



CDS 140b: Control of Bifurcations and Limit Cycles



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Caltech Control and Dynamical Systems
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Goals

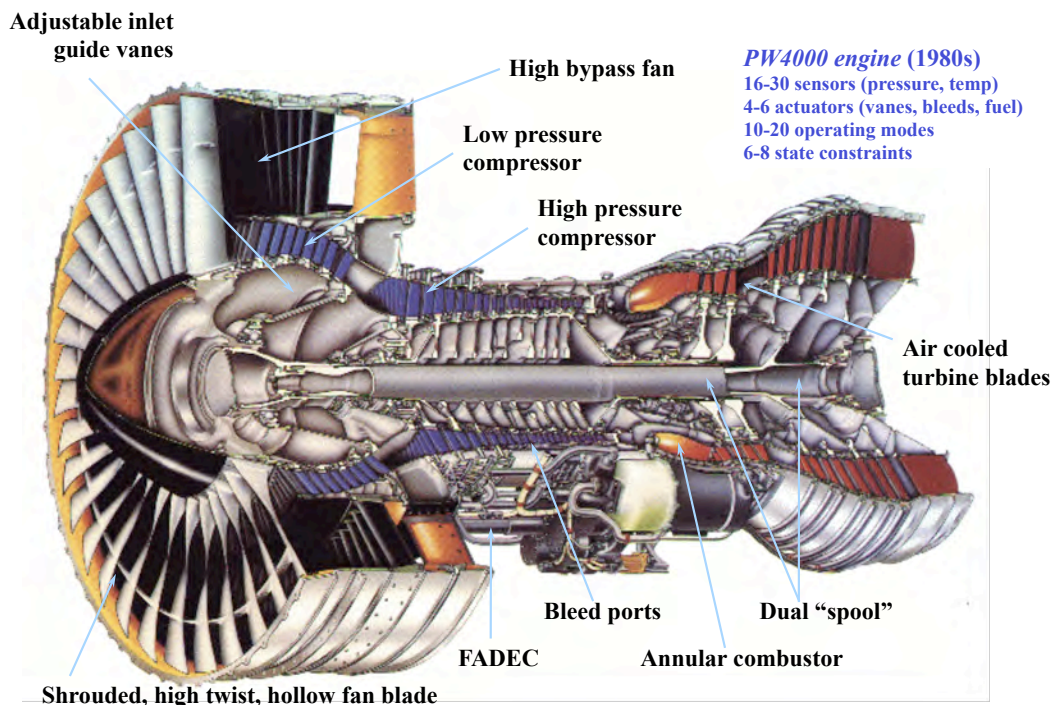
- Describe how bifurcations & limit cycles arise in engineering applications
- Review some tools for characterizing bifurcations and limit cycles
- Show how feedback can be used for design of (nonlinear) dynamics

Outline

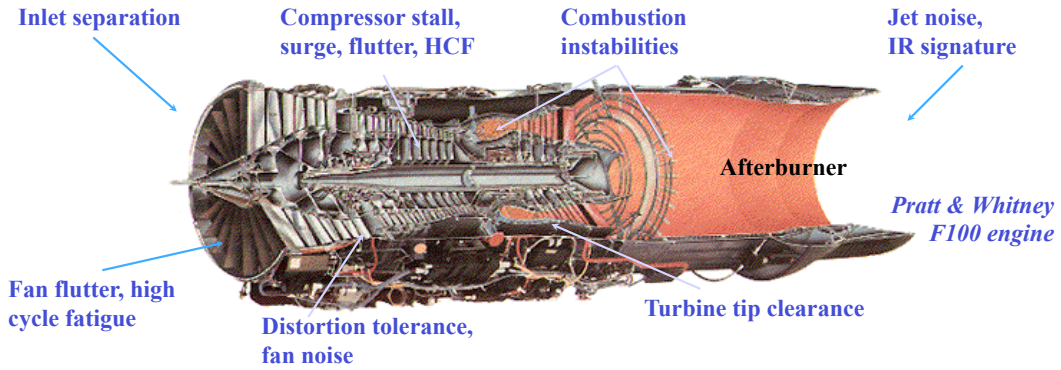
- Lecture 1: Introduction and background
- Lecture 2: Analysis and control of bifurcations
- Lecture 3: Modeling and control of limit cycles
- Lecture 4: Describing function analysis

