Lecture 6: Cooperative Control Systems

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17 March 2009

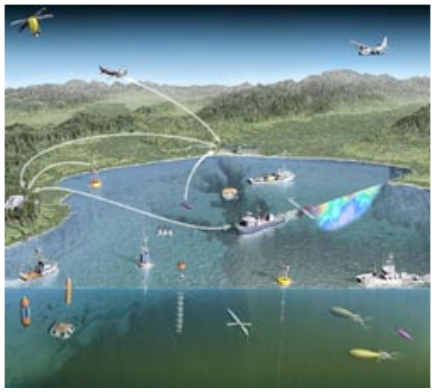
Goals:

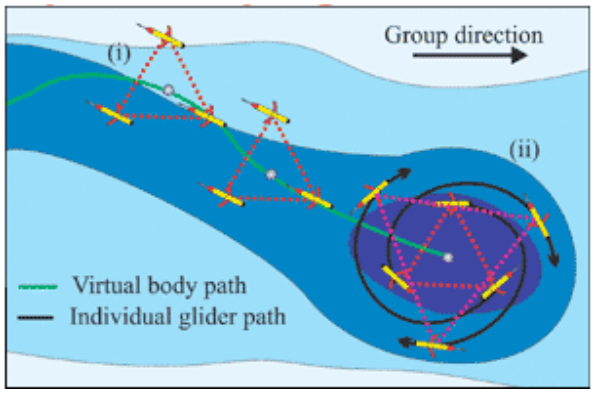
- Definition and examples of cooperative control systems
- Distributed receding horizon control
- Survey of other results in cooperative control: formations, coverage, ...

Reading:

- R. M. Murray, "Recent Research in Cooperative Control of Multi-Vehicle Systems", *Journal of Dynamic Systems, Measurement and Control*, 129(5):571-583, 2007

Adaptive Ocean Sampling Network

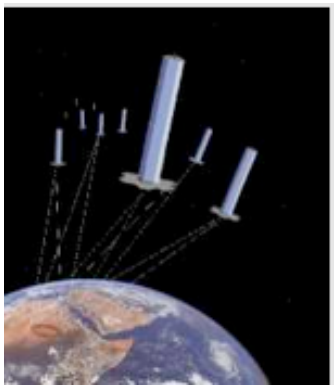


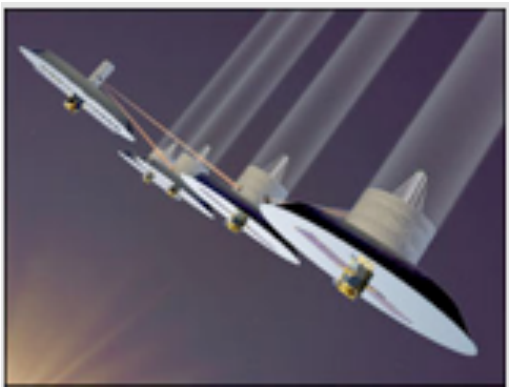


Goal: track the important events and dynamics in the Monterey Bay (Ca)

- Motion of vehicles is based on the observations taken by the vehicles
- Allows sensors to be positioned in the areas in which they can do the most good, as a function of the data already collected
- Cooperative control strategy is used to control the motion of the vehicles
- Summer, 2006: 10 gliders were controlled over 4 weeks to collect data
- More info: http://www.mbari.org/aosn + Leonard et al (TAC, 2007)

Distributed Aperature Observing Systems





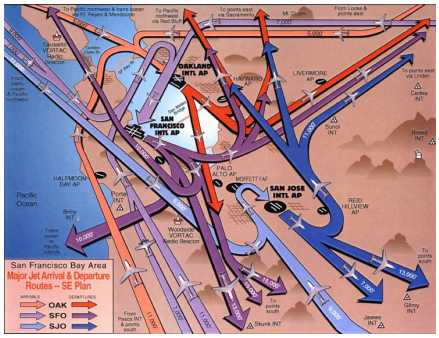
TechSat 21 (AFRL)

- Collection of ``microsatellites'' that would be used to form a ``virtual'' satellite with a single, large aperture antenna
- Project cancelled in 2003 due to funding limits (12 satelites -> 3 sats -> 1 sat)

Terrestrial Planet Finder (NASA)

- Use optical interferometry to image distance stars and to detect slight shifts in the stars positions that indicate presence of planets orbiting the stars

Transportation Systems



California Partners for Advanced Transit and Highways (PATH)

- System for allowing cars to be driven automatically down a freeway at close spacing
- Idea: reduce speed of collision via close spacing; need to worry about string stability

Next generation air traffic control

- Move from a human-controlled, centralized structure to a more distributed system
- Enable ``free flight'' technologies allowing aircraft to travel in direct paths rather than staying in pre-defined air traffic control corridors.
- Improve the current system by developing cockpit ``sensors'' such as augmented

