

1. Perko Section 2.9 Problem 3
2. Perko Section 2.9 Problem 4
3. (Khalil 4.8) Consider

$$\dot{x}_1 = \frac{-6x_1}{u^2} + 2x_2, \quad \dot{x}_2 = \frac{-2(x_1+x_2)}{u^2}$$

where  $u = 1 + x_1^2$ . Let  $V(x) = \frac{x_1^2}{1+x_1^2} + x_2^2$ .

- (a) Show that  $V(x) > 0$  and  $\dot{V}(x) < 0$  for all  $x \in \mathbb{R}^2 - \{0\}$ .
- (b) Consider the hyperbola  $x_2 = \frac{2}{x_1 - \sqrt{2}}$ . Show, by investigating the vector field on the boundary of this hyperbola, that trajectories to the right of the branch in the first quadrant cannot cross the branch.
- (c) Show that the origin is not globally asymptotically stable.

Hint: In part (b), show that

$$\frac{\dot{x}_2}{\dot{x}_1} = \frac{-1}{1 + 2\sqrt{2}x_1 + 2x_1^2}$$

on the hyperbola, and compare with the slope of the tangents to the hyperbola.

4. Perko Section 2.12 Problem 2
5. Perko Section 2.12 Problem 7
6. Consider the following system in  $\mathbb{R}^2$ :

$$\begin{aligned}\dot{x} &= -\frac{3}{2}(x^2 + y^2) + 3(x + y) - 3 \\ \dot{y} &= -3xy + 3(x + y) - 3\end{aligned}$$

- (a) Determine the stable, unstable and center manifold of the equilibrium point at  $x = 1, y = 1$ .
- (b) Determine the stability of the equilibrium point at  $x = 1, y = 1$ .

*Hint: There are two 1-dimensional linear invariant manifolds of  $(1, 1)$  (i.e., two manifold of the form  $M = \{(a_1, a_2) \in \mathbb{R}^2 | a_2 = ka_1\}$ ). Find them and determine the flow in these invariant manifolds.*