<http://www.cds.caltech.edu/~murray/wiki/cds140-bifctrl>

Evolution of Gas Turbine Aeroengines



Performance Limitations in Aircraft Engines



Inlet separation

- Separation of flow from surface
- Possible use of flow control to modify

Distortion

- Major cause of compressor disturbances

Rotating stall and surge

- Control using BV, AI, IGVs demonstrated
- Increase pressure ration \Rightarrow reduce stages

Flutter and high cycle fatigue

- Aeromechanical instability
- Major source of maintenance, failures

Combustion instabilities

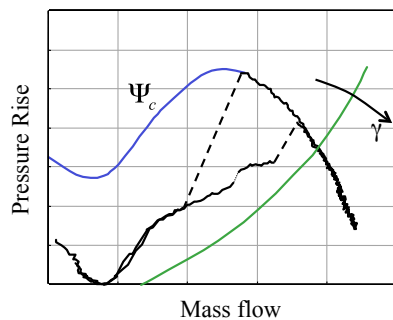
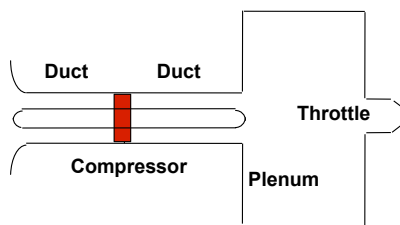
- Large oscillations cannot be tolerated
- Typically discovered late in development

Jet noise and shear layer instabilities

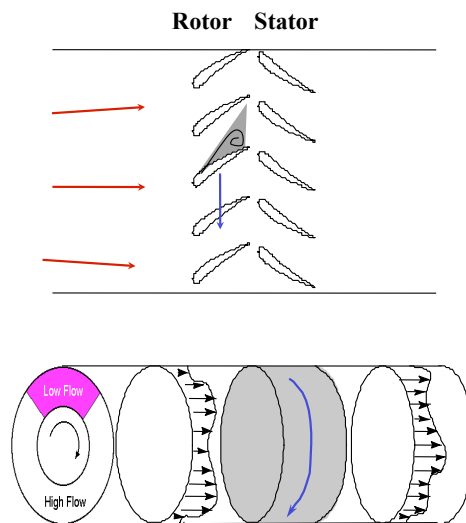
- Gov't regulations driving new innovation

Rotating Stall Dynamics

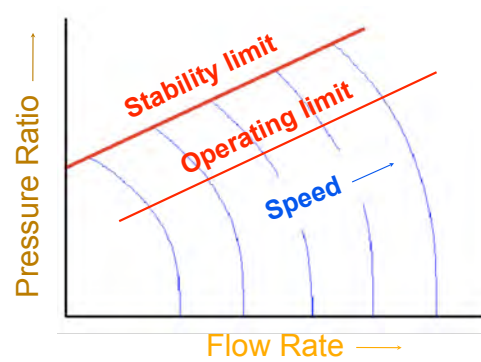
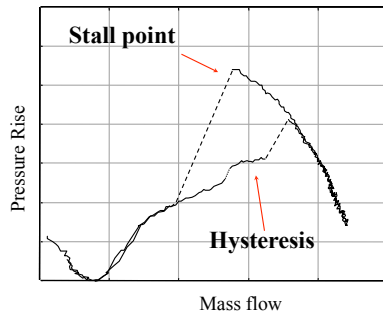
Compression System Dynamics



Emmons model (1952)