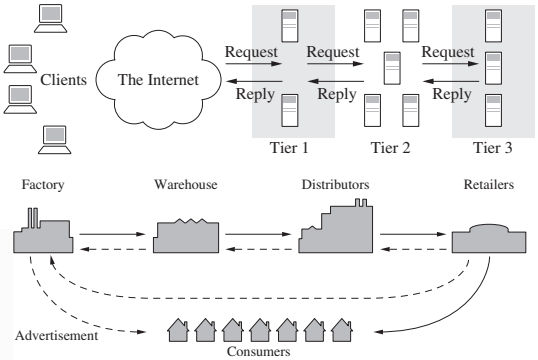
Other Cooperative Control Systems

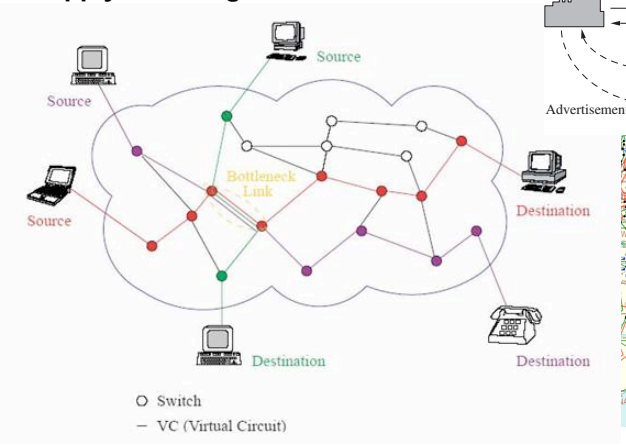
Power grid

Communication networks

- Networking/congestion control
- Routing/queue management
- Servers/resource allocation

Supply chain mgmt





M
JGCD, 2007

Cooperative Control Systems Framework

Agent dynamics

$$\dot{x}^i = f^i(x^i, u^i) \quad x^i \in \mathbb{R}^n, u^i \in \mathbb{R}^m$$
$$y^i = h^i(x^i) \quad y^i \in \mathbb{R}^q$$

Vehicle "role"

- $\alpha \in \mathcal{A}$ encodes internal state + relationship to current task
- Transition $\alpha' = r(x, \alpha)$

Communications graph $\mathcal{G}$

- Encodes the system information flow
- Neighbor set $^i(x, \alpha)$

Communications channel

- Communicated information can be lost, delayed, reordered; rate constraints

$$y_j^i[k] = \gamma y^i(t_k - \tau_j) \quad t_{k+1} - t_k > T_r$$

- γ = binary random process (packet loss)

Task

- Encode as finite horizon optimal control

$$J = \int_0^T L(x, \alpha, \mathcal{E}(t), u)\, dt + V(x(T), \alpha(T)),$$

- Assume task is *coupled,* env't estimated

Strategy

- Control action for individual agents

$$u^i = k^i(x, \alpha) \quad \{g_j^i(x, \alpha) : r_j^i(x, \alpha)\}$$

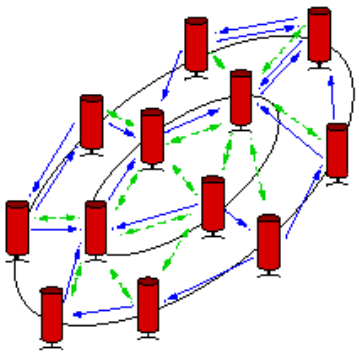
$$\alpha^{i\prime} = \begin{cases} r_j^i(x, \alpha) & g(x, \alpha) = \text{true} \\ \text{unchanged} & \text{otherwise.} \end{cases}$$

***Decentralized* strategy**

$$u^i(x, \alpha) = u^i(x^i, \alpha^i, y^{-i}, \alpha^{-i}, \hat{\mathcal{E}})$$
$$y^{-i} = \{y^{j_1}, \ldots, y^{j_{m_i}}\}$$
$$j_k \in \mathcal{N}^i \quad m_i = |\mathcal{N}^i|$$

- Similar structure for role update

Information Flow in Vehicle Formations



Example: satellite formation

- Blue links represent *sensed* information
- Green links represent *communicated* information

Sensed information

- Local sensors can see some subset of nearby vehicles
- Assume small time delays, pos'n/vel info only

Communicated information

- Point to point communications (routing OK)
- Assume limited bandwidth, some time delay
- Advantage: can send more complex information

Topological features

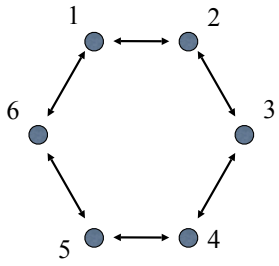
- Information flow (sensed or communicated) represents a directed graph
- Cycles in graph ⇒ information feedback loops

Question: How does topological structure of information flow affect stability of the overall formation?

Sample Problem: Formation Stabilization

Goal: maintain position relative to neighbors

- "Neighbors" defined by graph
- Assume only sensed data for now
- Assume identical vehicle dynamics, identical controllers?



