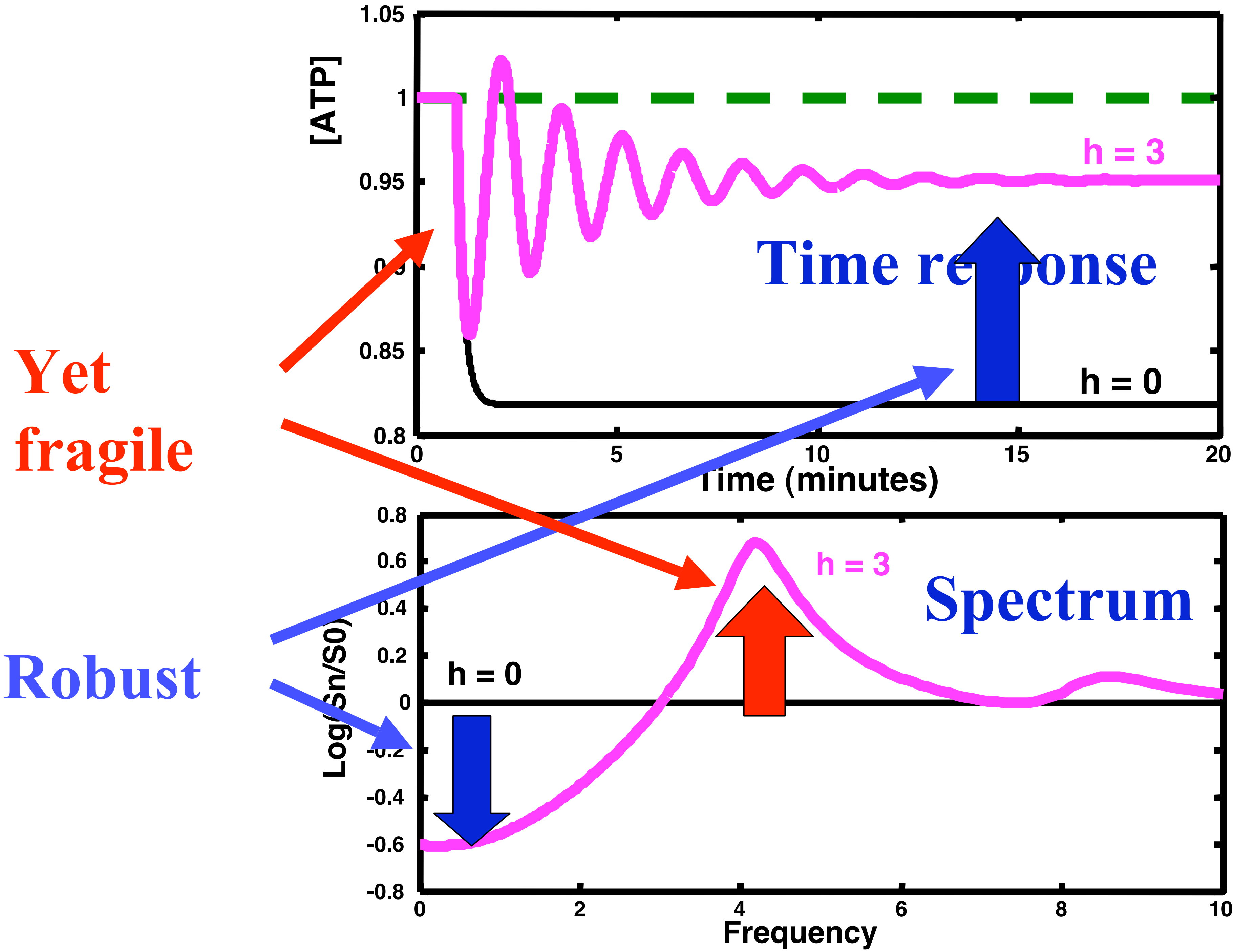
- Are these tradeoffs an artifice of this model?
- Does it matter if the model is nonlinear, stochastic, distributed, PDEs, etc? Does it depend on the model at all?
- Are these tradeoffs due to a frozen accident of evolution and not an absolute necessity?

- The answer to all these questions is ***no***.

$$S_h = \mathrm{F}(x)\big|_h$$

Fourier Transform of error

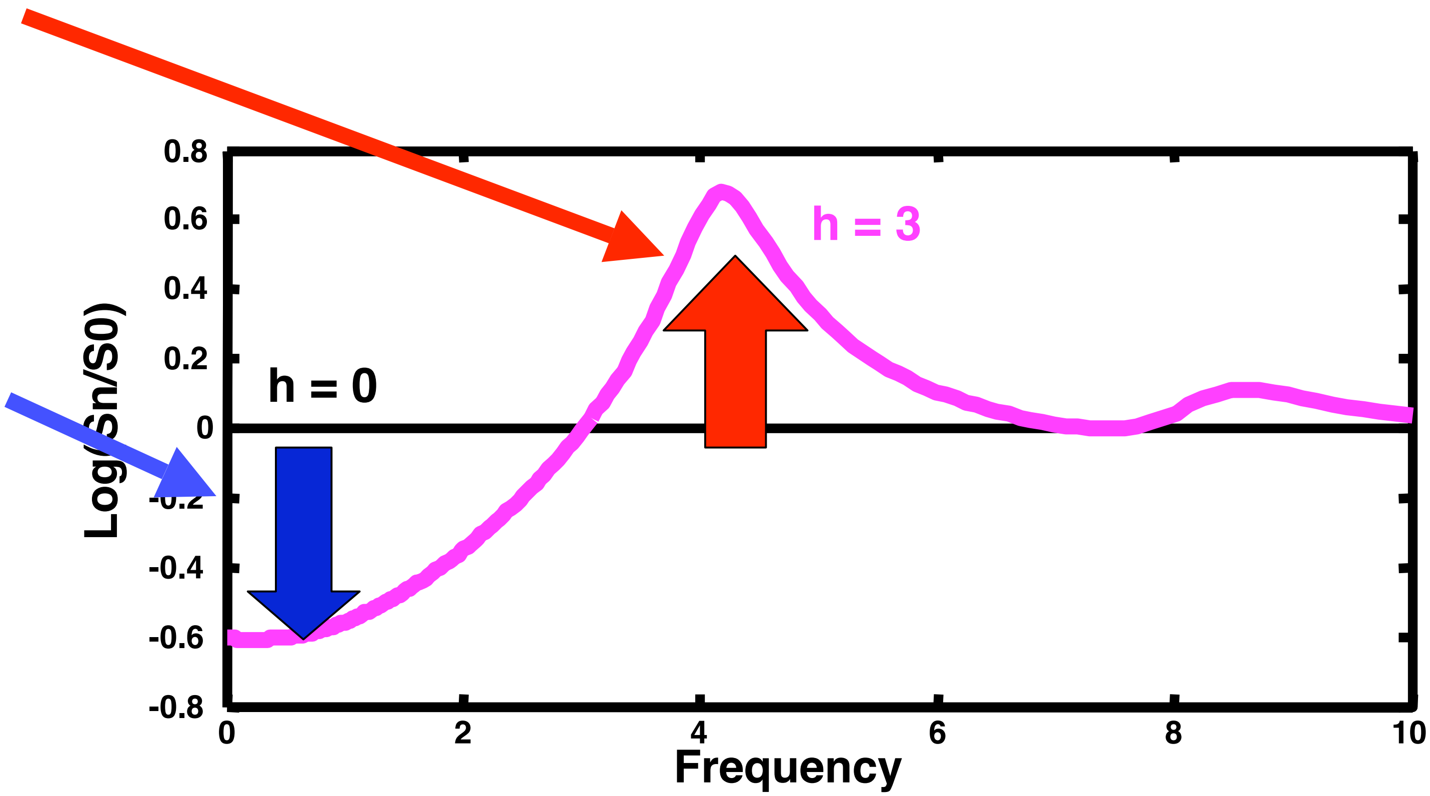




Yet fragile
Robust
[ATP]
1.05
1
0.95
0.9
0.85
0.8
0
5
10
15
20
Time (minutes)
h = 3
h = 0
Time response
Log(Sn/S0)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
2
4
6
8
10
Frequency
h = 3
h = 0
Spectrum