Impact of Stall and Surge on Engine Performance



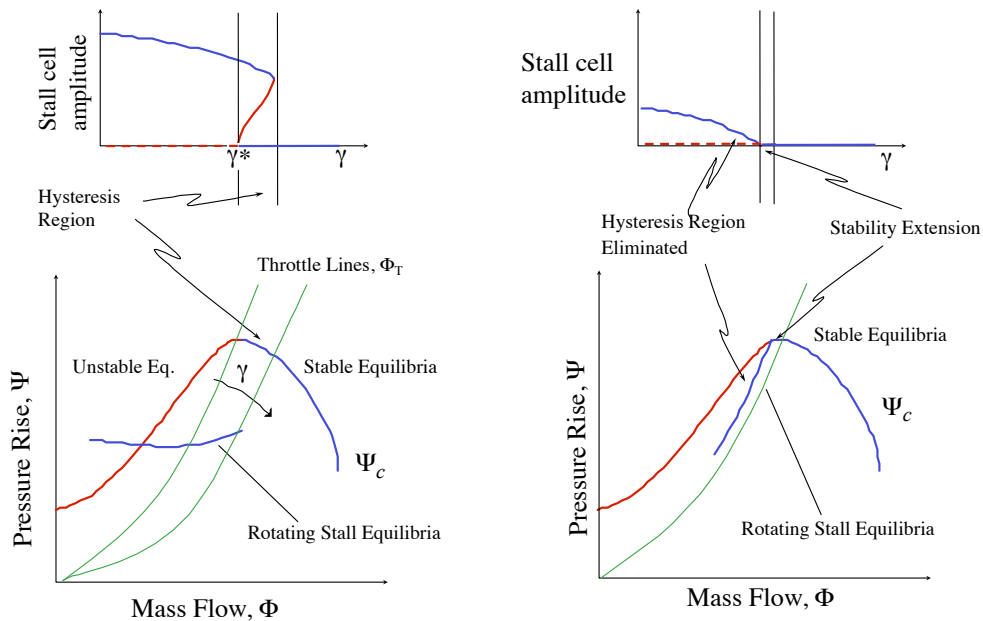
System performance limited by instability

- Number of rotors/stators required to deliver pressure set by instability limit
- Hysteresis loop forces operation away from peak pressure rise

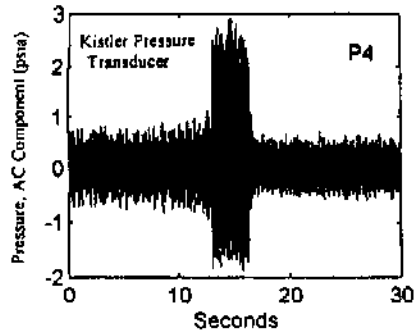
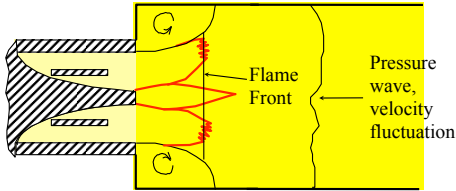
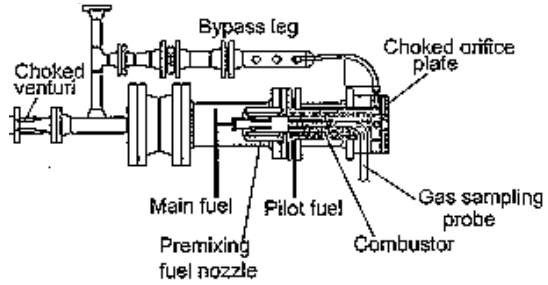
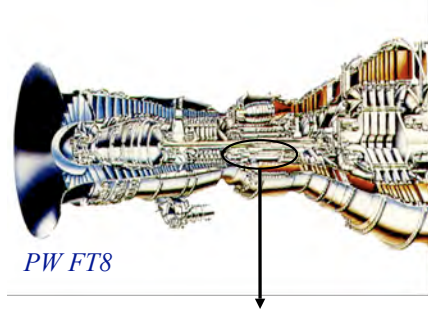
Benefits of active control of stall/surge