Example: hexagon formation

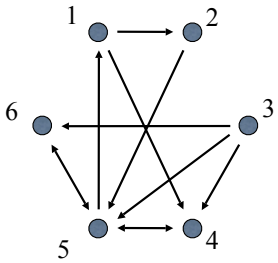
- Maintain fixed relative spacing between left and right neighbors

$$e_i = \sum_{j \in N_i} w_j (y_i - y_j - h_{ij})$$

relative position

weighting factor

offset

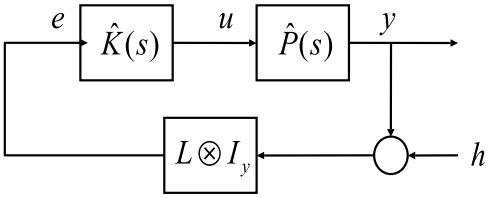


Can extend to more sophisticated "formations"

- Include more complex spatio-temporal constraints

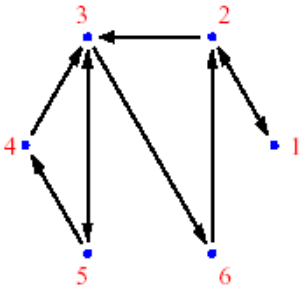
Fax and M
IEEE TAC 2004

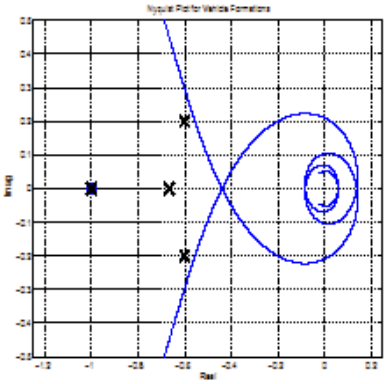
Stability Condition



Theorem *The closed loop system is (neutrally) stable iff the Nyquist plot of the open loop system does not encircle* $-1/\lambda_i(L)$, *where* $\lambda_i(L)$ *are the nonzero eigenvalues of* L.

Example $P(s) = \dfrac{e^{-s\tau}}{s^2}$ $K(s) = K_d s + K_p$

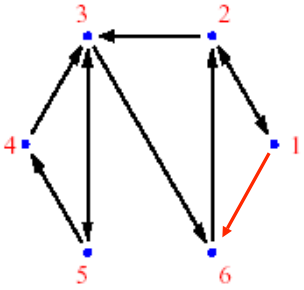


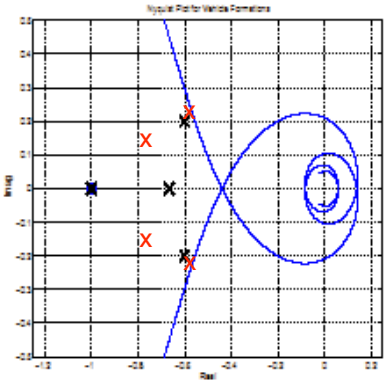


Example Revisited

Example

$P(s) = \dfrac{e^{-s\tau}}{s^2}$ $K(s) = K_d s + K$





- Adding link increases the number of three cycles (leads to “resonances”)
- Change in control law required to avoid instability
- Q: Increasing amount of information available *decreases* stability (??)
- A: Control law cannot ignore the information ⇒ add’l feedback inserted

Fax and M
IEEE TAC 2004

Improving Performance through Communication

Baseline: stability only

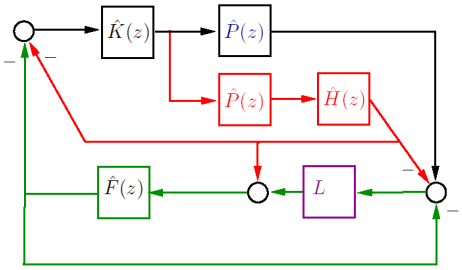
- Poor performance due to interconnection

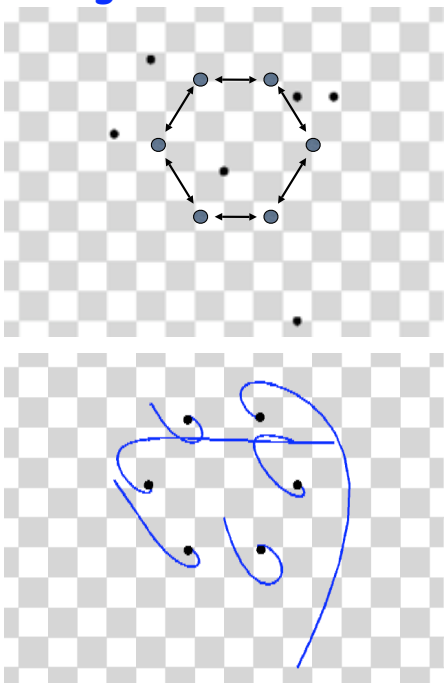
Method #1: tune information flow filter

- Low pass filter to damp response
- Improves performance somewhat

Method #2: consensus + feedforward

- Agree on center of formation, then move
- Compensate for motion of vehicles by adjusting information flow

