Coordinated Control Scheme for Multi-Agents Systems

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Outline

Goals and References

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- Formulate the coordinated control of multi-agent systems.
- Stability and performance analysis of formation control.
- Double-graph control scheme.

Reference

- "Information Flow and Cooperative Control of Vehicle Formations", J. Alexander Fax and Richard M. Murray, IEEE T. Automatic Control, 49(9):1465-1476, 2004;
- "Double-Graph Control Strategy of Multi-Vehicle Formations", Z. Jin and R. M. Murray, the 43rd IEEE Conference on Decision and Control, Dec. 14-17, 2004, Paradise Island, Bahamas.

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Distributed Coordinated Control Strategy

Assumptions

- Agents are decoupled and have identical dynamics (LTI) in physical level.
- Local controllers have same structures and can exchange states by onboard sensors or communication channels.

Objectives

- Stability conditions with respect to local controller, interactive topology, and information flow.
- Performance evaluation (leader-follower case).

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Where did such problems come from?

- Understanding the group behavior of wild animals.

- Formation Control (Automated highway systems and UAVs).

Dynamics of Formation Motion

- A set of N agents whose (identical) linear dynamics are:

$$\dot{x}_i = Ax_i + Bu_i;$$

- Each agent's sensed information is defined as:

$$\begin{aligned} y_i &= C_1 x_i \\ z_{ij} &= C_2(x_i - x_j - d_{ij}), j \in N_i; \end{aligned}$$

- Decentralized controller:

$$\begin{aligned} \dot{v}_i &= K_A v_i + K_{B1} y_i + K_{B2} z_i \\ u_i &= K_C v_i + K_{D1} y_i + K_{D2} z_i \end{aligned}$$

where $z_i = \sum z_{ij}/|N_i|$.

Stability Theorem

Theorem

A local controller stabilizes the formation dynamics if and only if it simultaneously stabilizes the set of N systems:

$$\begin{aligned}\dot{x} &= Ax + Bu\\ y &= C_1 x\\ z &= \lambda_i C_2 x\end{aligned}$$

where λ_i are the eigenvalues of $D^{-1}L$.

Proof

Using Kronecker matrix product and Schur decomposition.

Dynamics of Formation Motion

- A set of N agents whose (identical) linear SISO transfer functions are $P(s)$ and local controller are $C(s)$.
- For each agent, we have

$$\begin{aligned} H(s) &= \tfrac{P(s)C(s)}{1+P(s)C(s)} \\ y_i(s) &= H(s) \cdot \textstyle\sum_j y_{ij}(s)/|N_i|; \end{aligned}$$

- The whole system is a MIMO system:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_n(s) \end{bmatrix} = (I - H(s) \cdot \Theta)^{-1} \cdot \begin{bmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_N(s) \end{bmatrix}$$

where $\Theta = D^{-1}A$ is the average adjacency matrix.

Stability Theorem

Theorem

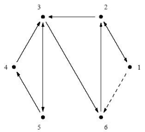
The distributed local controller stabilizes the formation dynamics iff the net encirclement of $-\lambda_i^{-1}$ by the Nyquist plot of P(s)C(s) is zero for all nonzero λ_i.

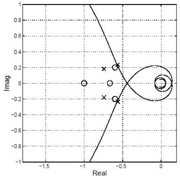
Proof

The determinant is: $\det(I - H(s)\Theta) = \prod_{i=1}^{N}(1 - \lambda_{\Theta i}H(s))$;
The char. poly. is: $\prod_i (1 + (1 - \lambda_{\Theta i})P(s)C(s))$;
Critical point -1 changes to multiple critical points $-(1 - \lambda_{\Theta i})^{-1}$.

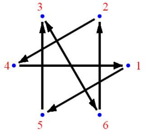
Performance

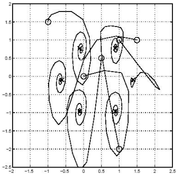
- Limited stability region





- Slow convergence speed

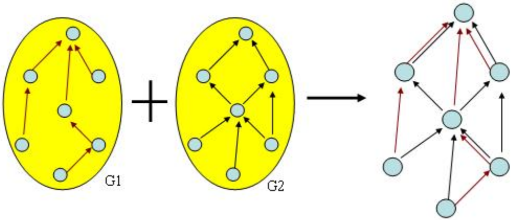




- Unscalable overshoots (string stability)

Double-Graph Model of a leader-follower system

- It's important for each follower that adjust their motions according to the leader as well as its neighbors.
- A connected digraph $\mathcal{G}_1$ is used to describe the information flow of group agreement (comm. network).
- Another connected digraph $\mathcal{G}_2$ is used to describe the neighbor states flow (onboard sensor).