- 10% decrease in stalling mass flow can lead to 2% increase in fuel efficiency (!)
- Requires system redesign, not retrofit
- Complexity, weight, reliability are important (mostly unaddressed) issues

Active Control Concepts: Stabilization + Bifurcation Control



Combustion Instabilities: Lean, Premixed, Liquid Fuel

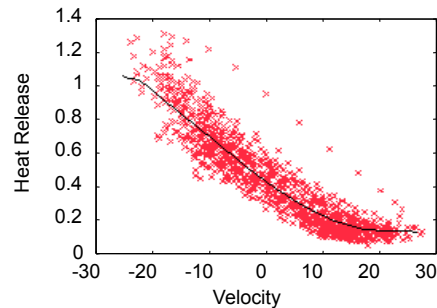
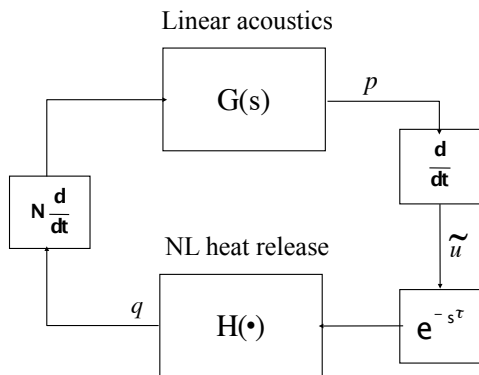
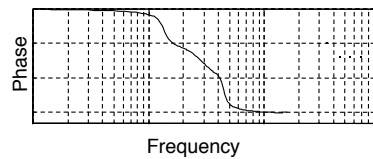
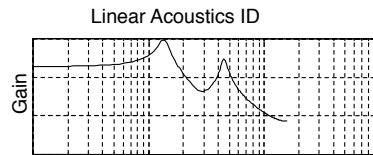
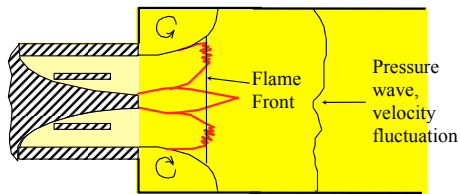


Thermoacoustic instability at lean limit

- Positive feedback between heat release and acoustic oscillations

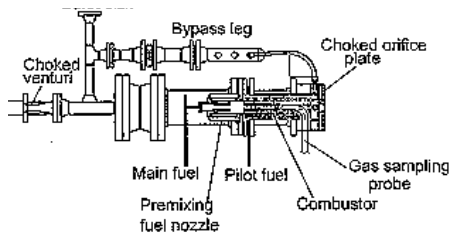
Essential Physics (Culick Model)

Murray, Jacobson, et al (ACC, 1998)



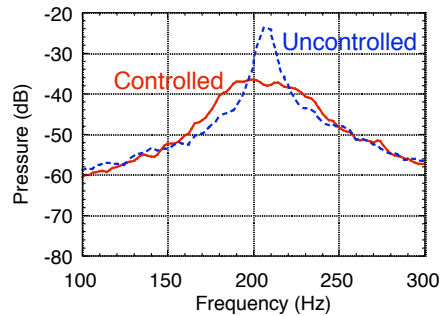
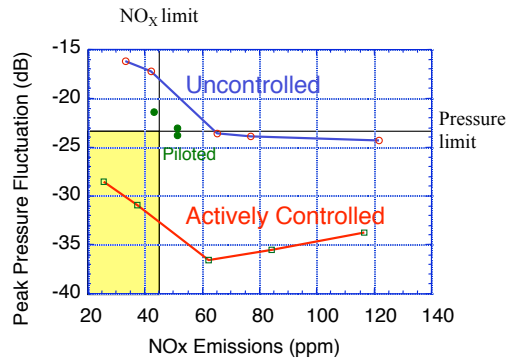
Combustion Instability Control

Proscia, Cohen, Jacobson et al (UTRC)