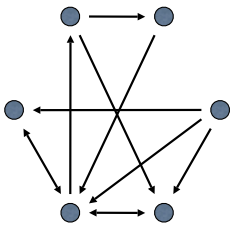


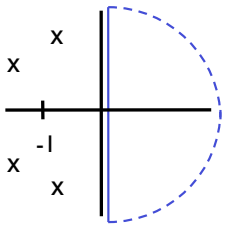


Special Case: (Asymptotic) Consensus

$$\dot{x}_i = \sum w_{ij}(x_j - x_i)$$

$$\dot{x} = -Lx$$





Consensus: agreement between agents using information flow graph

- Can prove asymptotic convergence to single value if graph is connected
- If w_{ij} = 1/(in-degree) + graph is *balanced* (same in-degree for all nodes) ⇒ all agents converge to average of initial condition

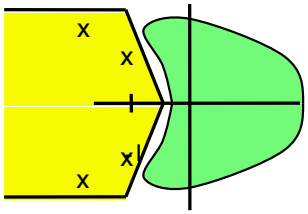
Extensions (Jadbabaie/Morse, Moreau, Olfati-Saber, Xiao, Chandy/Charpentier, ...)

- Switching (packet loss, dropped links, etc),time delays, plant uncertainty
- Nearest neighbor graphs, small world networks, optimal weights
- Nonlinear: potential fields, passive systems, gradient systems
- Distributed Kalman filtering, distributed optimization
- Self-similar algorithms for operation with varying connectedness
- See also: gossip algorithms, load balancing, distributed computing (Tsitsiklis)

Open Problems: Design of Information Flow (graph)

How does graph topology affect location of eigenvalues of L?

- Would like to separate effects of topology from agent dynamics



- Possible approach: exploit form of characteristic polynomial

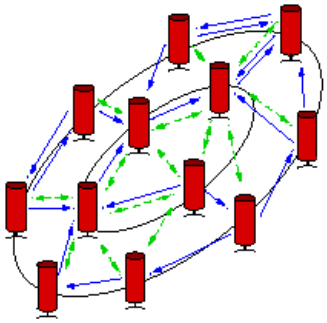
$$\lambda(s) = \text{“}s^n + \Big(\sum w_i\Big)s^{n-} + \Big(\sum_{[redacted]} w_i w_j\Big)s^{n-} + \Big(\sum_{[redacted]} w_i w_j w_k\Big)s^{n-} + \cdots + \Big(\sum_{[redacted]} w_i \ldots w_N\Big)\text{”}$$

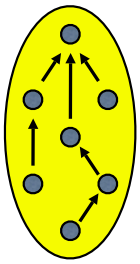
- Exploit structure of quadratic invariance (Rotkowitz and Lall)

Jin and M
CDC 04

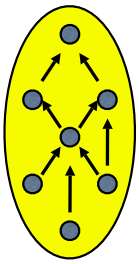
Performance

Look at motion between selected vehicles





G_1 - Control



G_2 - Performance

Theorem

For a leader-follower formation with double-graph strategy, If $\|(1-\alpha)\cdot H(s)\|_\infty = M < 1$, *then*

$$\frac{\|E\|_\infty}{\|V\|_\infty} \le \left(\frac{1-M^n}{1-M} + \frac{\rho(\Theta_2)M^n}{1-\rho(\Theta_2)M}\right)\cdot\|N(s)\|_\infty$$

where $\rho(\Theta_2)$ *is the spectral radius of* Θ_2 *and* $N(s) = \frac{e_1(s)}{v_1(s)}$.

String Stability

Goal: string of vehicles following each other

- Transients can grow as you pass down the line
- System is *string stable* if for every ε there exist a δ such that

$$\sup_i \|x^i(0)\| < \delta \quad \Longleftrightarrow \quad \sup_i \|x^i(\cdot)\|_\infty < \epsilon$$

  (∞ norm with respect to time)
- Problem can get worse when there are cycles of information (due to performance specs)

One solution: mix in global information

- Allow some centralized information to be used to provide stability and robustness

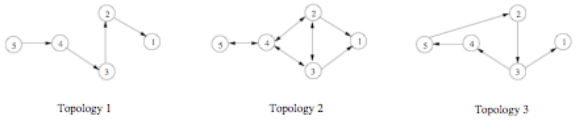
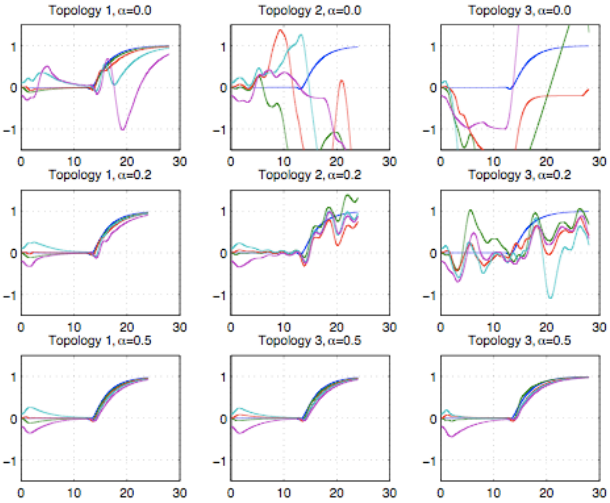