Yet fragile

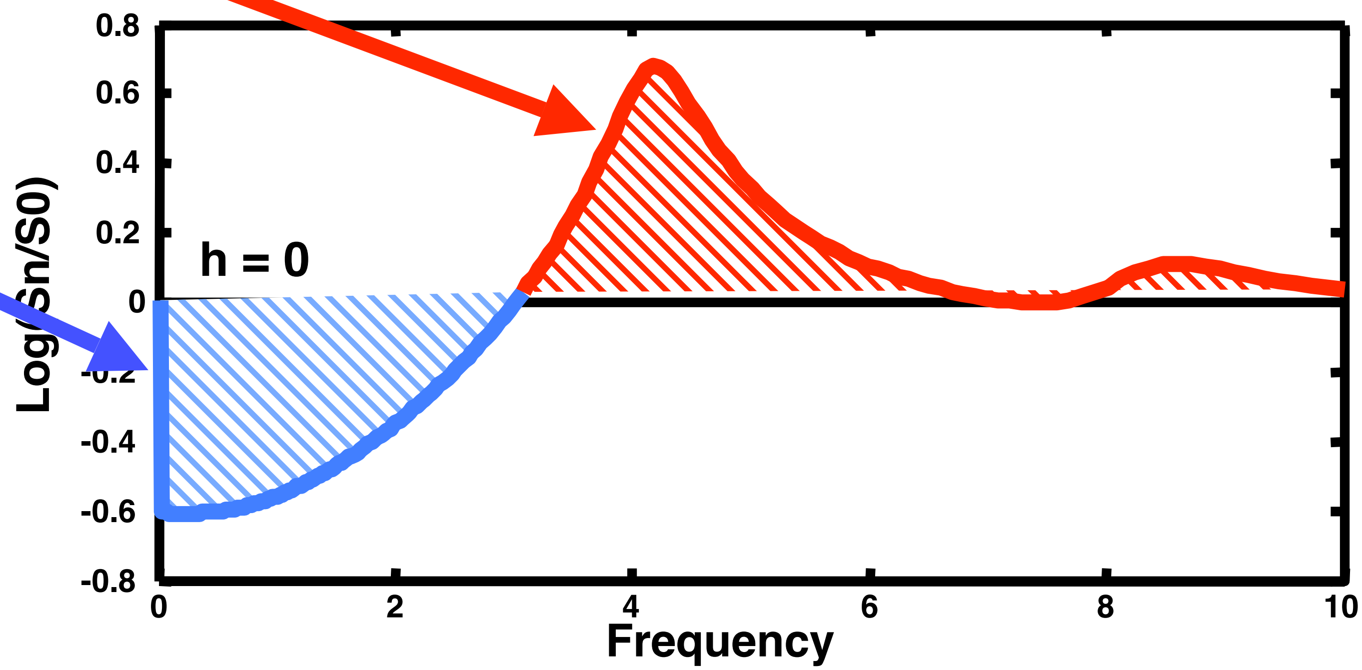
Robust



$$\int \log \left| \mathsf{F}(x_n) \right| d\omega \quad = \quad constant?$$

Yet fragile

Robust

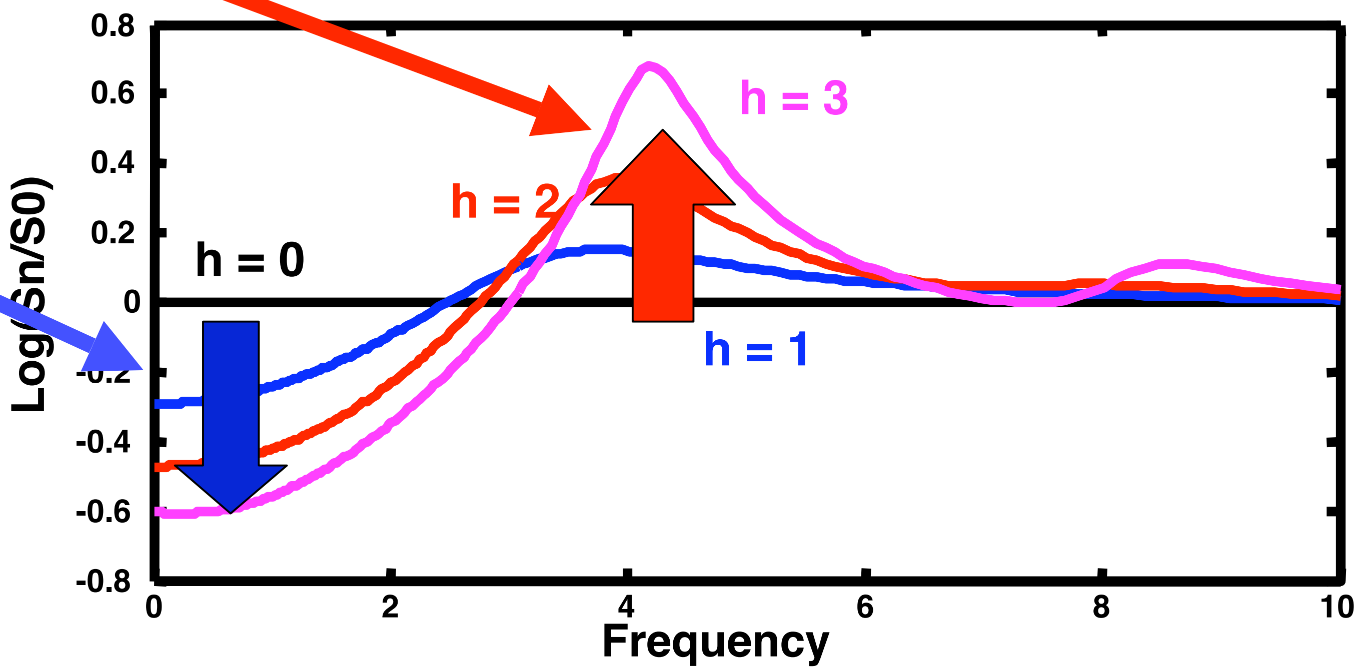


Theorem

$$\int \log\left|F(x_n)\right| d\omega \quad = \quad constant$$

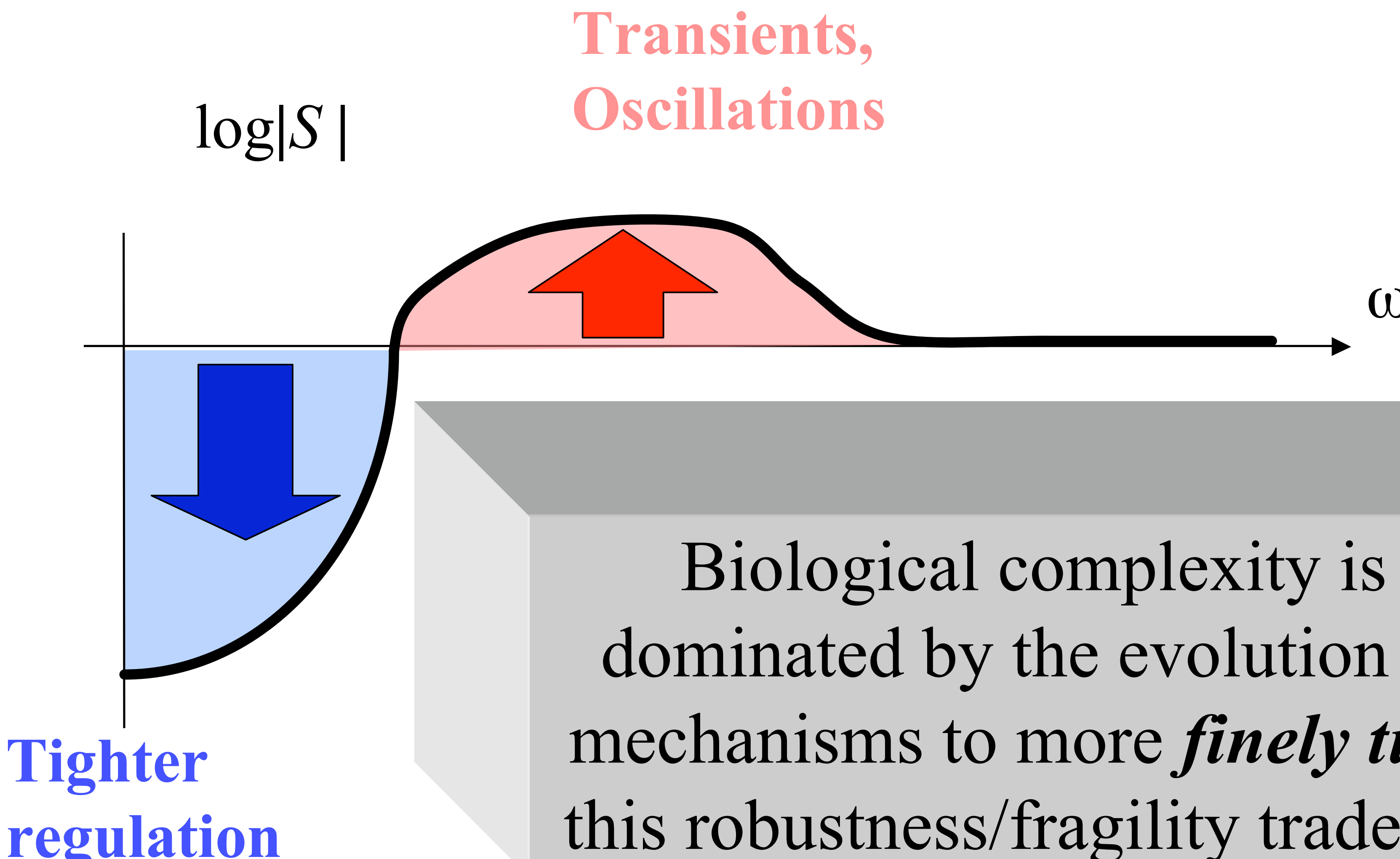
Transients, Oscillations

Tighter steady-state regulation



$$\log\left|F(x_n)\right|$$

This tradeoff is a ***law***.



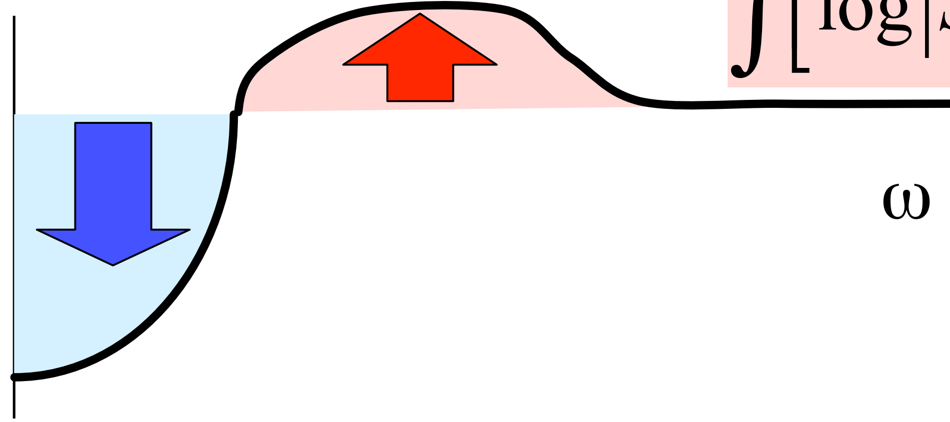
Biological complexity is dominated by the evolution of mechanisms to more ***finely tune*** this robustness/fragility tradeoff.

cost = amplification

goal: make this small

$$\int [\log|S|]_+ \, d\omega$$

$$\int [\log|S|]_- \, d\omega$$

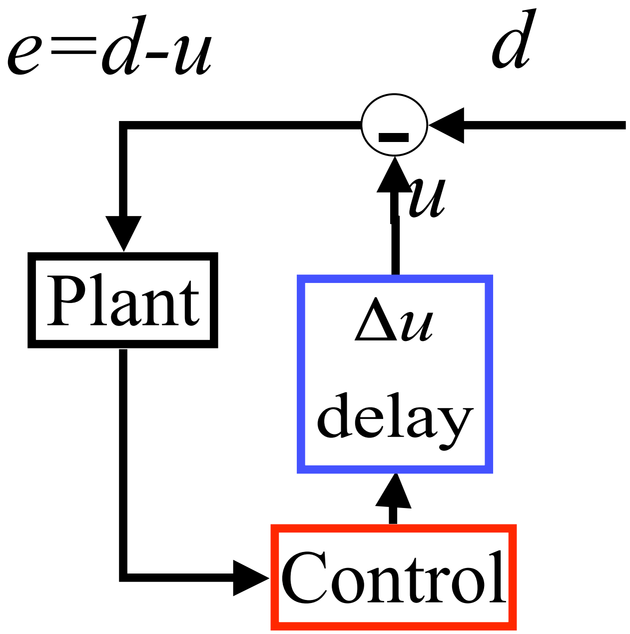


- benefits = attenuation of disturbance
- goal: make this as negative as possible

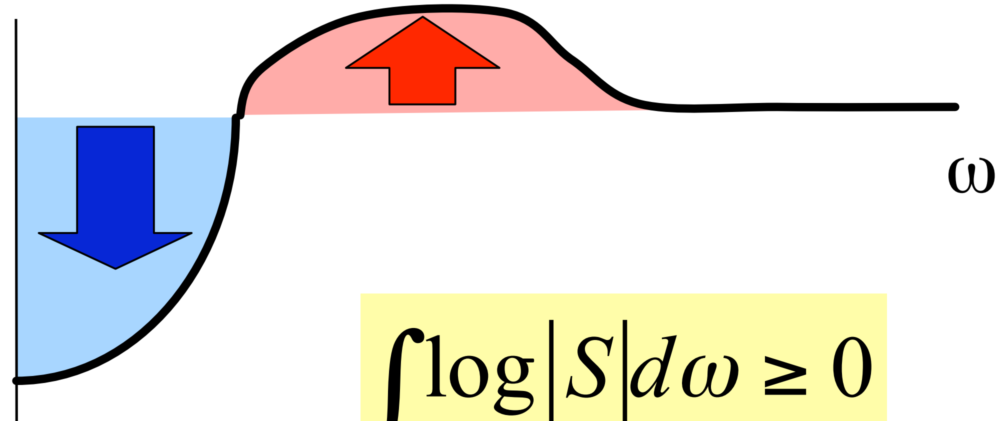
Constraint: $$\int [\log|S|]_- \, d\omega \geq -\int [\log|S|]_+ \, d\omega$$

benefits **costs**

Bode



$$S(\omega) = \frac{E(\omega)}{D(\omega)}$$



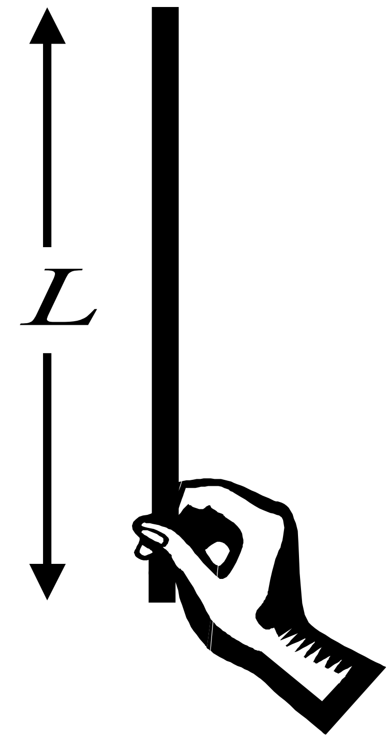
$$\int \log|S|d\omega \geq 0$$

- **What helps or hurts this tradeoff?**
- **Helps**: remote sensing
- **Hurts**: instability, remote control

Control demo

$$\int_0^\infty \log\left|S(\omega)\right| d\omega \geq \pi / \sqrt{L}$$

Note: assumes continuous time

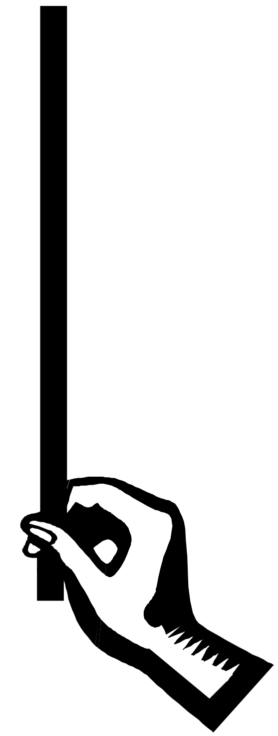


- Unstable plants are intrinsically more difficult to control than stable ones, and are generally avoided unless the instability confers some great functional advantage, which it often does.

- A classic illustration of instability and control, the simple inverted pendulum experiment, illustrates the essential point.
- Here the pendulum is the plant, and the human is the controller. The experiment can be done with sticks of different lengths or with an extendable pointer, holding the proximal tip between thumb and forefinger so that it is free to rotate but not otherwise slip.

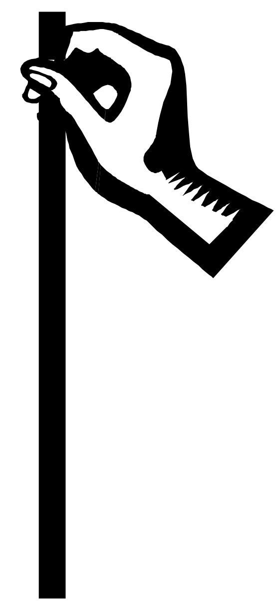
• With the controlling hand fixed, this system has two equilibria, down and up, which are stable and unstable, respectively. By watching the distal tip and controlling hand motion, the up case can be stabilized if the stick is long enough.

• For an external disturbance, imagine that someone is throwing objects at the stick and you are to move so that the stick remains roughly vertical and avoids the thrown object. Alternatively, imagine that the distal tip is to track some externally driven motion.



• You will soon find that it is much easier to control the distal tip down than up, even though the components in both cases are the same.

• Because the up configuration is unstable, certain hand motions are not allowed because they produce large, unstable tip movements. This presents an obstacle in the space of dynamic hand movements that must be avoided, making control more difficult.

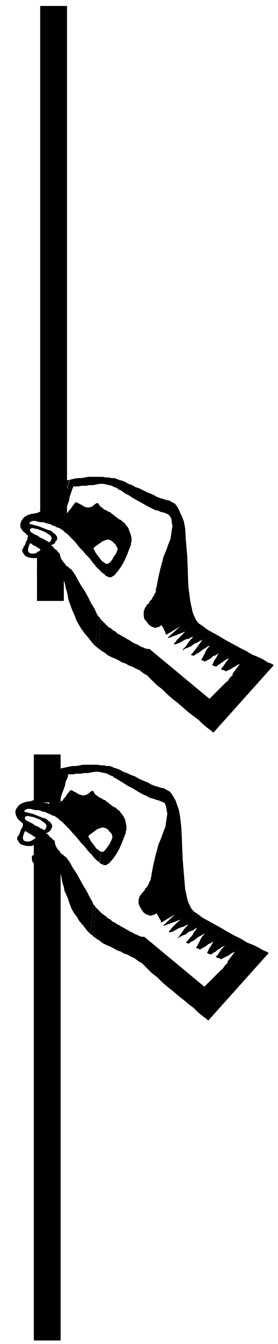


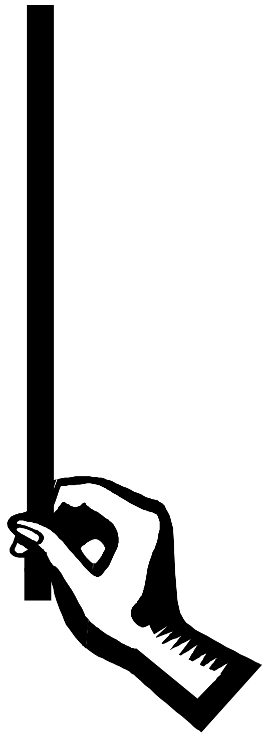
• If you make the stick shorter, it gets more unstable in the up case, evident in the short time it takes the uncontrolled stick to fall over.

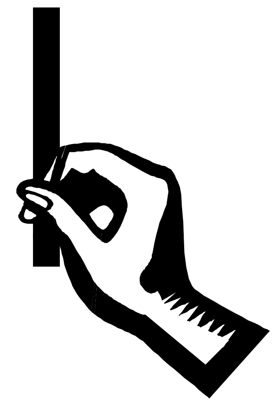
• Shorter pendulums get harder and ultimately impossible to control in the up case, while length has little such effect on the down case.

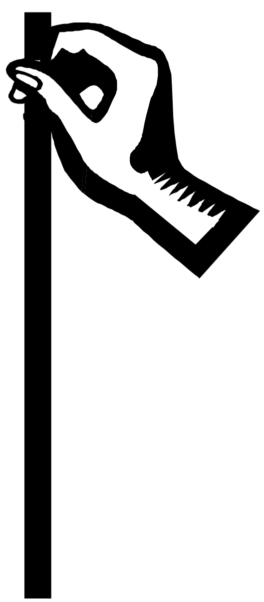
• Also, the up stick cannot be stabilized for any length if only the proximal tip is watched, so the specific sensor location is crucial as well.

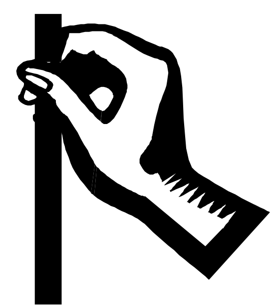
• This exercise is a classical demonstration of the principle that the more unstable a system the harder it is to control robustly, and control theory has formally quantified this effect in several ways.



