CDS 270-2: Lecture 8-3
Distributed Receding Horizon Control

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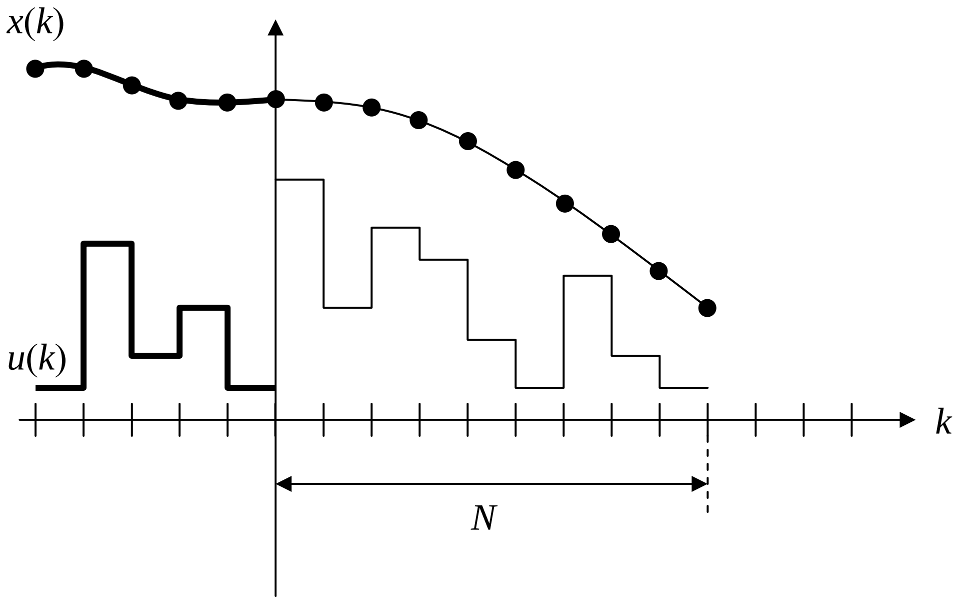
Goals:

- Investigate the use of receding horizon control in a distributed setting
- Discuss a practical discrete-time framework that can be used to study the key requirements for stability
- Look at different approaches to ensuring stability and constraint fulfillment

Reading:

- T. Keviczky, F. Borrelli and G.J. Balas, “Stability analysis of decentralized RHC for decoupled systems”, CDC-ECC’05, Seville, Spain.
- W.B. Dunbar and R.M. Murray, “Distributed receding horizon control for multi-vehicle formation stabilization”, *Automatica*, 2006, Vol. 42, pp. 549-558.

Receding horizon control principle



Receding Horizon Control (RHC) is a form of control, in which:

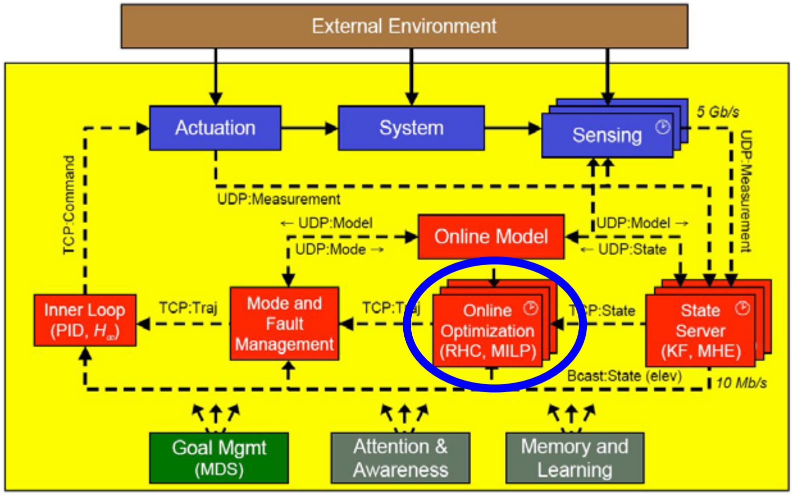
- The current control action is obtained by solving on-line, at each sampling instant, a finite horizon open-loop optimal control problem, using the current state of the plant as the initial state.

- The first part of the optimal control sequence is then applied to the plant and the procedure repeated for future sampling times.