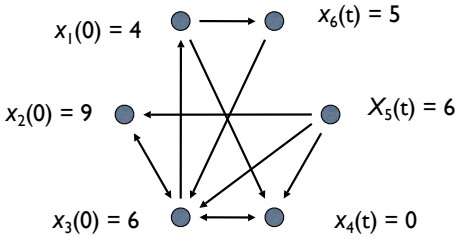
Fig. 9. Three topologies of the five-vehicle formation



Gupta, Langbort and M CDC 06

Robustness

What happens if a single node "locks up"



- Single node can change entire value of the consenus
- Desired effect for "robust" behavior: $\Delta x_l = \delta/N$

Different types of robustness (Gupta, Langbort & M)

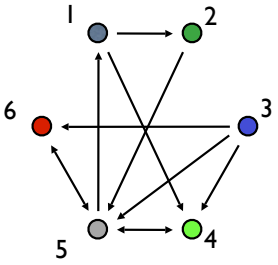
- Type I - node stops communicating (stopping failure)
- Type II - node communicates constant value
- Type III - node computes incorrect function (Byzantine failure)

Related ideas: delay margin for multi-hop models (Jin and M)

- Improve consensus rate through multi-hop, but create sensitivity to communications delay

Cremean and M
CDC 2003

Stability of (Heterogeneous) Nonlinear Systems



Model as affine nonlinear system

$$\dot{x}_i = f_i(x_i) + B_i u$$

- allow agents to have different dynamics

Stability conditions

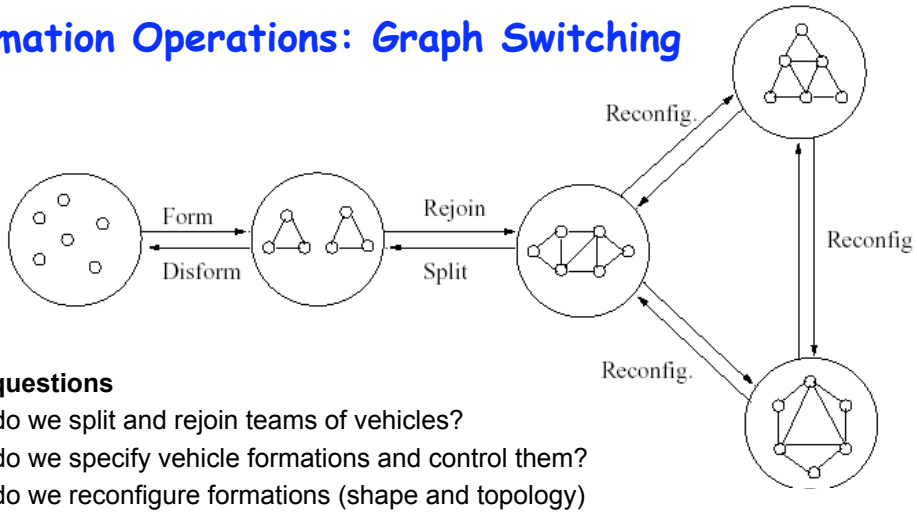
- Asy stable if

$$x_i^T f_i(x_i) < 0 \quad \text{for all } i$$

$$L \otimes BC \geq 0$$

- Fairly weak set of conditions: tells us when interconnection doesn't *destabilize* system

Formation Operations: Graph Switching



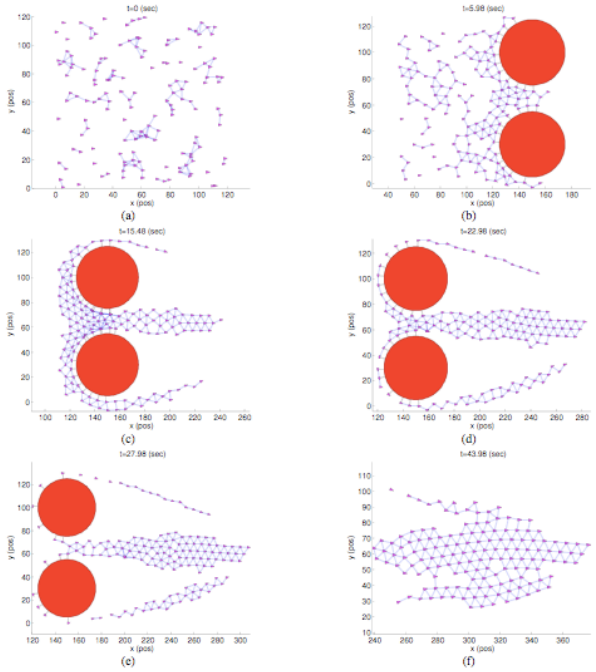
Control questions

- How do we split and rejoin teams of vehicles?
- How do we specify vehicle formations and control them?
- How do we reconfigure formations (shape and topology)

Consensus-based approach using balanced graphs

- If each subgraph is balanced, disagreement vector provides common Lyapunov fcn
- By separately keeping track of the flow in and out of nodes, can preserve center of mass of of subgraphs after a split manuever

Extensions to Flocking (Olfati-Saber)



