

CALIFORNIA INSTITUTE OF TECHNOLOGY  
Control and Dynamical Systems

**CDS 202**

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**Problem Set #9**

Issued: 5 Mar 09  
Due: 12 Mar 09

Reading: Abraham, Marsden, and Ratiu (MTA), Sections 7.4–7.5, 8.1–8.2

Problems:

1. Calculate exterior derivatives of the following forms in  $\mathbb{R}^3$ :
  - (a)  $z^2 dx \wedge dy + (z^2 + 2y) dx \wedge dz$
  - (b)  $z^2 dx \wedge dy + (z^2 + 2y) dx \wedge dz$
  - (c)  $f dg$ , where  $f$  and  $g$  are functions on  $\mathbb{R}^3$
  - (d)  $(x + 2y^3)(dz \wedge dx + \frac{1}{2} dy \wedge dx)$
2. MTA, 7.4-3: practice with exterior derivatives
3. MTA, 7.5-6: divergence of a Lie bracket
4. MTA, 8.2-1: pullback and exterior derivatives
5. MTA, 8.2-2: exterior derivative on a manifold without boundary
6. Consider the locomotion system given by a disk rolling on the plane from homework #7. Using the exterior derivative of the kinematic connection, determine if the system is totally controllable and/or fiber controllable.