Use the benefits of RHC in large scale systems

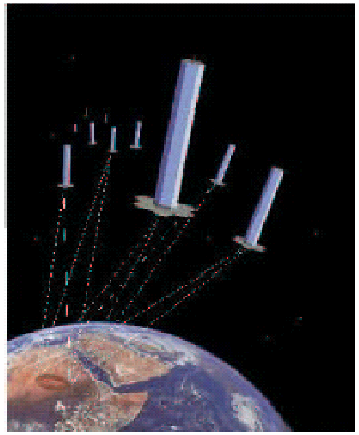
RHC definition, advantages, features

- Current control action is obtained by solving a finite horizon optimal control problem, using current state of the plant as initial state and model-based prediction
- Ability to cope with hard state and control constraints
- Centralized, online optimization (comp. and comm. intensive)



Distributed RHC

- Formulate and solve smaller, local finite horizon optimal control problems.
- We would like the overall system to behave in a desired, meaningful way
- We need stability guarantees and constraint fulfillment



Special class of systems with a structure

Common features

- Large scale systems
- Dynamically decoupled
- Independently actuated
- Constrained subsystems

Coupling arises from

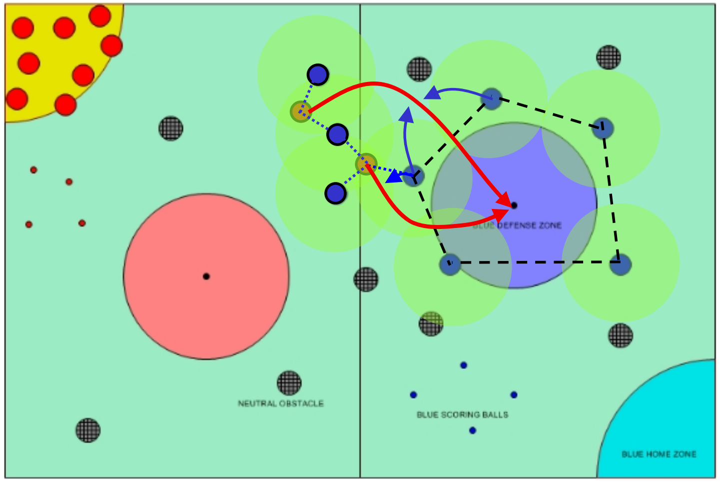
- Performance objective (common task)
- Interconnection constraints (e.g. collision avoidance)

Objective

- Decentralized control design, respect communication constraints
- Minimize a certain performance index
- Satisfy interconnection constraints

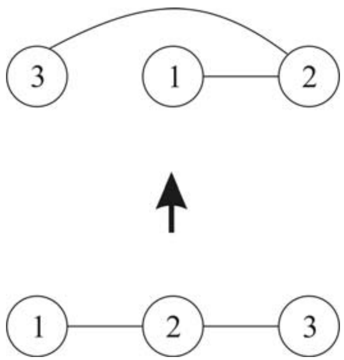
Example

- RoboFlag formation control (maintain relative positions to guard defense zone)
- Cooperation by coupling terms in cost function

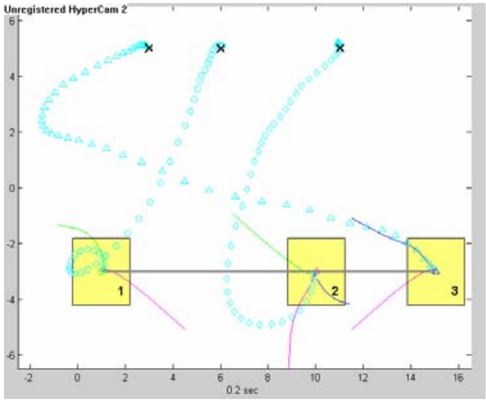


Simulation examples

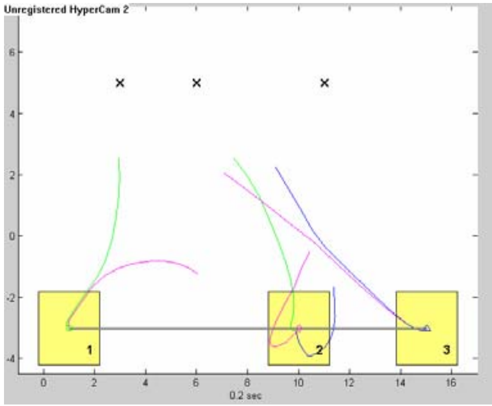
3-vehicle formation



Using the ingredients learned in Lecture 3-2, let's investigate the overall system behavior using decentralized RHC solutions.