Modulation of main fuel flow

- Modulate main fuel flow using combustor pressure
- Simple control law (gain + phase) provides significant reduction in pressure oscillations
- Fundamental limits determined by actuator constraints (magn + BW)



Rotorcraft Separation Control (DARPA MAFC)

Must exploit dynamic effects to achieve low authority actuation

Goal

- Alleviate separation as constraint on design / performance for rotorcraft
 - Retreating blade stall
 - Airframe separation
 - Improved engine integration
 - UAV performance

Objectives

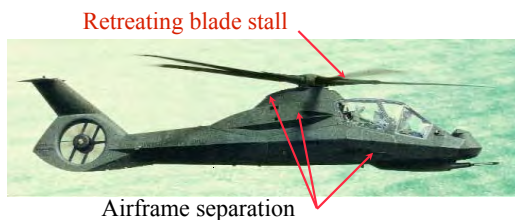
- 10% maneuverability improvement
- 50% reduction in high speed drag
 - ⇒ 20% fuel savings or +15 knot max speed

Technical Challenges

- High speed compressible flow
- Complex 3D geometries
- Actuator power and weight
- Dynamic modeling to guide design

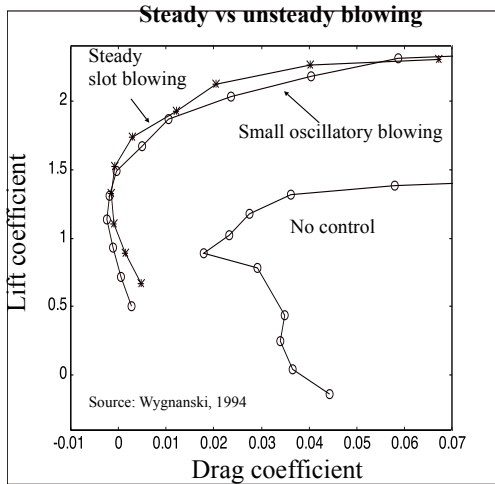
Approach

- Prioritize & downselect applications
- Minimize actuator authority
 - Model low-order dynamics of flow physics
 - Optimize location, frequency, etc.



Active Control of Separation Using Unsteady Forcing

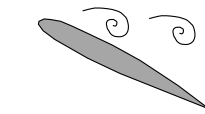
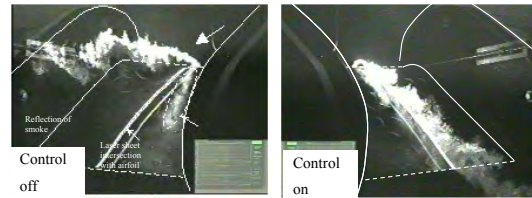
McCormick, Lorber et al (UTRC)



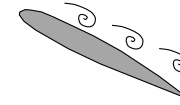
Advantages of oscillating slot blowing

- Zero mean actuation
- 10X reduction in power
- Exploits natural flow instabilities

UTRC/DARPA Airfoil Tests



Separated flow \Rightarrow loss of lift



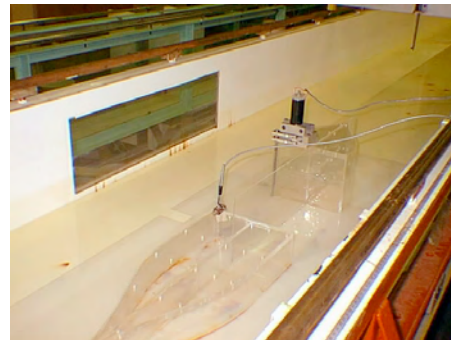
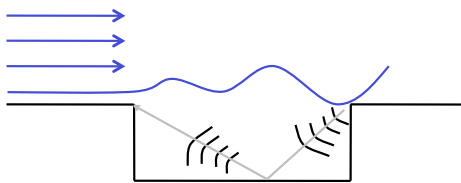
Separation reduced

Open Questions

- How much actuator authority is required (magn, rate, spacing)?
- What is the underlying mechanism that we are exploiting?
- McCormick, Aerospace Sciences, Jan 00

Cavity Flow Instabilities

C. Rowley (Princeton), T. Colonius, D. MacMynowski, R. Murray (CIT), D. Williams (MIT)



Phenomena

- Shear layer instability above cavity
- Self-excited via acoustic reflections
- Generates large oscillations

Applications

- Landing gear, bomb bays
- Railroad cars (?)