Swarm behavior using nearest neighbor rules

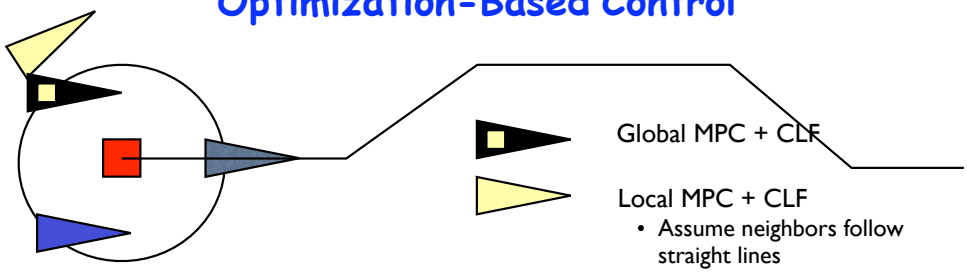
- Implement a control law of the form

$$u^i = f_g^i + f_d^i + f_\gamma^i.$$

- $f_g^i = -V(y^i, y^{-i})$ is a gradient-based term where V is a potential function
- $f_d^i = \alpha(q)(v^i - v^j)$ is a damping term based on the relative velocities of neighboring vehicles
- f_γ^i is a navigational feedback term that takes into account a group objective, such as moving to a given rendezvous point
- Use potential function to keep vehicles away from obstacles but near each other

Optimization-Based Control

Dunbar and M
Automatica, 2006

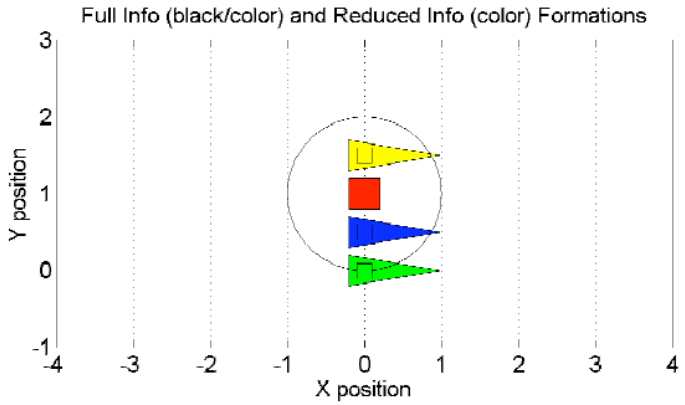


Task:

- Maintain equal spacing of vehicles around circle
- Follow desired trajectory for center of mass

Parameters:

- Horizon: 2 sec
- Update: 0.5 sec



Main Idea: Assume Plan for Neighbors

Individual optimization:

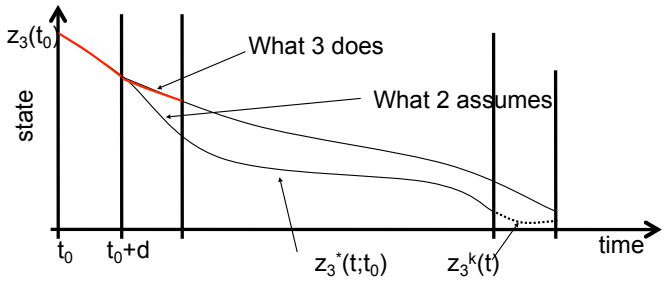
$$\min_{u_3(\cdot)} \left\{ \int_{t_k}^{t_k+T} L_3(z_3(\tau), \hat{z}_2(\tau), u_3(\tau))\, d\tau + G_3(z_3(t_k+T)) \right\}$$

$$\text{s.t.} \quad \dot{z}_3(t) = f_3(z_3(t), u_3(t))$$
$$u_3(t) \in U_3, \quad z_3(t_k+T) \in Z_{f3}$$
$$||z_3(t) - \hat{z}_3(t)|| \le \delta^2 \kappa$$

Compatibility constraint:
- each vehicle transmits plan to neighbors
- stay w/in bounded path of what was transmitted

Theorem. Under suitable assumptions, vehicles are stable and converge to globally optimal solution.

Pf Detailed Lyapunov calculation (Dunbar thesis)



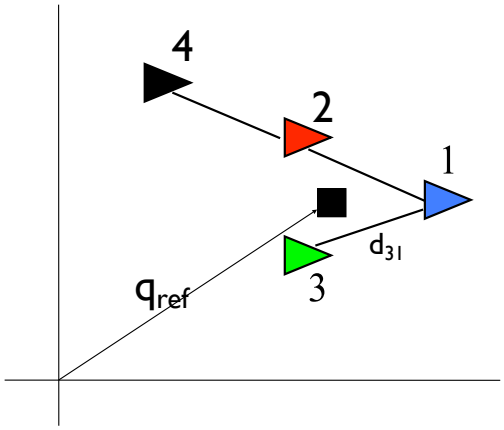
Example: Multi-Vehicle Fingertip Formation

Four vehicles with $\ddot{q}_i = u_i$.

Constraints:
$U_i = \{(v_1, v_2) \in \mathbb{R}^2 : -1 \le v_{1,2} \le 1\}$.