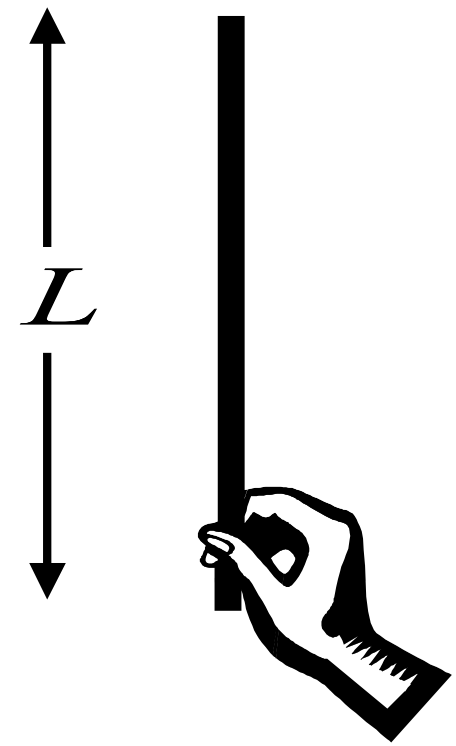




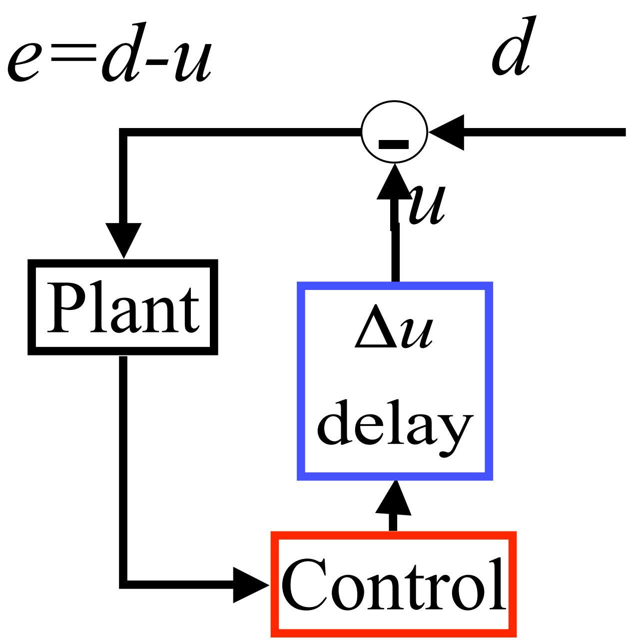


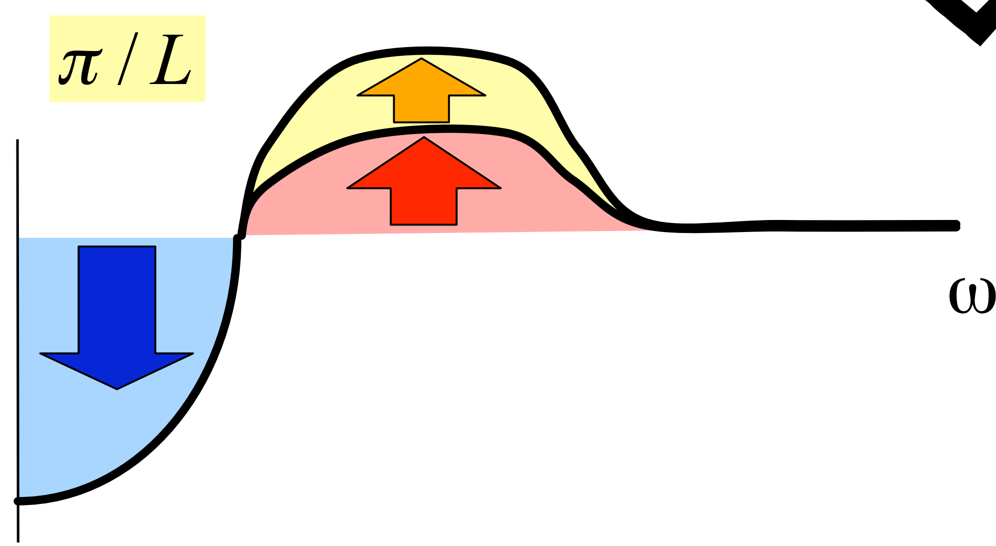


$$\int \log |S| d\omega \geq \frac{\pi}{\sqrt{L}}$$



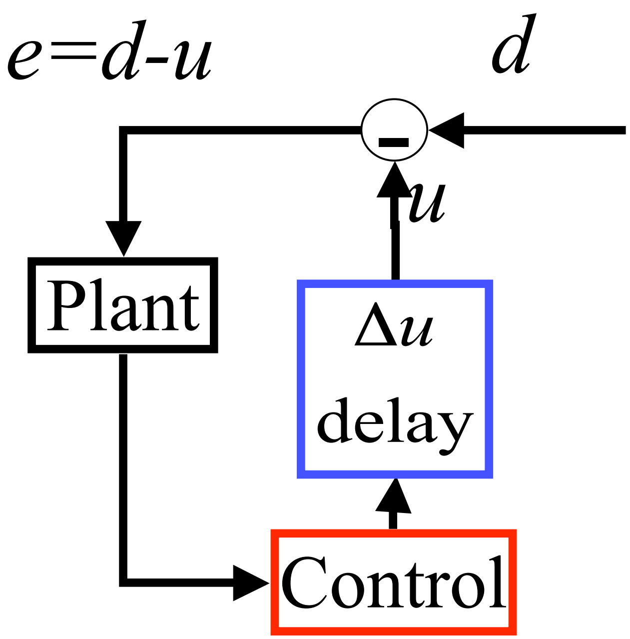
$$S(\omega) = \frac{E(\omega)}{D(\omega)}$$



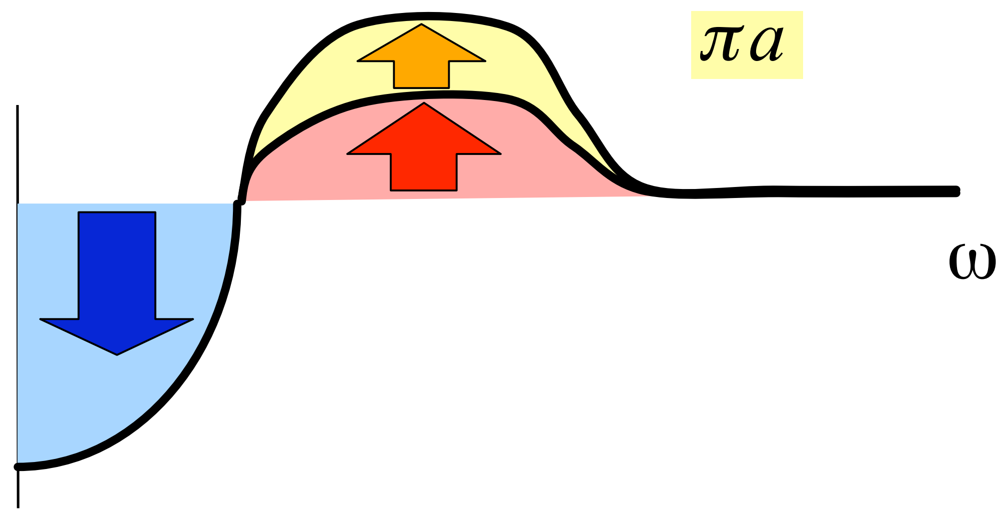


Freudenberg and Looze, 1984

Bode



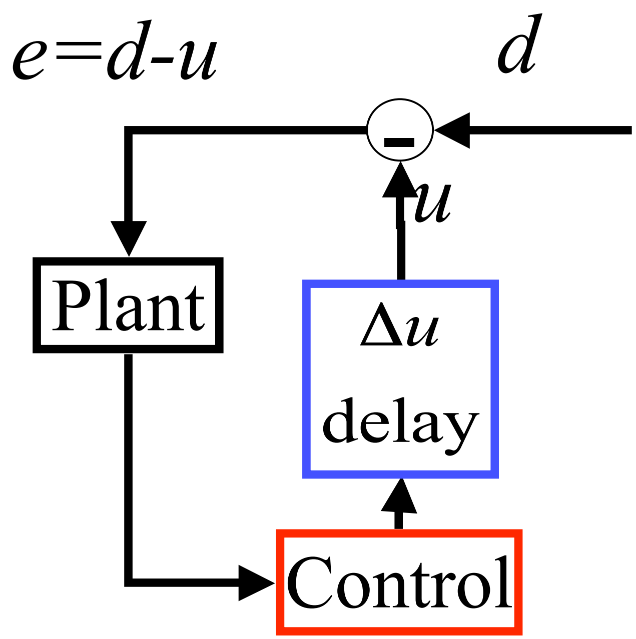
$$S(\omega)=\frac{E(\omega)}{D(\omega)}$$



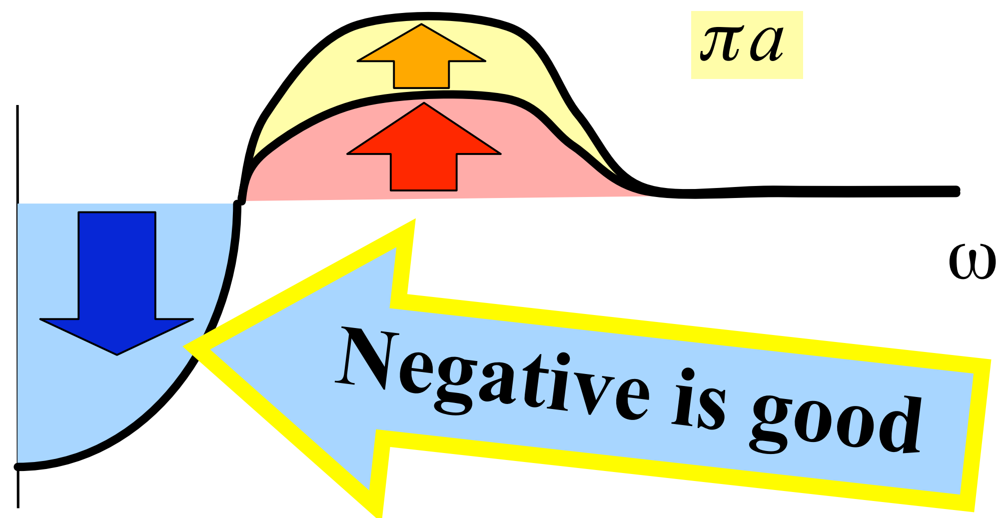
$$\int\left[\log|S|\right]_{-}d\omega \geq -\int\left[\log|S|\right]_{+}d\omega \qquad +\pi a$$

benefits **costs** **stabilize**

Bode

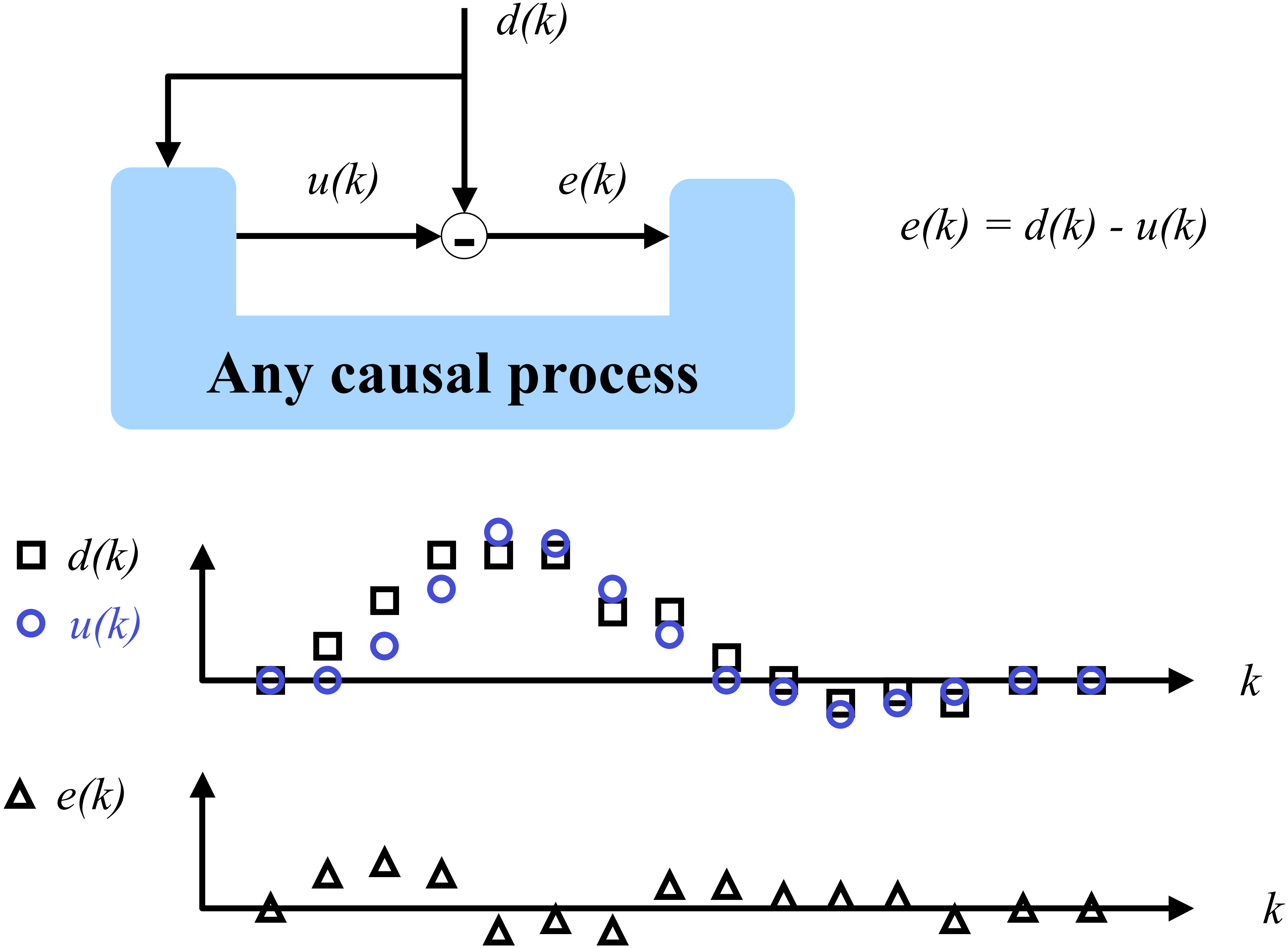


$$S(\omega)=\frac{E(\omega)}{D(\omega)}$$

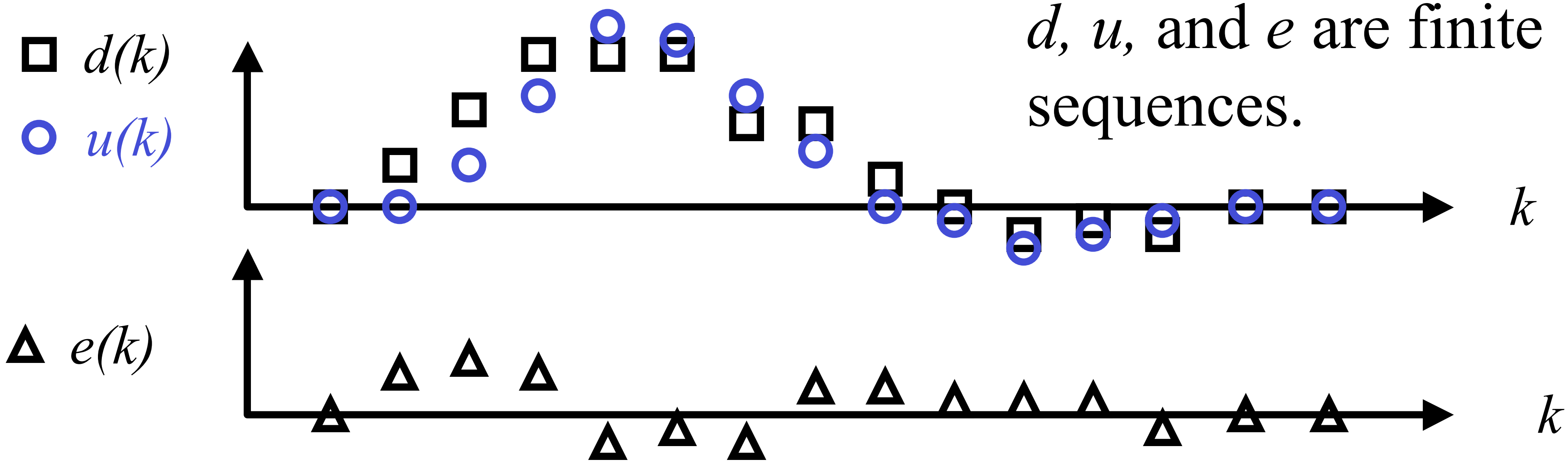


$$\int[\log|S|]_-\,d\omega \geq -\int[\log|S|]_+\,d\omega \qquad +\pi a$$

benefits **costs** **stabilize**



$$e(k) = d(k) - u(k)$$



For simplicity, assume d, u, and e are finite sequences.

$$X(z) = \sum_{k=0}^{N} x(k) z^{-k}$$

Then the discrete Fourier transform D, U, and E are polynomials in the transform variable z.