Approach

- Verify instability mechanism using CFD (Colonius, May 99)
- Build control-oriented model
 - Capture essential physics
 - Integrate actuation, sensing
- Test control in CFD, water tunnel

Common Features & Observations on Control of Fluids (ca 2001)

1. Effective control of flows in engineering applications relies on the existence of a *low order phenomenon* that control can affect

- Limits in sensing and actuation will restrict us to these cases
- Experiments leading theory; *many* examples with coherent structures

Rotating stall
Rotor stall
Cavity flows
Combustion

2. Actuator placement and limits are *critical*

- Minimize *spatial* and *temporal* authority of actuators (and sensors)
- Application specs include cost, weight, reliability, complexity, wiring
- Exploit dynamics to achieve reduced authority control

Bleed valves
Air injection
Slot blowing
Flap actuation
Fuel modulat'n

3. Stabilization of steady flows is not the most important problem

- Most examples give unsteady controlled behavior (eg, small oscillations)
- Limited spatial authority often makes linearization uncontrollable

Rotating stall
Airfoil stall
Cavity flows
Combustion

4. Need better tools

- Data-driven, control-oriented modeling & analysis
- Stabilization of unsteady flows \Rightarrow *operability enhancement*
- Need better tools for analysis and synthesis

Criticality ctrl
Fund. limits
System ID
Robustness

Bifurcations of Dynamical Systems

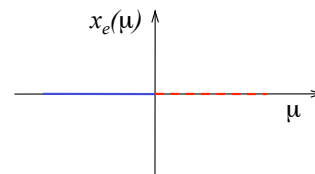
Consider a *family* of differential equations:

$$\dot{x} = f(x, \mu) \quad f(x_e(\mu), \mu) = 0$$

Defn The system has a *bifurcation* at $\mu = \mu^*$ if the flow of the system changes quantitatively at μ^* .