Formation defined by:

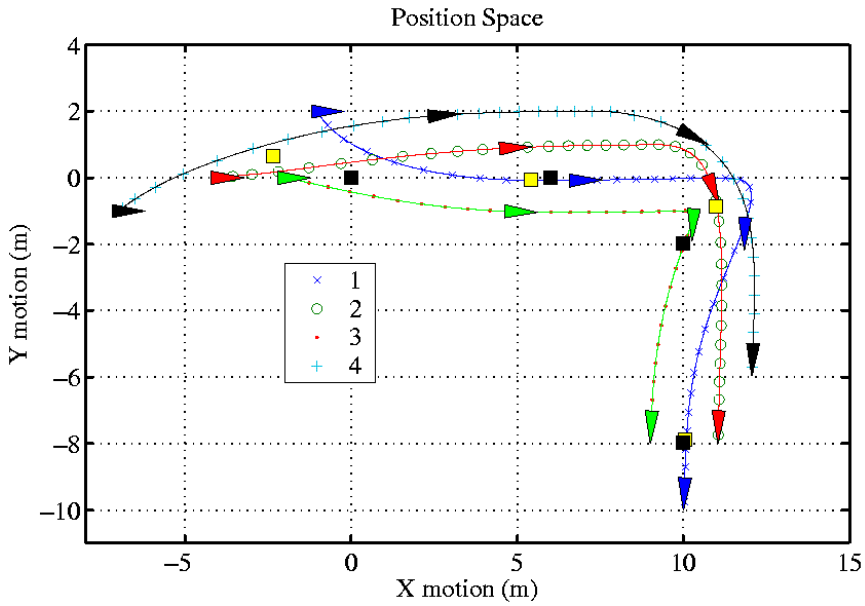
- Relative vectors d_{ij}
- COM of $\{1,2,3\}$ tracking signal $(q_{\text{ref}}, \dot{q}_{\text{ref}})$.



Coupled objective function

$$L(z,u) = ||q_3 - q_1 + d_{31}||^2 + ||q_2 - q_1 + d_{21}||^2 + ||q_4 - q_2 + d_{42}||^2$$
$$+ ||q_{\text{ref}} - (q_1+q_2+q_3)/3||^2 + \sum_{i=1}^{4} ||\dot{q}_{\text{ref}} - \dot{q}_i||^2 + ||u_i||^2.$$

Simulation Results



Cooperative Tasking (Richards et al)

Formulate UAV tasking problem as cooperative control problem

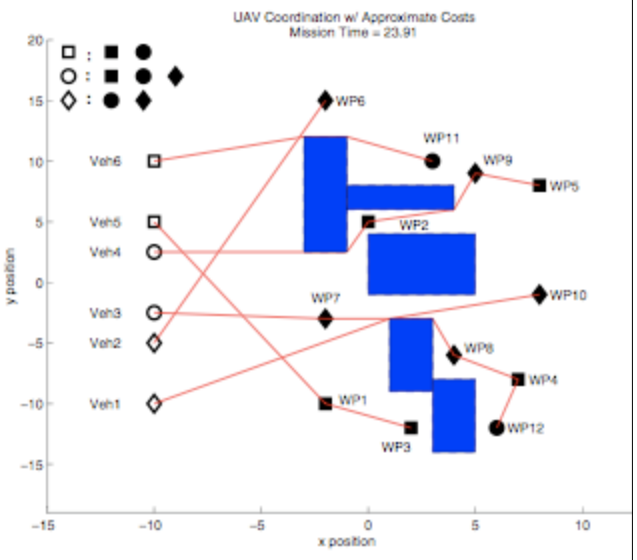
- t^p is the time at which the *p*th vehicle completes its task
- $\bar{t}$ is the time at which the last vehicle completes its task
- Cost function trades off input forces on vehicles with time that the overall task is completed + tasks of the individual vehicles

$$J = \bar{t} + \rho_1 \sum_{p=1}^{N} \left(t^p + \rho_2 \sum_{t=0}^{T} \left(|u_1(t)| + |u_2(t)| \right) \right)$$

Solution approach: mixed integer linear programming (MILP)

$$\sum_{t=0}^{T} \sum_{p=1}^{N} K_{pi} b_{ipt} = 1 \quad \forall \text{ waypoints } i$$

- K_{pi} is the suitability of vehicle *p* to visit waypoint *i*
- b_{ipt} is 1 if vehicle *p* visits waypoint *i* and time *t* zero otherwise



Rendezvous (Tiwari et al)