If we set $z = e^{i\omega}$, $\omega \in [0,\pi]$ then X(ω) measures the frequency content of x at frequency ω.

- Denote by $\{z_k\}$ the complex zeros for $|z| > 1$ of $X(z)$
- Jensen's theorem

$$\int_0^{\pi} \log\left|X(\omega)\right| d\omega = \sum \log\left|z_k\right| + \log\left|X(\infty)\right|$$

Proof: Contour integral

A useful measure of performance is in terms of the sensitivity function *S(z)* defined by Bode as

$$\boxed{S(z) = \frac{E(z)}{D(z)} = \frac{D(z) - U(z)}{D(z)} = 1 - \frac{U(z)}{D(z)}}$$

If we set $z = e^{i\omega}$, $\omega \in [0,\pi]$ then $|S(\omega)|$ measures how well ***C*** does at each frequency. (If ***C*** is linear then S is independent of d, but in general S depends on d.)

It is convenient to study $\log |S(\omega)|$

- Denote by $\{\varepsilon_k\}$ and $\{\xi_k\}$ the complex zeros for $|z| > 1$ of $E(z)$ and $D(z)$, respectively. Then

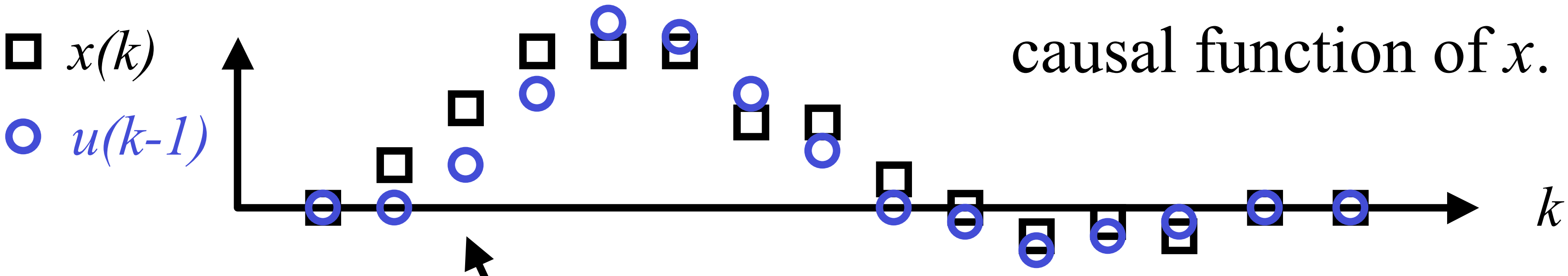
$$\boxed{\int_0^{\pi} \log\left|S(\omega)\right| d\omega = \sum \log\left|\varepsilon_k\right| - \sum \log\left|\xi_k\right| + \log\left|S(\infty)\right|}$$

Proof: Follows directly from Jensen's formula.

If d is chosen so that $D(z)$ has no zeros in $|z| > 1$ (this is an open set), then

$$\int_0^{\pi} \log\left|S(\omega)\right| d\omega = \sum \log\left|\varepsilon_k\right| - \cancelto{0}{\sum \log\left|\xi_k\right|} + \log\left|S(\infty)\right|$$

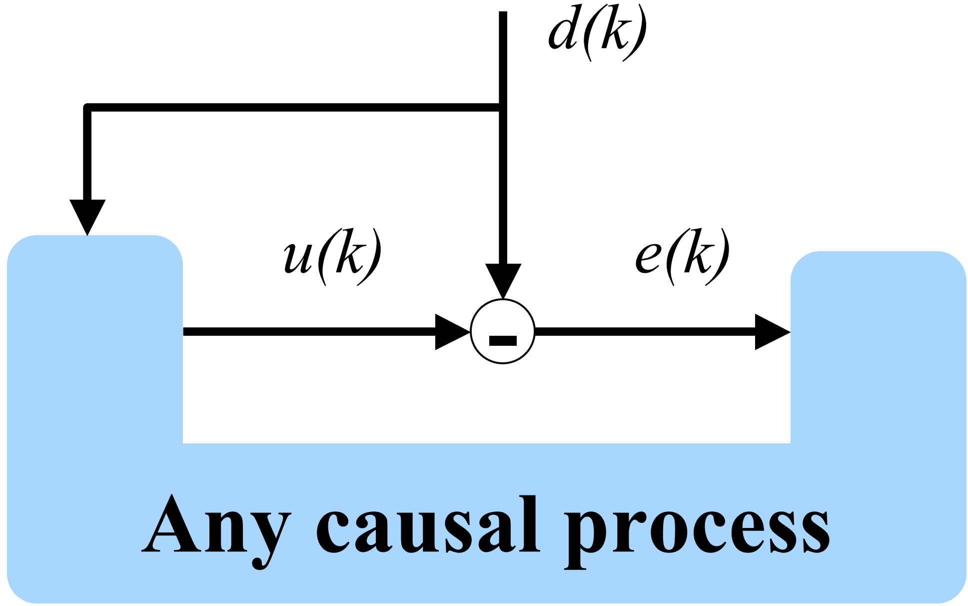
Assume u is a causal function of x.



Then the first nonzero element of u is delayed at least one step behind the first nonzero element of x.

$$S(\infty) = 1 - \frac{u(0)}{d(0)} = 1 \quad \Rightarrow \quad \log\left|S(\infty)\right| = 0$$

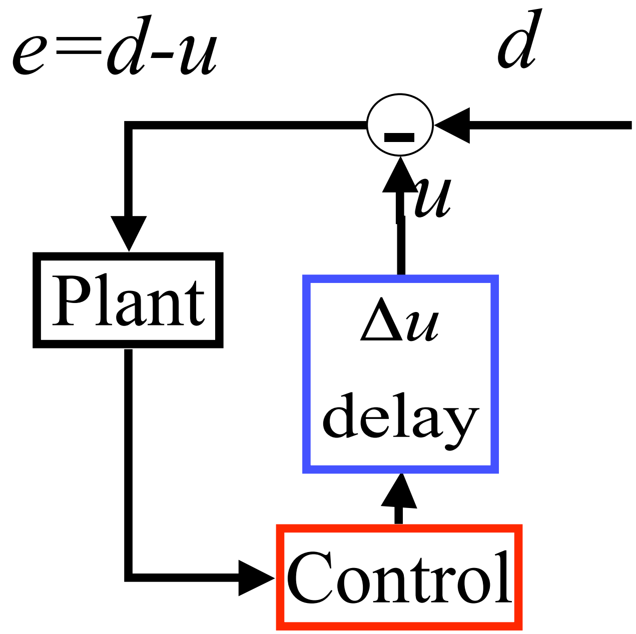
$$\int_0^{\pi} \log\left|S(\omega)\right| d\omega = \underbrace{\sum \log\left|\varepsilon_k\right|}_{\geq 0} + \cancelto{0}{\log\left|S(\infty)\right|}$$



$$e(k) = d(k) - u(k)$$

$$\int_0^\pi \log|S(\omega)|\,d\omega = \int_0^\pi \log|E(\omega)|\,d\omega - \int_0^\pi \log|D(\omega)|\,d\omega$$

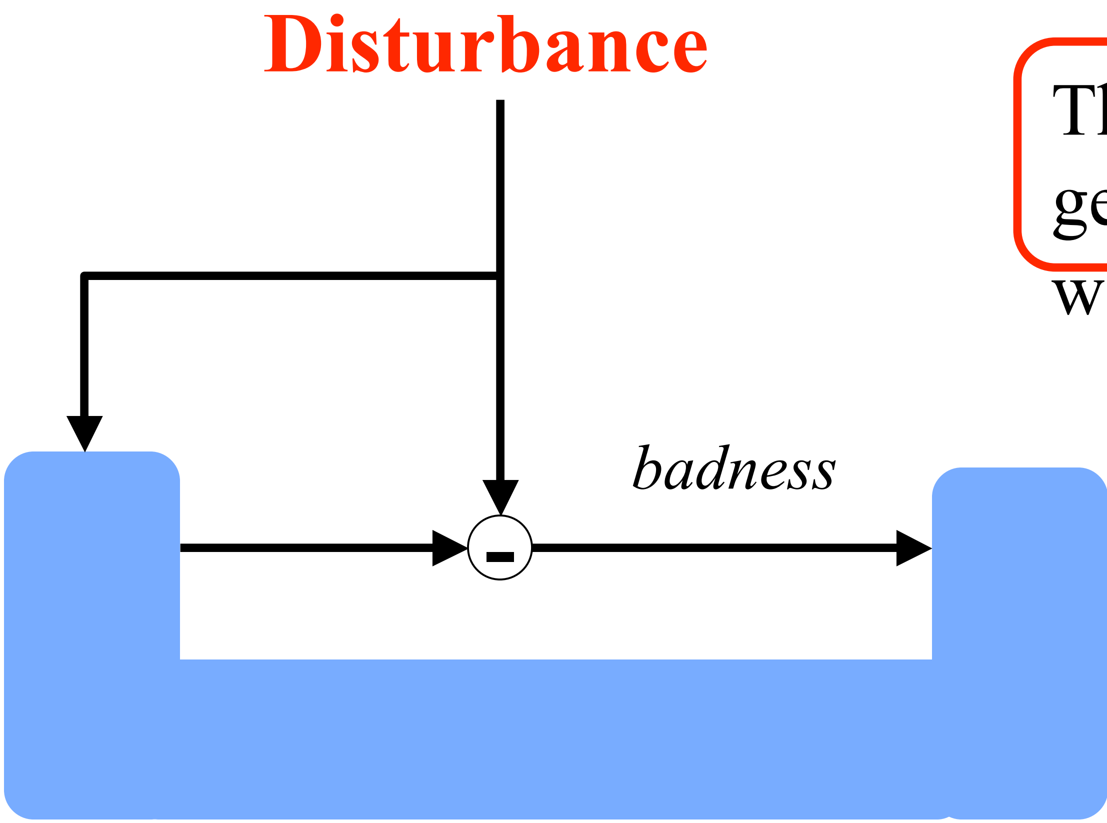
$$\int_0^\pi \log|S(\omega)|\,d\omega = \sum \log|\varepsilon_k| \geq 0$$



If the plant has (linear) instabilities, then e must have zeros at the plant poles.

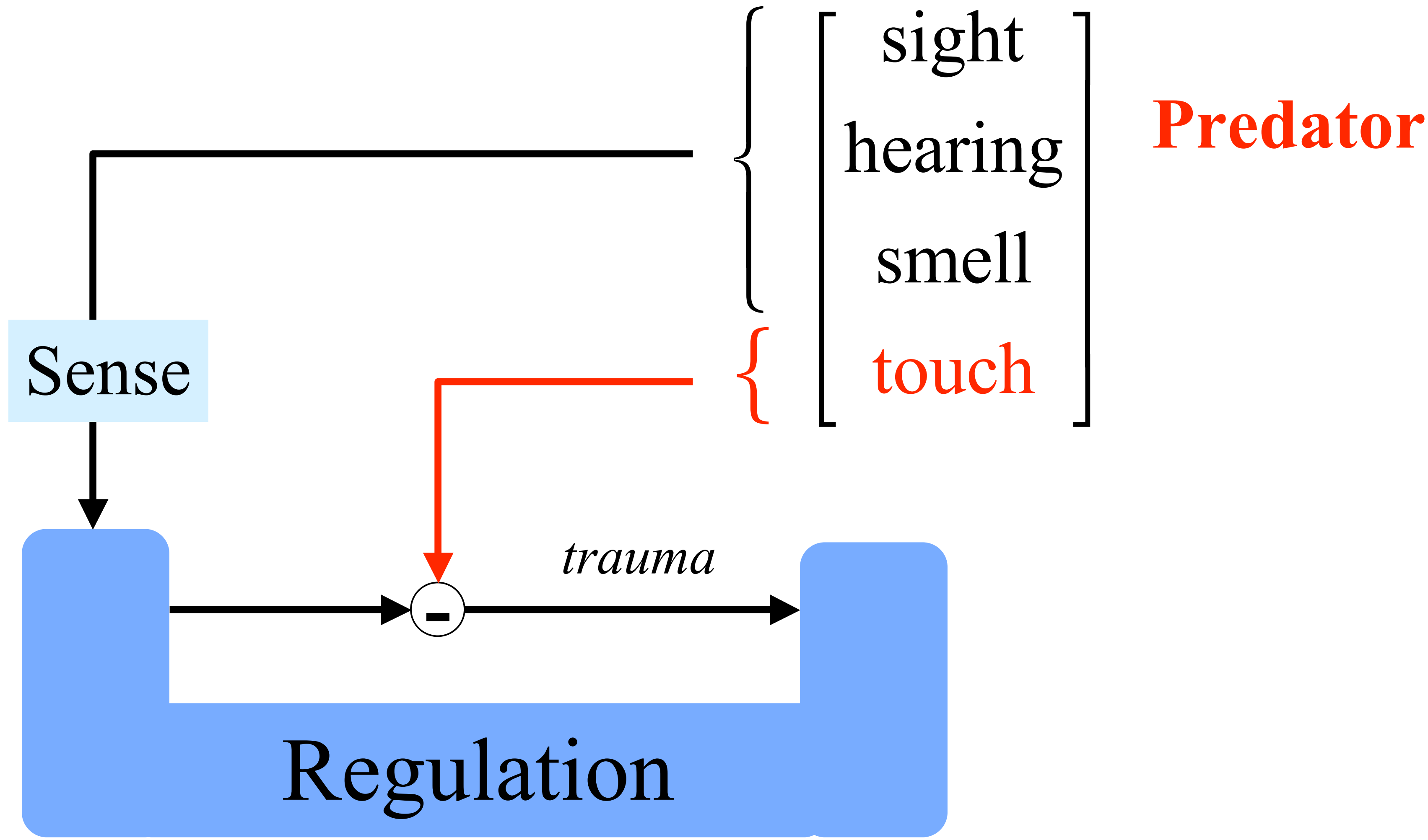
$$\int_0^{\pi} \log\left|S(\omega)\right| d\omega = \boxed{\sum \log\left|\varepsilon_k\right|} \geq 0$$

This is where plant instability hurts.