Local Controller Structure

Transfer function of local controller

$$Y_i(s) = H_1(s) \cdot U(s) + H_2(s) \cdot \sum_j Y_{ij}(s)$$

where Y_{ij} is the outputs of vehicle i's neighbors in $\mathcal{G}_2$.

Simplify the analysis

$$\begin{aligned} H_1(s) &= \alpha \cdot H(s) \\ H_2(s) &= \frac{(1-\alpha)}{\Phi_i} \cdot H(s) \end{aligned}$$

where Φ_i is the out-degree of vehicle$i$ in $\mathcal{G}_2$ and $0 \leq \alpha \leq 1$.

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Stability Analysis

Transfer function matrix

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_n(s) \end{bmatrix} = (I - (1-\alpha)\cdot H(s)\cdot \Theta_2)^{-1} \cdot \begin{bmatrix} 1 \\ \alpha \\ \vdots \\ \alpha \end{bmatrix} \cdot H(s)U(s).$$

where $U(s)$ is the input of the leader.

Normalized adjacency matrix

A_2 is the adjacency matrix of the graph $\mathcal{G}_2$ and D_2 is a diagonal matrix with the out-degree of each node along the diagonal. Normalized adjacency matrix Θ_2 is defined by $\Theta_2 = D_2^{-1} \cdot A_2$.

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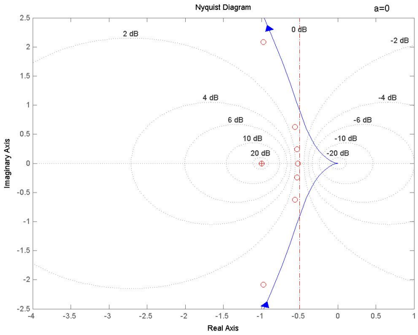
Stability Analysis

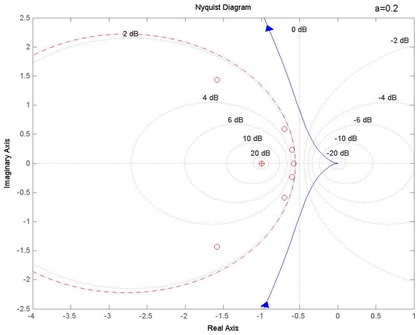
Nyquist plot, $\alpha = 0$



Critical point changed from -1 to several points. Determined by topology of $\mathcal{G}_2$ and α.

Stability Analysis

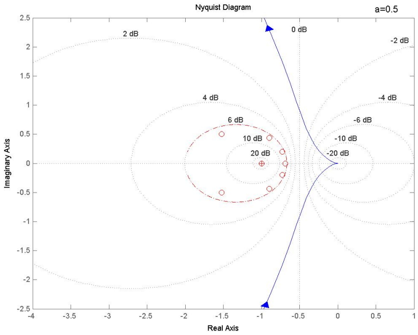
Nyquist plot, $\alpha = 0.2$



Critical point changed from -1 to several points. Determined by topology of $\mathcal{G}_2$ and α.

Stability Analysis

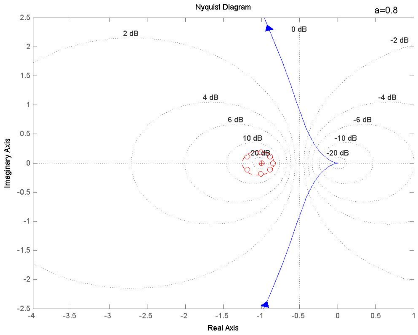
Nyquist plot, $\alpha = 0.5$



Critical point changed from -1 to several points. Determined by topology of $\mathcal{G}_2$ and α.

Stability Analysis

Nyquist plot, $\alpha = 0.8$



Critical point changed from -1 to several points. Determined by topology of $\mathcal{G}_2$ and α.

Performance Analysis

Theorem

For a leader-follower system with double-graph strategy, if $\|H(s)\|_\infty < 1/(1-\alpha)$, then for any disturbance introduced at the leader, we have

$$\|e_i(s)\|_2 < \|e_1(s)\|_2$$

where $e_i(s)$ is the perturbation signal of the follower $i \in [2, n]$, $e_1(s)$ is the perturbation signal of the leader.