Example 1: exchange of stability

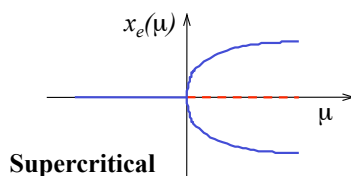
$$\dot{x} = \mu x \quad x_e(\mu) = 0$$



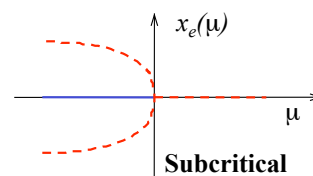
Example 2: pitchfork bifurcation

$$\dot{x} = \mu x - x^3 \quad x_e(\mu) = 0, \pm\sqrt{\mu}$$

$$\dot{x} = \mu x + x^3 \quad x_e(\mu) = 0, \pm\sqrt{-\mu}$$

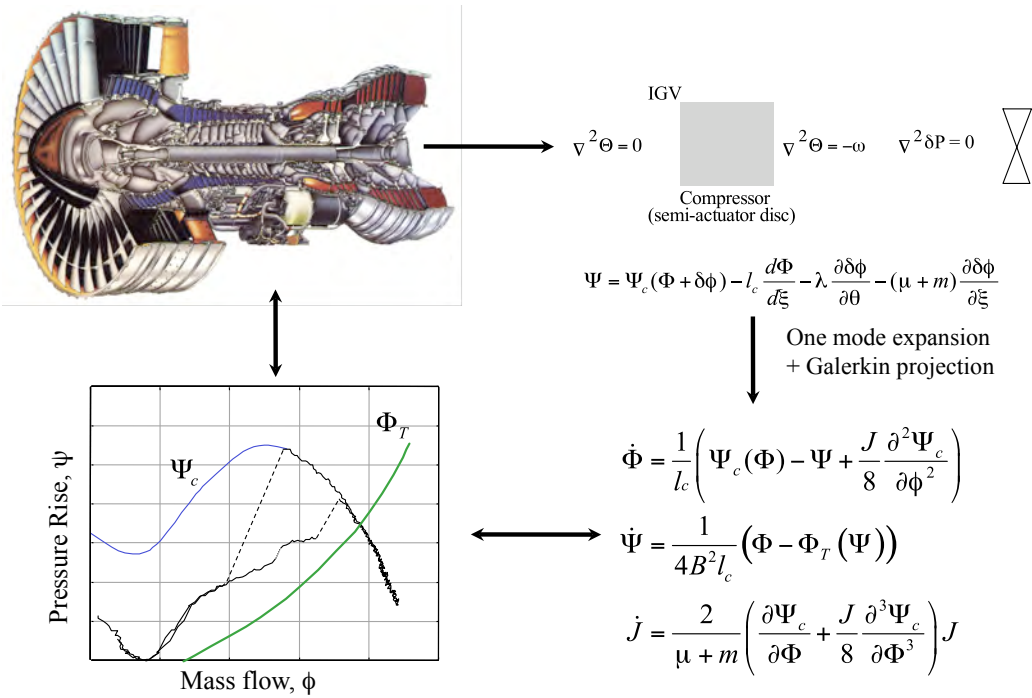


Supercritical



Subcritical

Moore-Greitzer Model (1986)



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Linear Stability Analysis of MG-3 Model

Linearization around J=0 equilibrium point:

$$\dot{\phi} = \frac{1}{l_c} (m_c \phi - \psi)$$

$$m_c = \frac{\partial \Psi_c}{\partial \Phi}$$

Slope of compressor characteristic

$$\dot{\psi} = k_B \left(\frac{1}{l_c} \phi - m_T \psi \right)$$

$$m_T = \frac{\partial \Phi_T}{\partial \Psi}$$

Slope of throttle

$$j = \frac{2}{m + \mu} m_c J$$

$$k_B = \frac{1}{4B^2}$$

B = Greitzer B-parameter

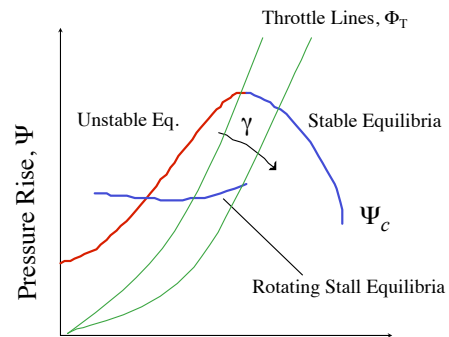
Stability conditions:

$$m_c < \frac{1}{4B^2 m_T}$$

Surge mode

$$m_c < 0$$

Stall mode



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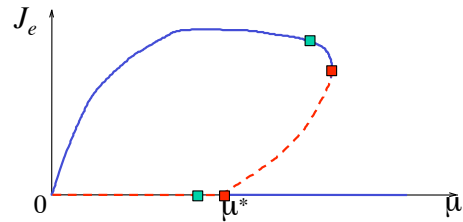
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Example: MG-3 model

$$\dot{\Phi} = \frac{1}{l_c} \left(\Psi_c(\Phi) - \Psi + \frac{J}{8} \frac{\partial^2 \Psi_c}{\partial \Phi^2} \right)$$

$$\dot{\Psi} = \frac{1}{4B^2 l_c} (\Phi - \Phi_T(\Psi))$$

$$\dot{J} = \frac{2}{\mu + m} \left(\frac{\partial \Psi_c}{\partial \Phi} + \frac{J}{8} \frac{\partial^3 \Psi_c}{\partial \Phi^3} \right) J$$