This motivates trying to get "advanced warning."

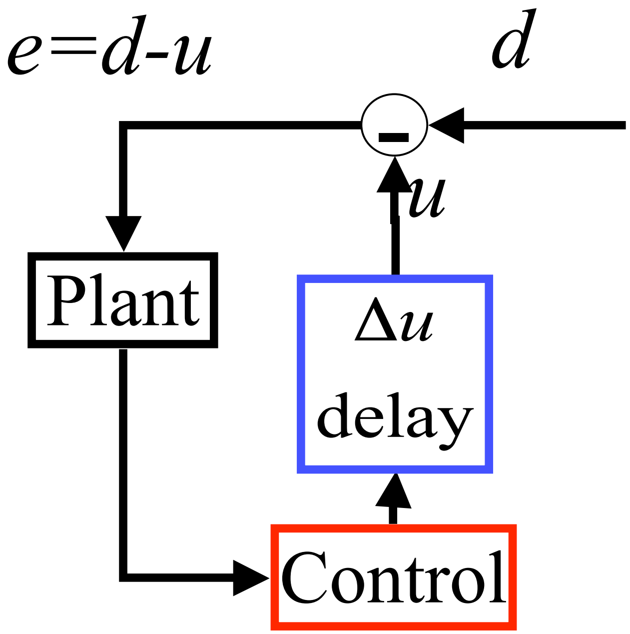
$$\int_0^\pi \log|S(\omega)|\,d\omega = \sum \log|\varepsilon_k| - \boxed{\sum \log|\xi_k| + \log|S(\infty)|}$$



$$\int_0^\pi \log\left|S(\omega)\right| d\omega = \sum \log\left|\varepsilon_k\right| - \boxed{\sum \log\left|\xi_k\right| + \log\left|S(\infty)\right|}$$

Usually this term is zero.

$$\int_0^{\pi} \log\left|S(\omega)\right| d\omega = \sum \log\left|\varepsilon_k\right| - \boxed{\sum \log\left|\xi_k\right| + \log\left|S(\infty)\right|}$$



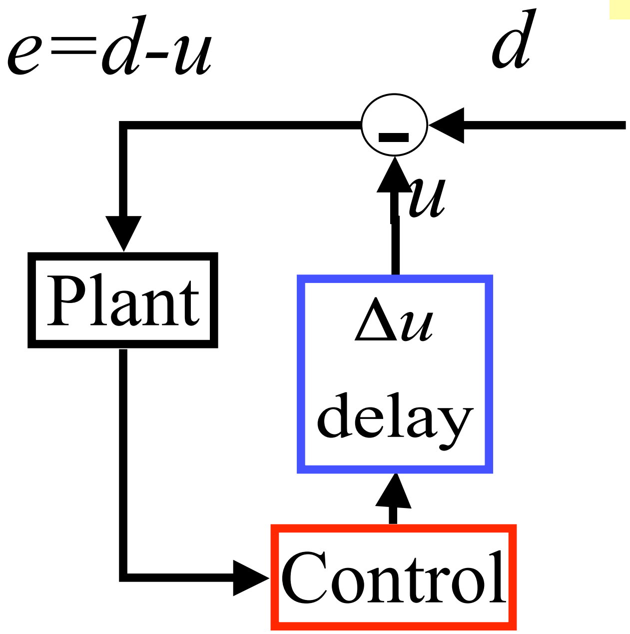
If the plant has (linear) instabilities

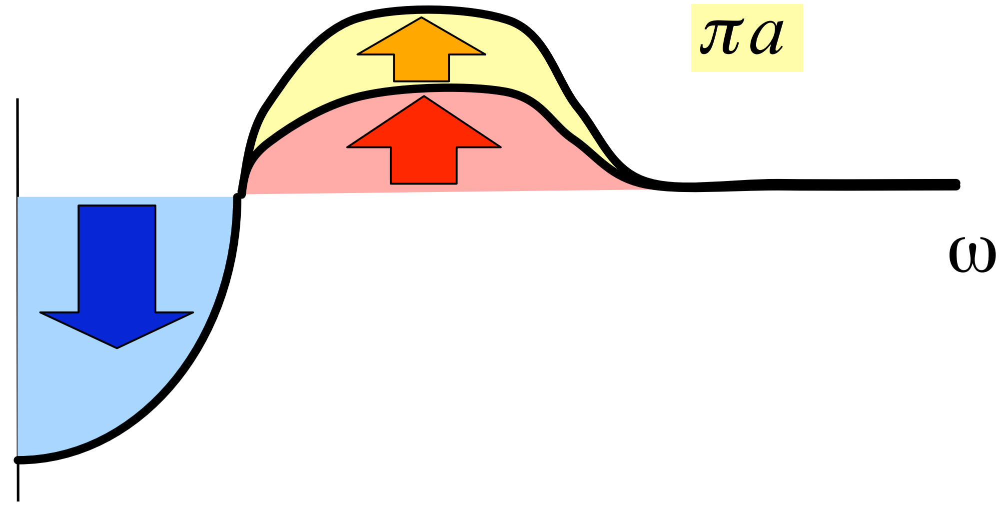
$$\int_0^{\pi} \log\left|S(\omega)\right| d\omega = \boxed{\sum \log\left|\varepsilon_k\right|} \geq 0$$

This is where plant instability hurts.

Bode

$$S(\omega) = \frac{E(\omega)}{D(\omega)}$$





$$\int \left[\log|S|\right]_{-} d\omega \geq -\int \left[\log|S|\right]_{+} d\omega \quad +\pi a$$

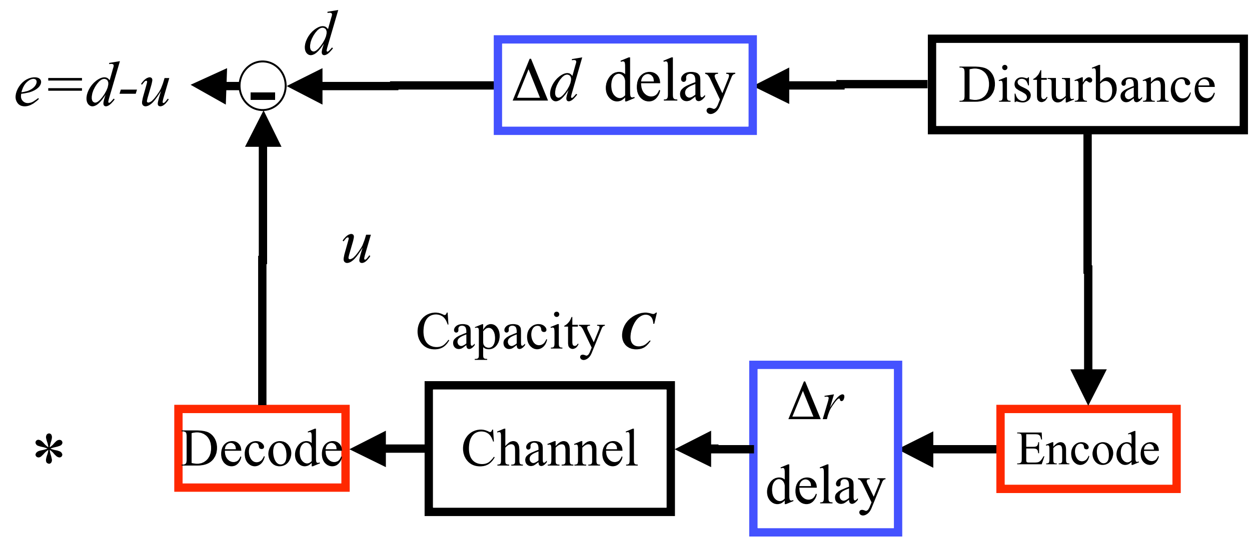
Recall

$$h(E) - h(D) \geq -C$$

As $\Delta d \to \infty$,

$$\{h(D) < C \Rightarrow e \approx 0\} \quad *$$

Shannon



Δr = total delay of encoding, decoding, and channel

Δd =disturbance arrival delay from where it is remotely sensed

Features of the theory:

1. Hard bounds
2. Achievable (⇐assumptions)
3. Solution decomposable (⇐assumptions)

* The interpretation of ≈ depends on the details of the model.

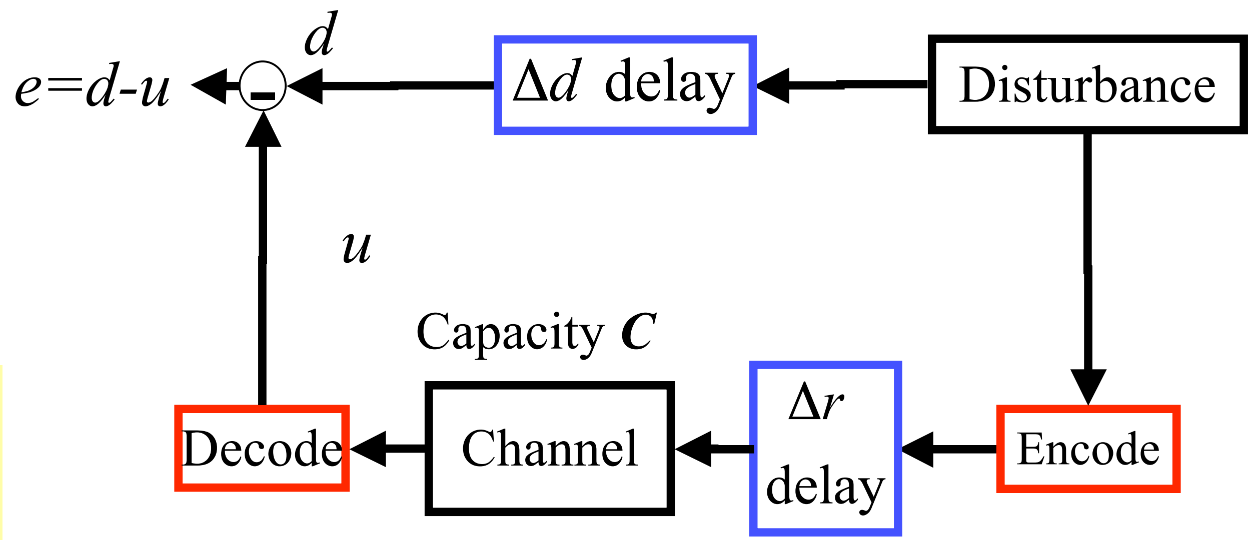
Recall

$$h(E) - h(D) \geq -C$$

As $\Delta d \to \infty$,

$$\{h(D) < C \Rightarrow e \approx 0\}$$

Shannon



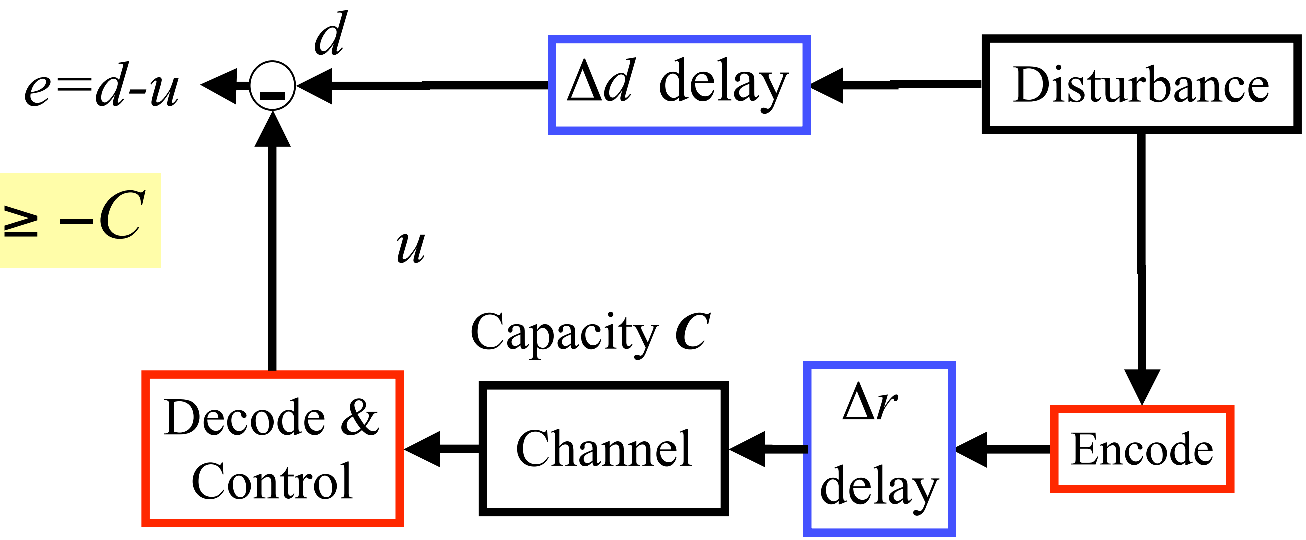
This is a nonstandard way of describing the results but will be convenient later.

Δr = total delay of encoding, decoding, and channel

Δd =disturbance arrival delay from where it is remotely sensed

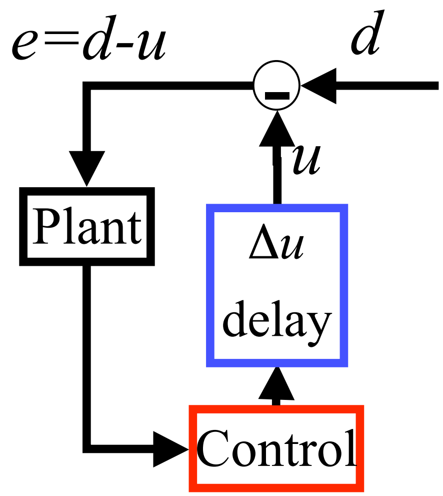
Shannon

$$h(E) - h(D) \geq -C$$



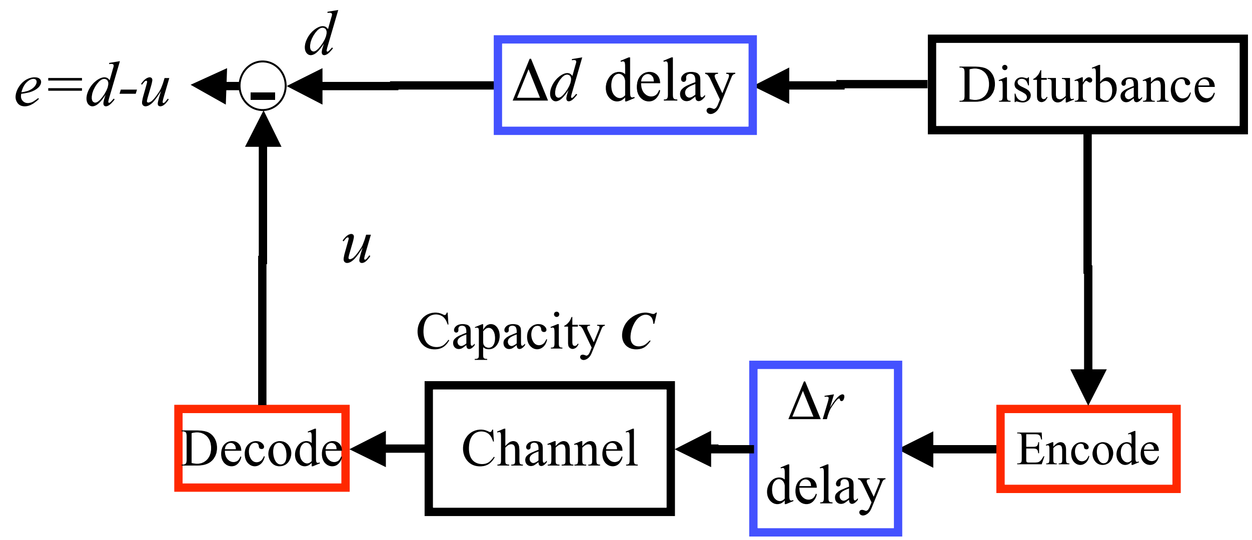
- We can think of this as a simple "network" control problem with a disturbance that can be remotely sensed.
- A "controller" decodes a signal sent over a noisy channel and attempts to make the error small.
- The entropy reduction in the error is bounded by the channel capacity.