Goal: collection of vehicle arrive in a given region at the same time

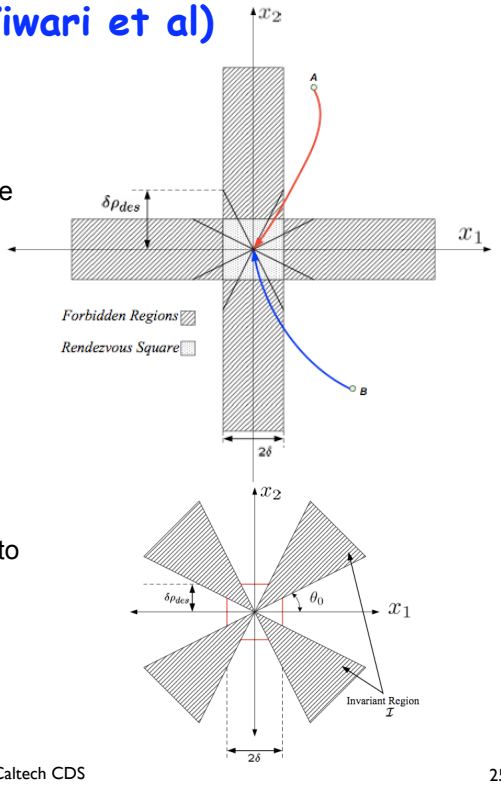
- = rendezvous point region, radio delta
- ρ = max and min distances of vehicles at the time ta that first of them enters rendezvous point:

$$\rho = \frac{\max(\|x_i(t_a)\|)}{\delta}$$

- Find a control law a such that from all initial conditions, $\rho \leq \rho_{\text{des}} \leq 1$.
- Perfect rendezevous: ρ = 1

Approach: create invariant regions (cones) outside of forbidden regions

- Solution is centralized: each vehicle needs to know where the others are at
- Can also formulate as optimization-based problem; possibly decentralize?



Coverage (Cortes, Martinez, Bullo)

Place N vehicles over a region to maximize sensor coverage

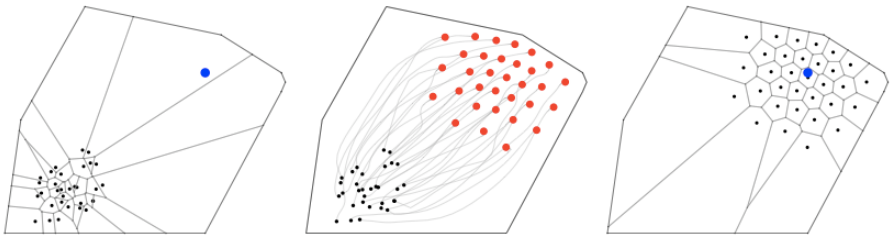
- Partition region Q into set of polytopes $\mathcal{W} = \{W^1, \ldots, W \ \}$ that cover Q
- Let $f^i : \mathbb{R}_+ \to \mathbb{R}_+$ represent sensing performance (small is good); $\phi(q)$ = distribution density function
- Represent coverage problem as minimizing the cost function

$$L = \sum_{i=1}^{n} \int_{W^i} f(\|q - y^i\|)\phi(q)dq,$$

- Can show that if environment is fixed, optimal sol'n is a (weighted) Voronoi partition

$$W^i = \{q \in Q | \|q - y^i\| \leq \|q - y^j\|, \forall j \neq i\}.$$

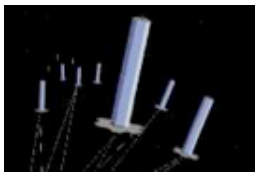
- Can implement coverage using a control law of the form $u^i = -k(y^i - C_{V^i})$
- Can achieve solution using nearest neighbor communications

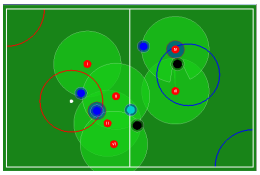


Summary

Information flow and stability

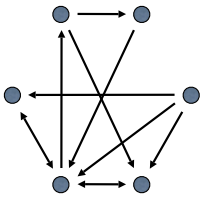
- Stability conditions for interconnected dynamical systems
- Extensions: centralized information, consensus, ...

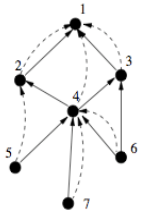


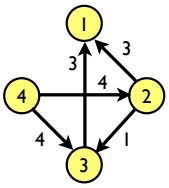


Distributed optimization

- Conditions on amount of information required to solve optimization problem
- Distributed receding horizon control







Asynchronous protocols

- Decentralized strategies for control, decision making and estimation

$$u^i = k^i(x,\alpha) \qquad \{g^i_j(x,\alpha) : r^i_j(x,\alpha)\}$$

$$\alpha^{i\,\prime} = \begin{cases} r^i_j(x,\alpha) & g(x,\alpha) = \text{true} \\ \text{unchanged} & \text{otherwise.} \end{cases}$$

Verification and Validation

NCS Lecture Schedule

	Mon	Tue	Wed	Thu	Fri
9:00	L1: Intro to Networked Control Systems	L5: Distributed Control Systems	L7: Distributed Estimation and Sensor Fusion	L11: Quantization and Bandwidth Limits	L13: Distributed Protocols and CCL
11:00	L2: Optimization-Based Control	L6: Cooperative Control	L8: Information Theory and Communications	L12: Estimation over Networks	L14: Open Problems and Future Research
12:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00	L3: Information Patterns		L9: Jump Linear Markov Processes		
16:00	L4: Graph Theory		L10: Packet Loss, Delays and Shock Absorbers		