Comments

The theorem solves the general "string stability" problem for any graph topology (or formation topology).

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Performance Analysis

Definition

Treat the formation as an MIMO system and

$$\begin{cases} \|\underline{E}\|_\infty = \max_{i=1}^n \left(\|e_i(s)\|_2\right) \\ \|\underline{V}\|_\infty = \max_{i=1}^n \left(\|v_i(s)\|_2\right). \end{cases}$$

Theorem

For a leader-follower formation with double-graph strategy, If $\|(1-\alpha)\cdot H(s)\|_\infty = M < 1$, *then*

$$\frac{\|\underline{E}\|_\infty}{\|\underline{V}\|_\infty} \leq \left(\frac{1-M^n}{1-M} + \frac{\rho(\Theta_2)M^n}{1-\rho(\Theta_2)M}\right)\cdot\|N(s)\|_\infty$$

where $\rho(\Theta_2)$ *is the spectral radius of* Θ_2 *and* $N(s) = \frac{e_1(s)}{v_1(s)}$.

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where $\rho(\Theta_2)$ *is the spectral radius of* Θ_2 *and* $N(s) = \frac{e_1(s)}{v_1(s)}$.

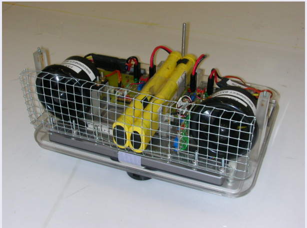
Trade Off in This Strategy

- So far, $\alpha = 1$ is the best choice for the stability issue and disturbance gains.
- Graph $\mathcal{G}_2$ provides the separation and cohesion feedbacks to keep the formation in certain shape. $\alpha = 0$ is good for this issue.
- There exists a trade off between the stability, disturbance resistance, and formation maintenance. The weight coefficient α is the indicator.

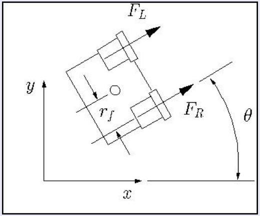
Quick review of Caltech MVWT

The Caltech Multi-Vehicle Wireless Testbed (MVWT) is an experimental platform for validating theoretical advances in multi-vehicle coordination and cooperation control.

Vehicle Kelly



Dynamics

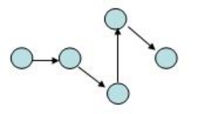


$$
\begin{aligned}
m\ddot{x} &= -\mu\dot{x} + (F_L + F_R)\cos\theta \\
m\ddot{y} &= -\mu\dot{y} + (F_L + F_R)\sin\theta \\
m\ddot{\theta} &= -\psi\dot{\theta} + (F_L - F_R)r_f
\end{aligned}
$$

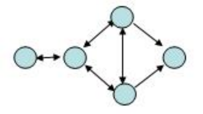
Simulations for different $\mathcal{G}_2$

Without double-graph strategy, or $\alpha = 0$

Topology 1



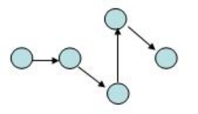
Topology 2



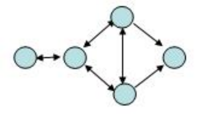
Simulations for different $\mathcal{G}_2$

With double-graph strategy, and $\alpha = 0.5$

Topology 1



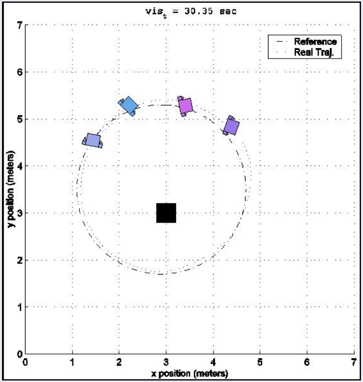
Topology 2



Four vehicle platoon experiment on MVWT

With double-graph strategy, and $\alpha = 0.6$

Experiment diagram



Experiment movie

Summary

- The stability of multi-agent systems is much complicated than consensus problem because of the agent dynamics. Local controller design is related to the interactive topology.
- The Double-graph strategy can greatly improve the stability and performance of large scale vehicle group. The cost we need to pay is an additional wireless communication network.
- Possible project ideas
  - Switch between different topologies and strategies (hybrid system).
  - Consensus problem in communication graph.