Bode



$$\int \log|S|\,d\omega \geq \log(a)$$

Shannon

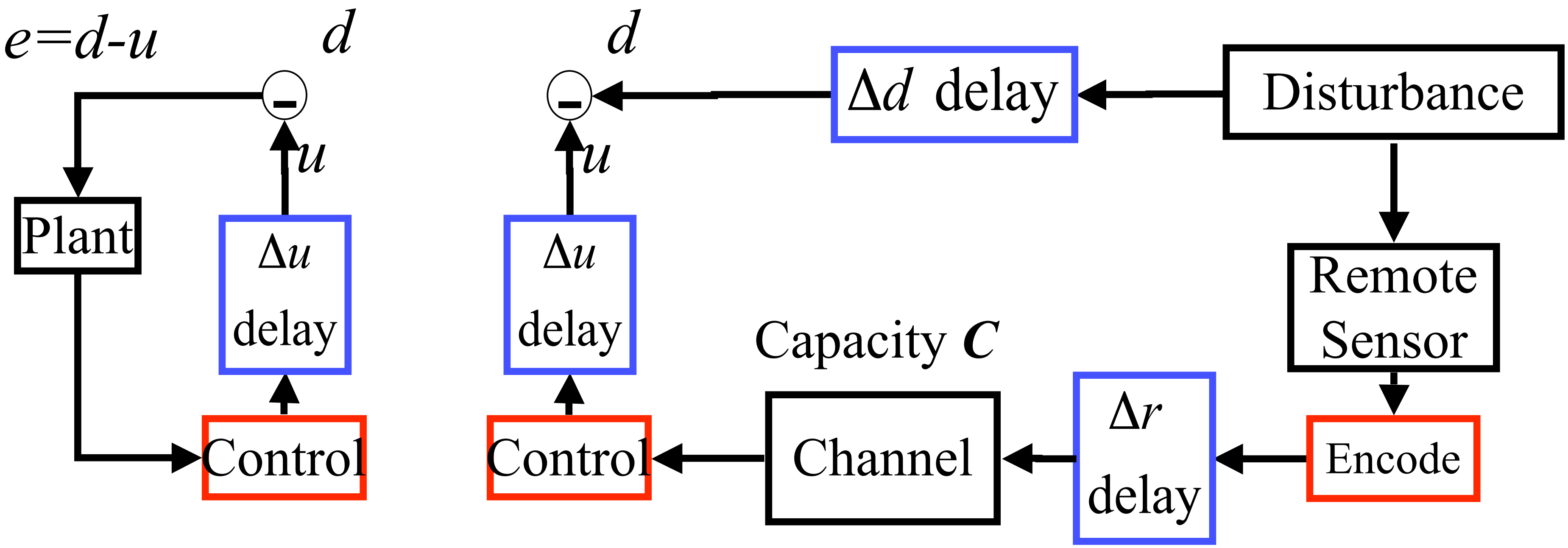


$$h(E) - h(D) \geq -C$$

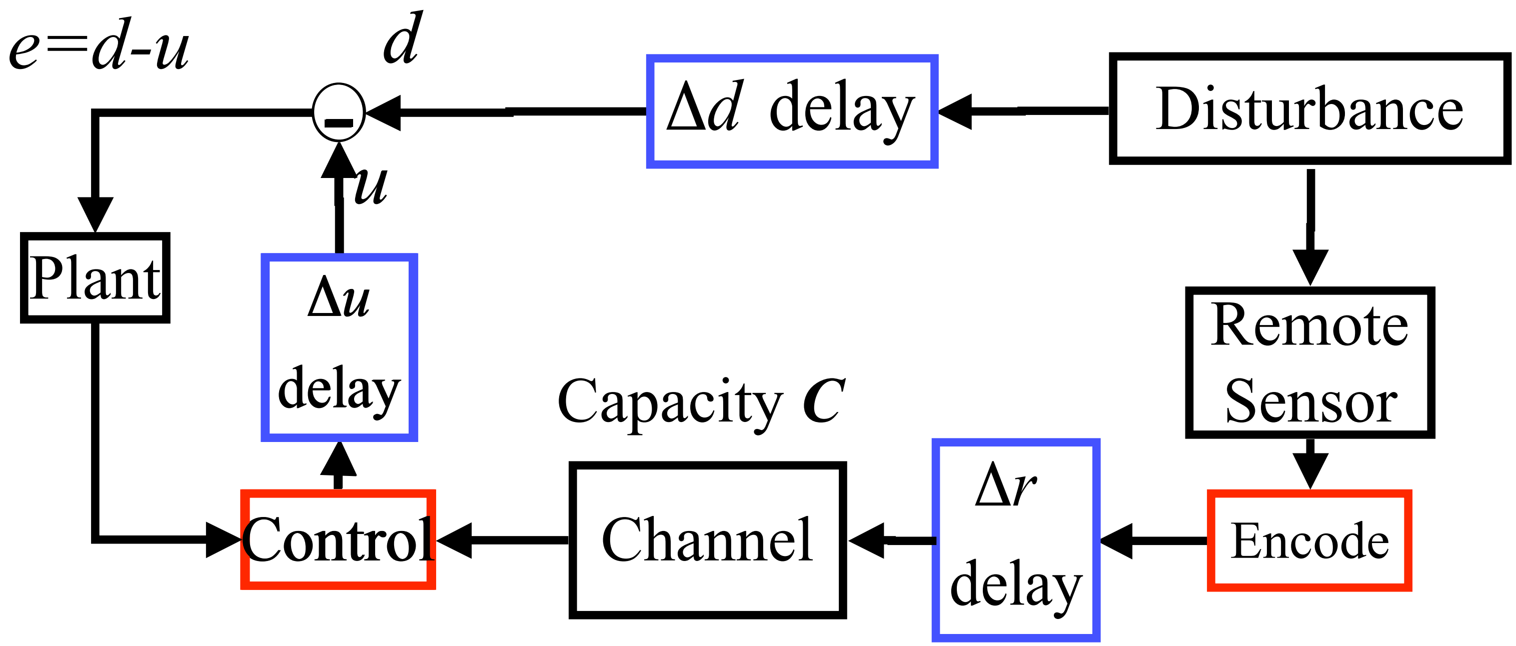
$$\text{As } \Delta d \to \infty, \quad \{h(D) < C \Rightarrow e \approx 0\}$$

1. Hard bounds
2. Achievable ($\Leftarrow$assumptions)
3. Solution decomposable ($\Leftarrow$assumptions)

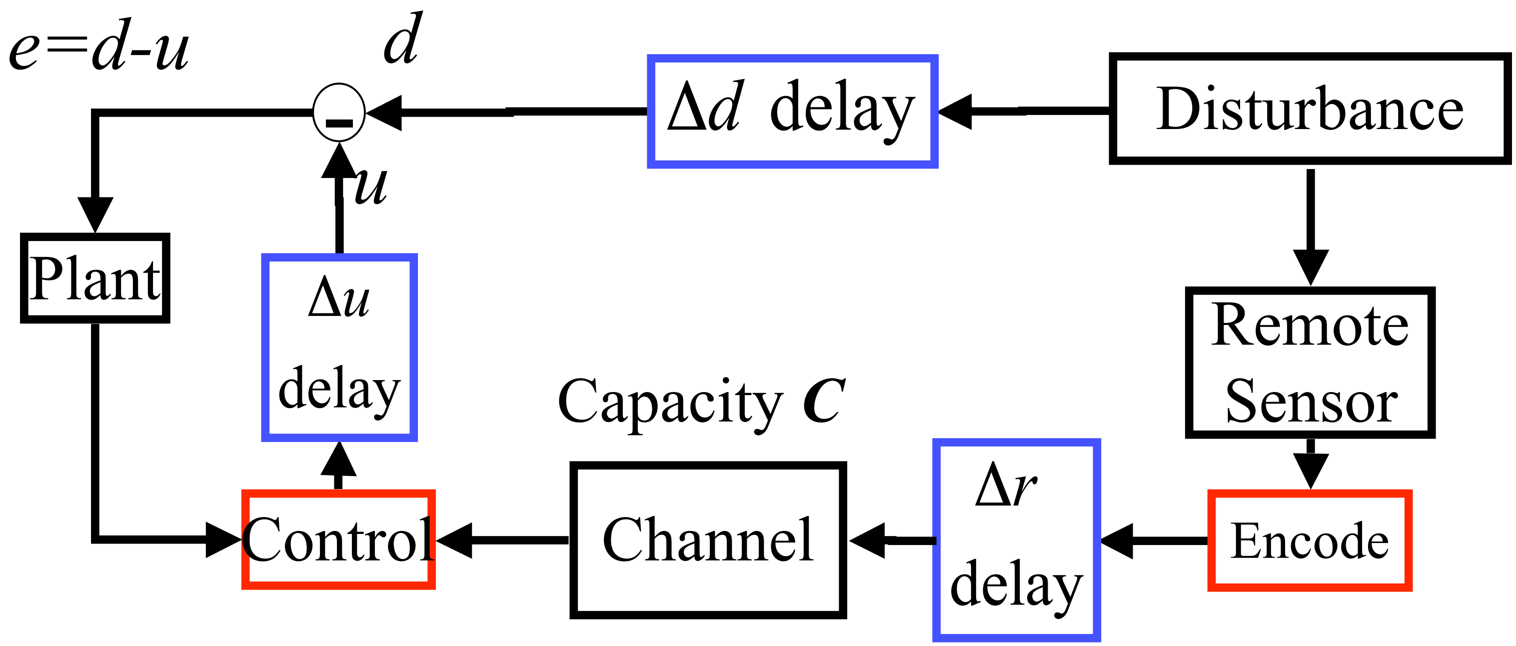
Incompatible assumptions (for 50+ years).



It's easy to ***pose*** a combined problem.

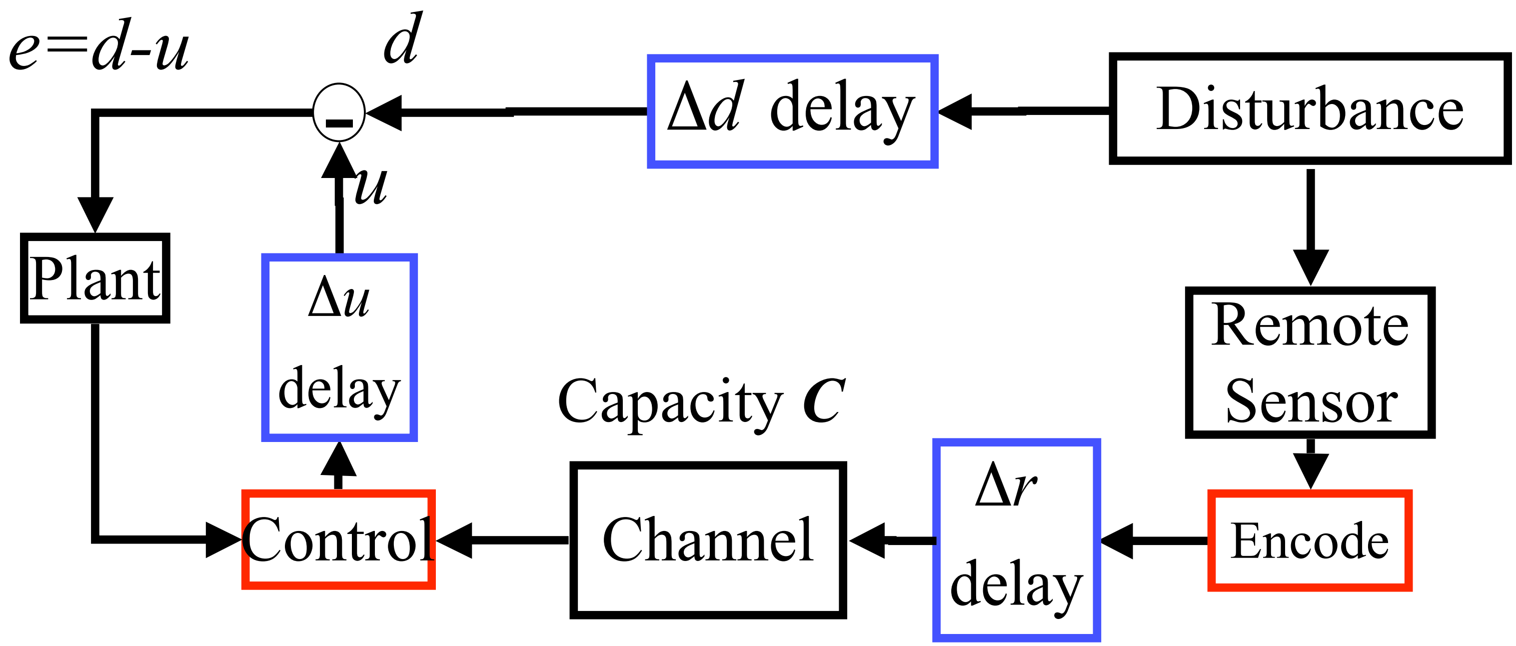


It's easy to ***pose*** a combined problem.



It's easy to ***pose*** a combined problem.