Bifurcation #1: transcritical bifurcation (subcritical)

- Basically the same as a subcritical pitchfork bifurcation

Bifurcation #2: saddle-node bifurcation

- New pair of equilibria appear; one stable, one unstable

Bifurcation #3, 4: Hopf bifurcation to surge (not analyzed)

- Linear stability condition:

$$m_c < \frac{1}{4B^2 m_T}$$

Net effect:
hysteresis loop

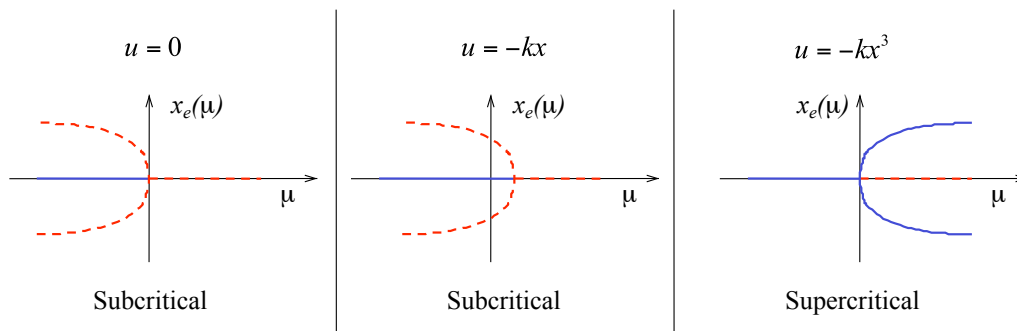
Bifurcation Control

$$\dot{x} = f(x, \mu, u) \quad u = \alpha(x)$$

Question: can we change the bifurcation behavior using $\alpha(x)$?

Example: bifurcation control of a pitchfork bifurcation

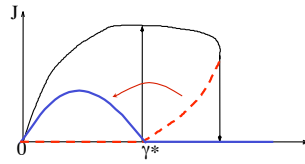
$$\dot{x} = \mu x + x^3 + u$$



Outline for Remaining Lectures

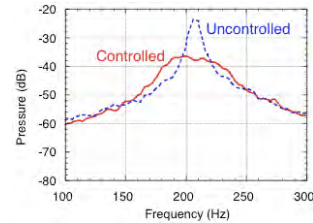
Lecture 2: Analysis and control of bifurcations

- Main idea: eliminate hysteresis loops and other global structures in the dynamics
- Key limitation: actuation magnitude and rate limits



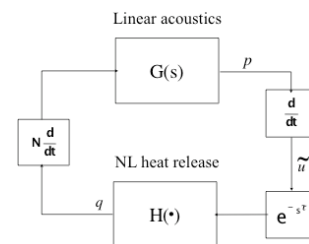
Lecture 3: Modeling and control of limit cycles

- Look at higher dimensional attracting sets
- Focus on control of amplitude of limit cycles
- Non-equilibrium behavior => trickier to control



Lecture 4: Describing functions (harmonic balance)

- Extension of loop analysis (Nyquist) to handle static nonlinearities that enter in a simple way
- Gives good intuition about how to detect limit cycles and how control can be used to change their amplitude (or eliminate them)
- Very useful technique, but often not taught...



Project Ideas

Survey papers

- Stabilization of homogeneous systems (M'Closkey et al)
- Stabilization in the presence of magnitude and rate constraints (Wang, Teel)
- Harmonic balance (generalization of describing functions)
- Nonlinear "peaking": what is it and how do you avoid it?

Case study: apply techniques to example with noise, uncertainty, etc

- Compression system instabilities with multiple modes (will do 1st mode in class)
- Cavity flow or combustion instabilities
- Mechanical example (pick something from CDS 140a)

Extensions of existing work (via analysis of a simple example)

- Robustness analysis for control of bifurcations, limit cycles, describing functions
- Disturbance/noise attenuation near bifurcation points

<http://www.cds.caltech.edu/~murray/wiki/cds140-bifctrl>