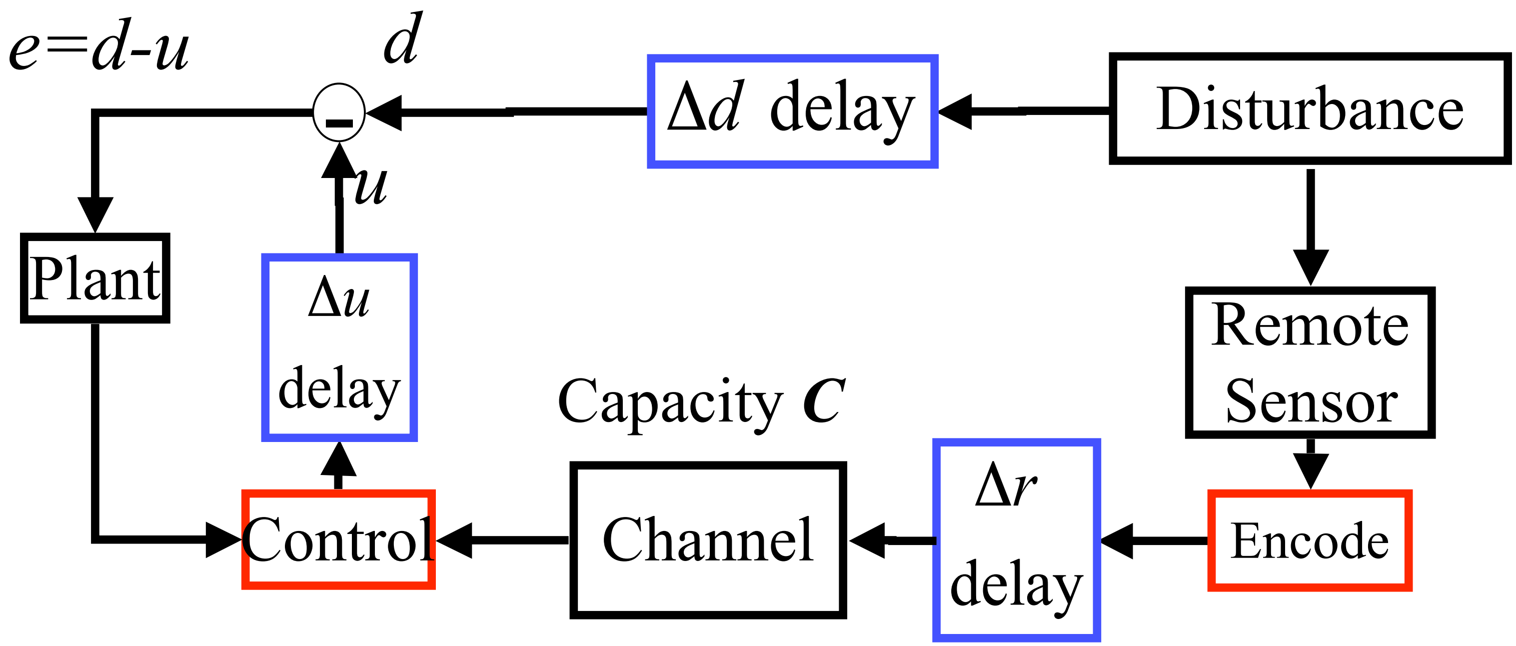
What is the benefit to control of remote sensing?



$$\int \log|S|\,d\omega \geq \log(a) - \begin{cases} 0 & \Delta d < \Delta u + \Delta r \\ C & \Delta d \geq \Delta u + \Delta r \end{cases}$$

This looks too good to be true?

What is the benefit to control of remote sensing?

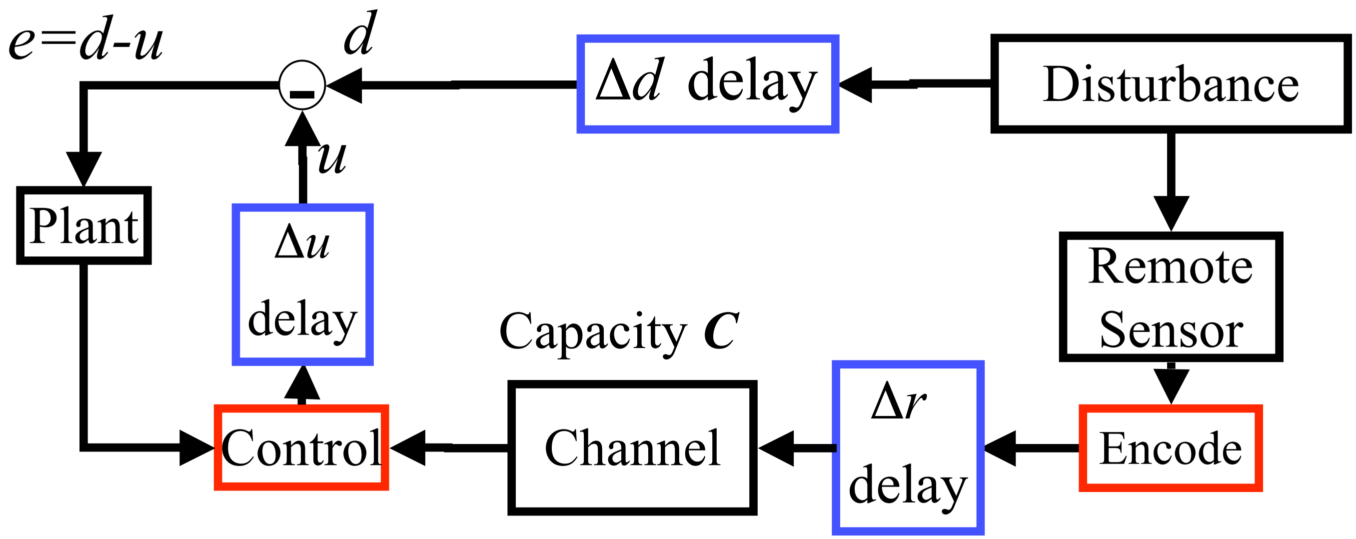


$$\int \log|S|\,d\omega \ge \log(a) - \begin{cases} 0 & \Delta d < \Delta u + \Delta r \\ C & \Delta d \ge \Delta u + \Delta r \end{cases}$$

Cost of stabilization

Benefits of remote sensing

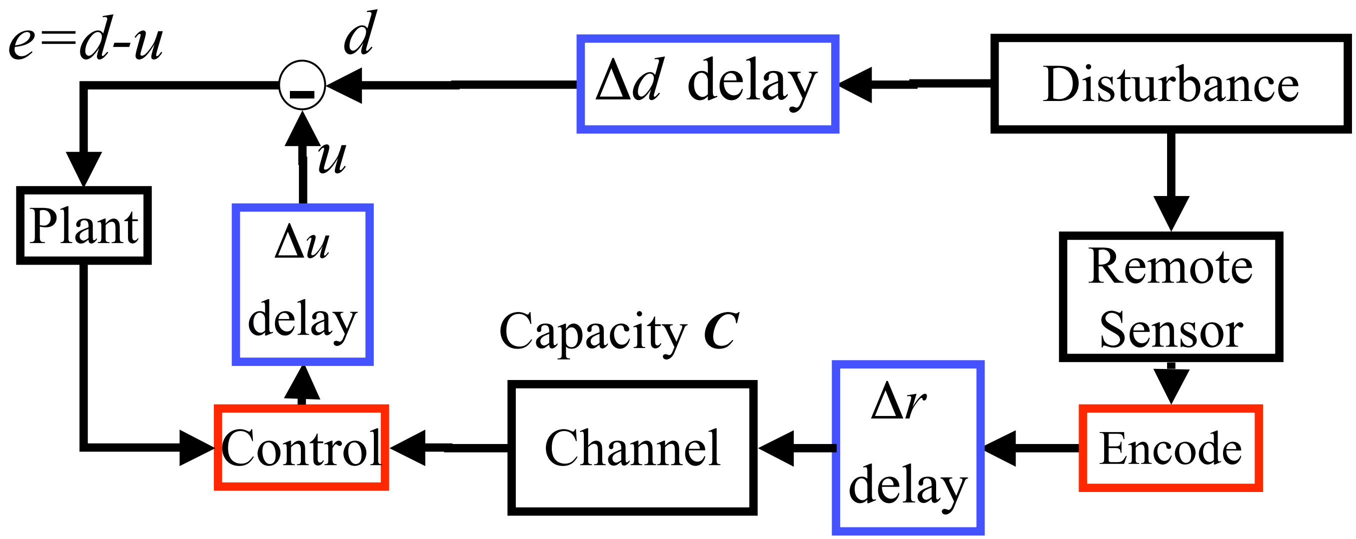
Need relatively low latency



$$\int \log|S|\,d\omega \geq \log(a) - \begin{cases} 0 & \Delta d < \Delta u + \Delta r \\ C & \Delta d \geq \Delta u + \Delta r \end{cases}$$

1. Hard bounds
2. Achievable (⇐assumptions)
3. Solution decomposable (⇐assumptions)

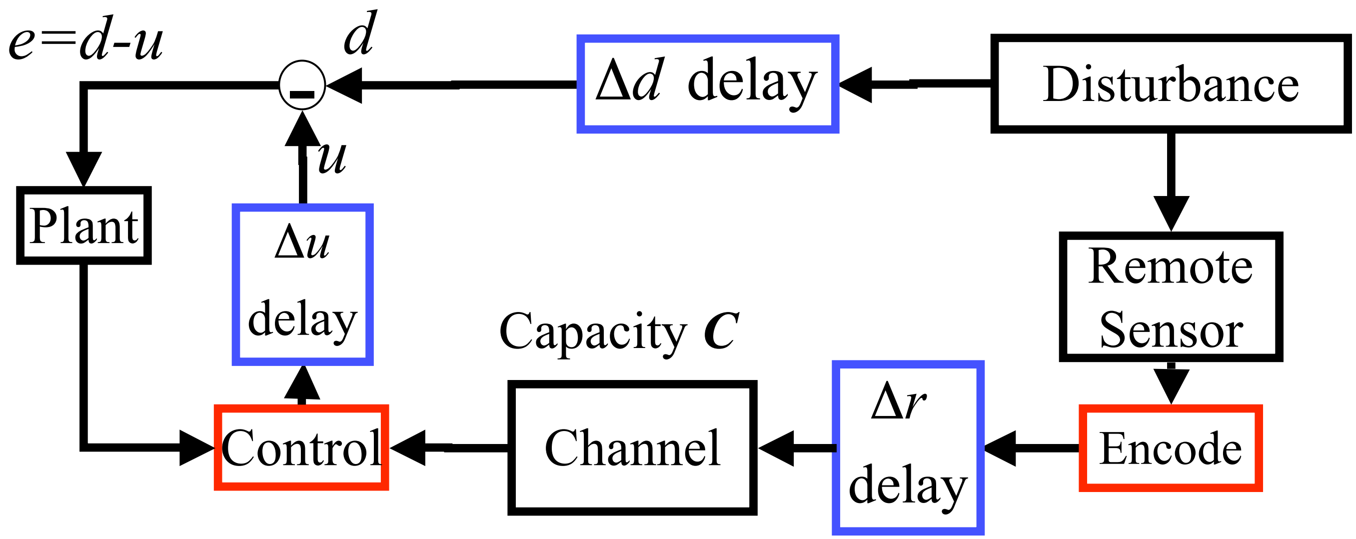
New unified comms, controls, and stat mech.?



$$\int \log|S|d\omega \geq \log(a) - \begin{cases} 0 & \Delta d < \Delta u + \Delta r \\ C & \Delta d \geq \Delta u + \Delta r \end{cases}$$

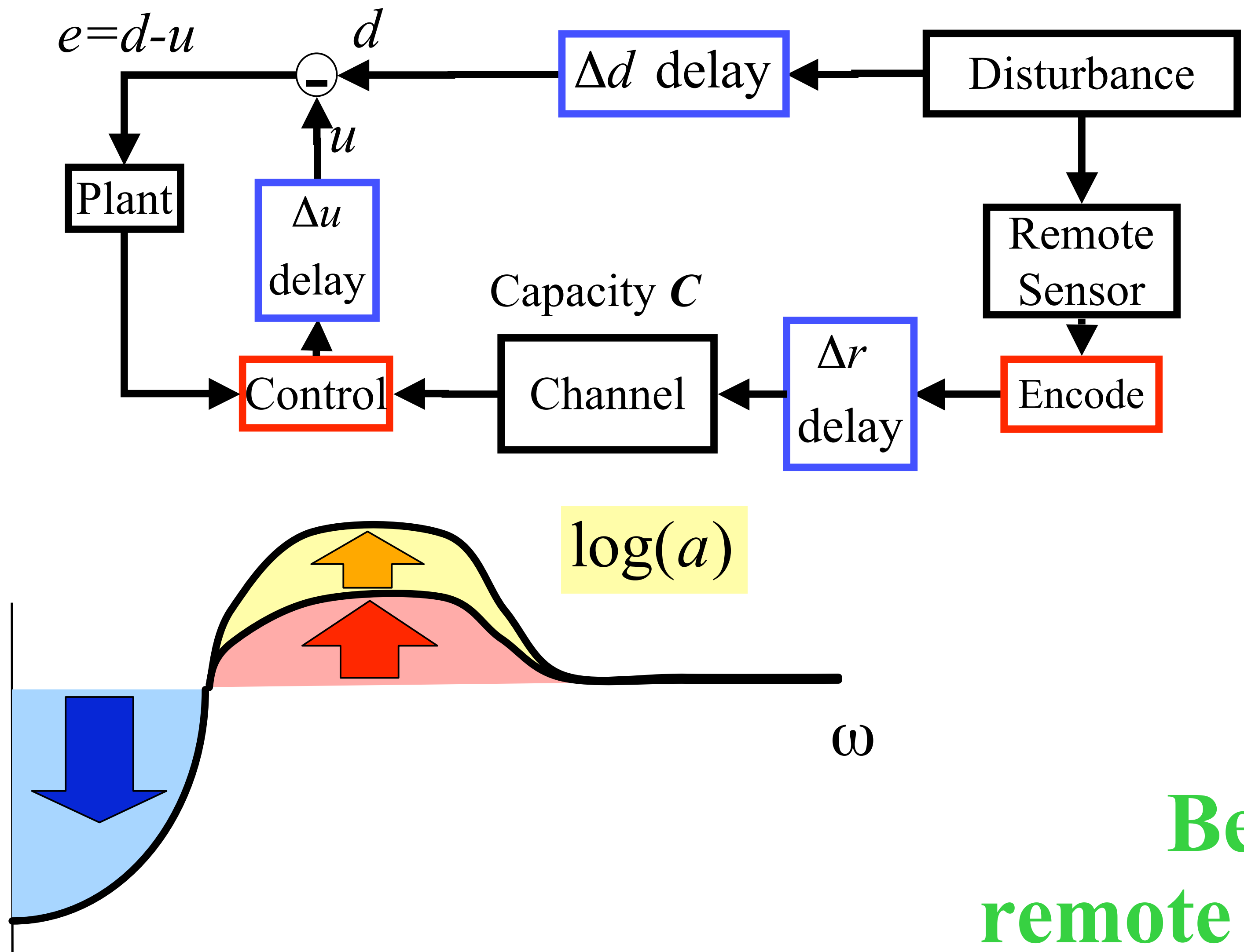
Claim (or irresponsible speculation?):

1. Biological complexity is dominated by the tradeoffs which are captured (simplistically) in this theorem.
2. Ditto for techno-networks.



$$\int \log|S|\,d\omega \geq \log(a) - \begin{cases} 0 & \Delta d < \Delta u + \Delta r \\ C & \Delta d \geq \Delta u + \Delta r \end{cases}$$

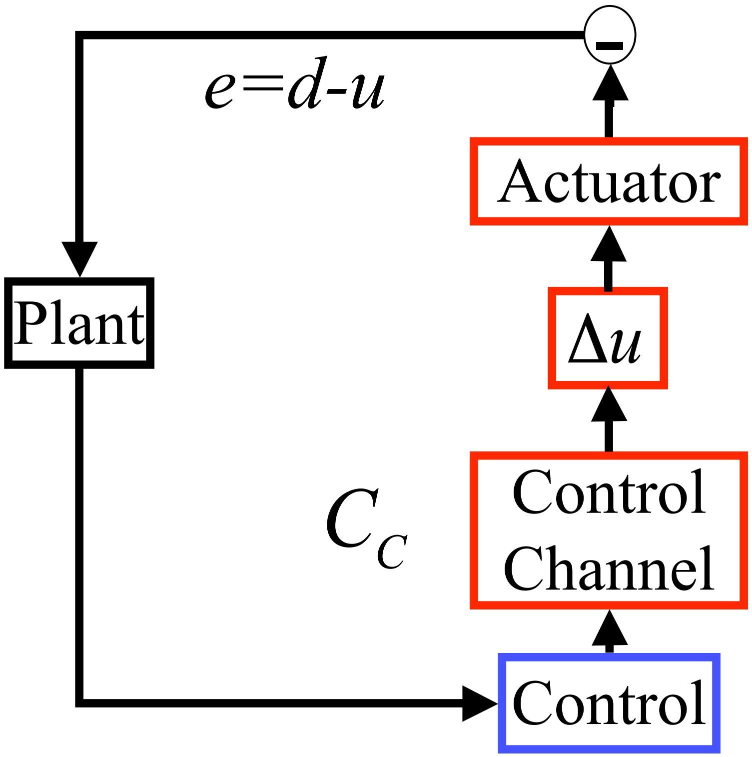
What is likely (though not agreed upon) is Bode/Shannon is a much better point-to-point communication theory to serve as a foundation for networks than either Bode or Shannon alone.



Benefit of remote sensing

$$\int [\log|S|]_- d\omega - \log(a) \geq -\int [\log|S|]_+ d\omega - C$$

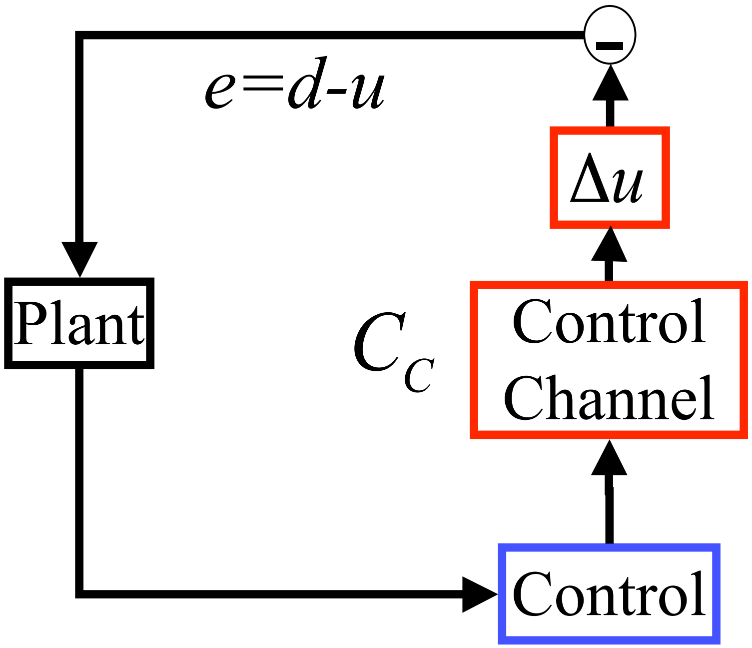
benefits **costs**



Cost of remote control

What is the cost if the control action must be done remotely and communication to an actuator is over a channel with capacity C_C ?

Note: Actuator not shown.



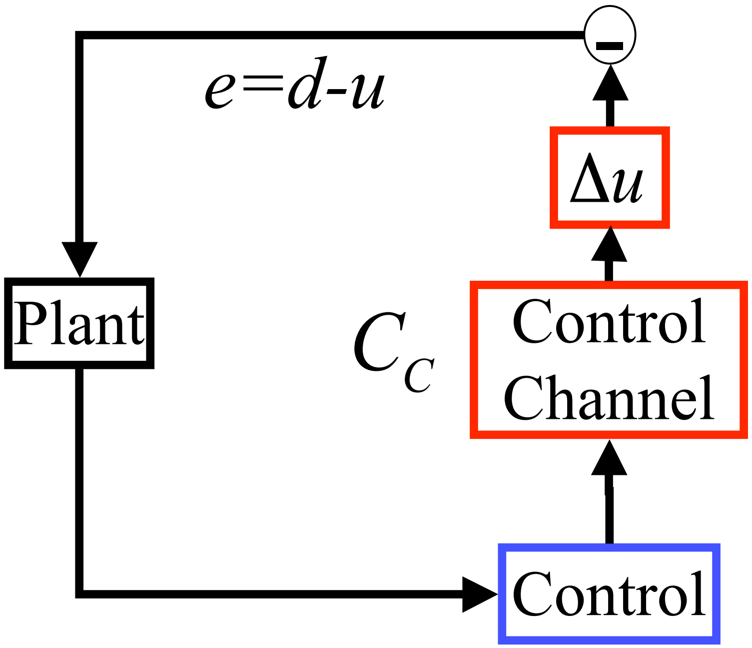
Cost of remote control

$$\int \left[\log\left|S\right|\right]_{-} d\omega \; - \log(a) \geq -C_C$$

benefits **costs**

(Note: needs directed information…)

Putting it all together



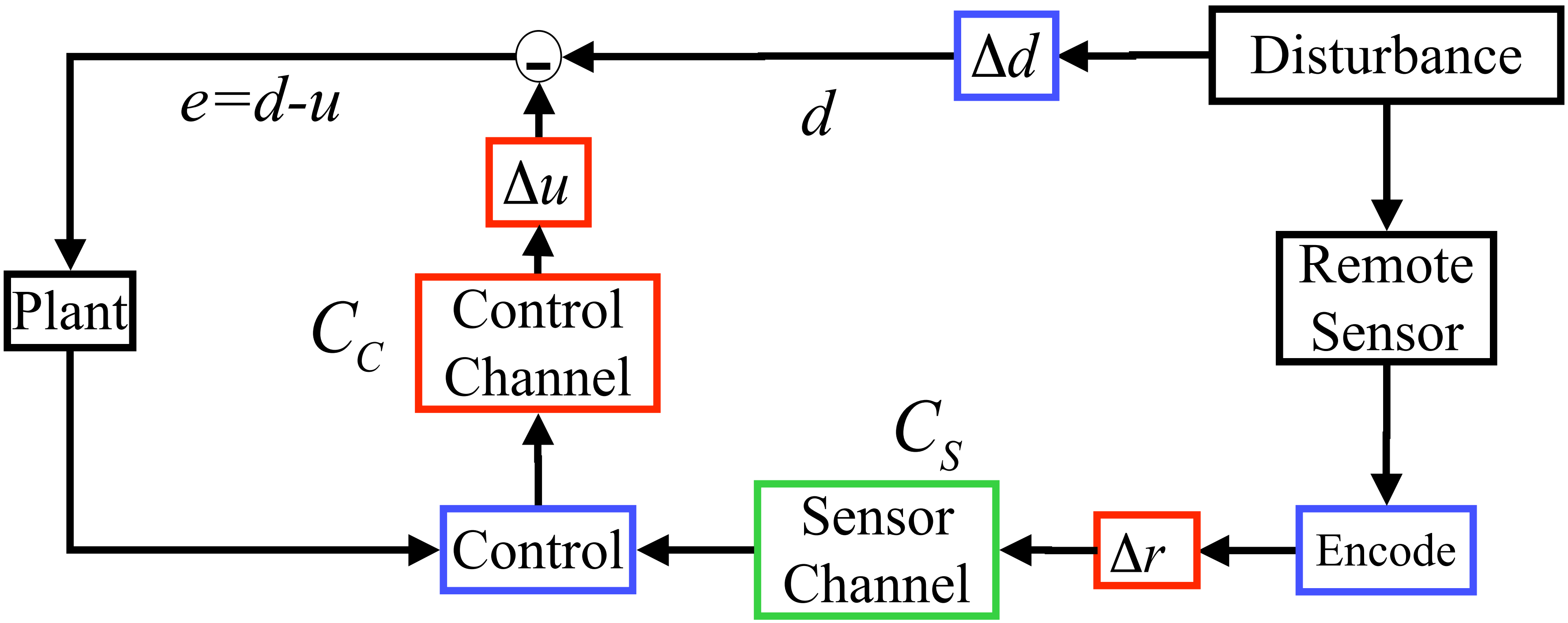
Cost of remote control

$$\int \left[\log|S|\right]_{-} d\omega - \log(a) \geq -C_C$$

Benefit of remote sensing

$$\int \left[\log|S|\right]_{-} d\omega - \log(a) \geq -\int \left[\log|S|\right]_{+} d\omega \;\; -C$$

benefits **costs**



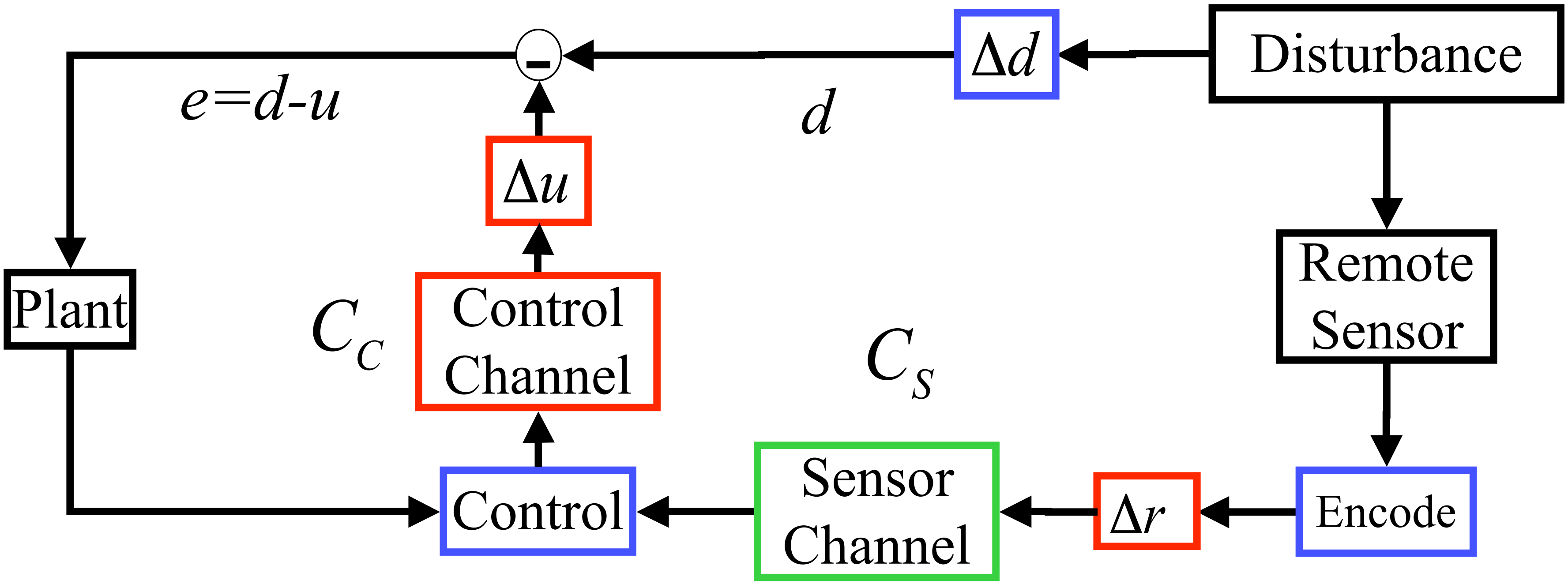
benefits stabilize

remote control

$$\int [\log|S|]_- \, d\omega \; - \log(a) \geq \begin{cases} -C_C \\ -\int [\log|S|]_+ \, d\omega \; - C_S \end{cases}$$

feedback remote sensing

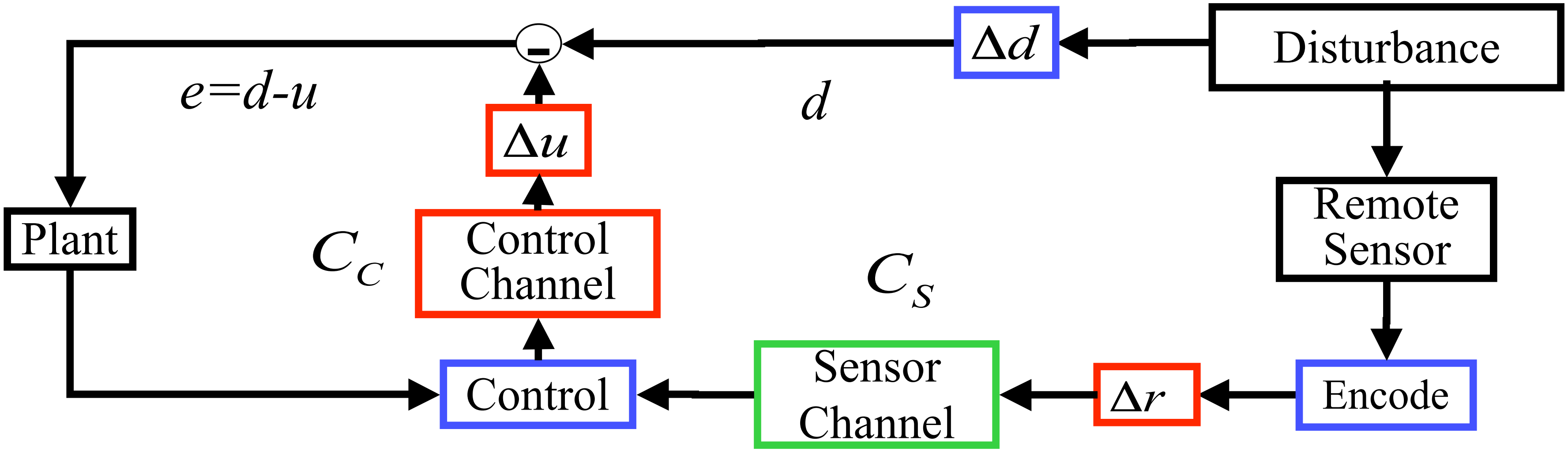
costs



$$\int [\log|S|]_- \, d\omega - \log(a) \geq \begin{cases} -C_C \\ -\int [\log|S|]_+ \, d\omega \;\; -C_S \end{cases}$$

Bode/Shannon is likely a better p-to-p comms theory to serve as a foundation for networks than either Bode or Shannon alone.

http://www.glue.umd.edu/~nmartins/



$$\int [\log|S|]_{-} d\omega - \log(a) \geq \begin{cases} -C_C \\ -\int [\log|S|]_{+} d\omega \;\; -C_S \end{cases}$$

Nuno C Martins and Munther A Dahleh, *Feedback Control in the Presence of Noisy Channels: "Bode-Like" Fundamental Limitations of Performance*.

(Submitted to the IEEE Transactions on Automatic Control) Abridged version in ACC 2005

Fundamental Limitations of Disturbance Attenuation in the Presence of Side Information

Nuno C. Martins, Munther A. Dahleh and John C. Doyle

(Submitted to the IEEE Transactions on Automatic Control) Abridged version in CDC 2005