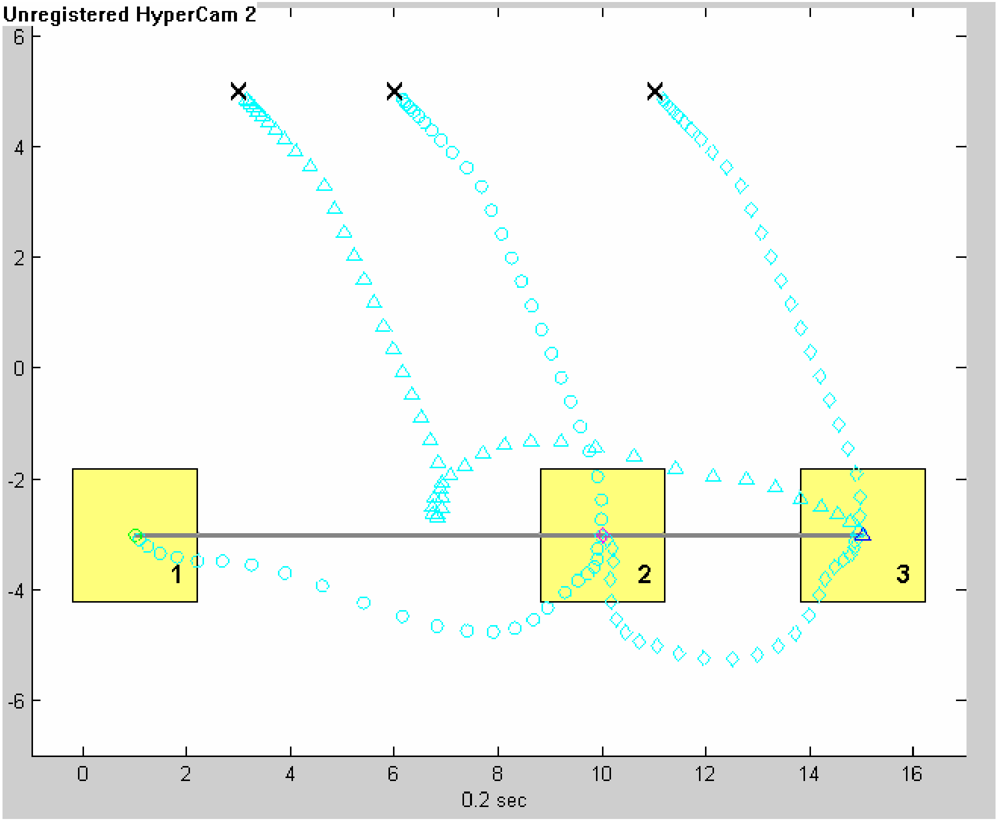


Feasible decentralized solution



Infeasible decentralized solution
(longer horizon length)

Simulation results



Centralized solution (increased weight on relative position)

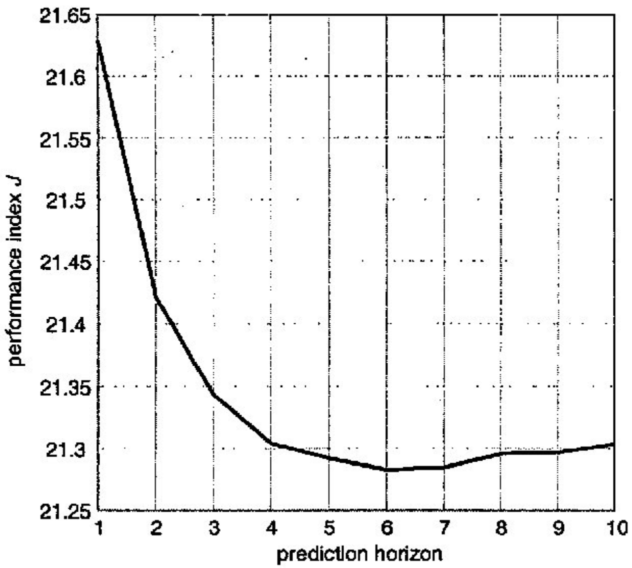
Simulation results

Interesting observations

- Decentralized RHC approach provides feasible solutions.
  (In many cases comparable to the centralized one.)

- Horizon length selection is less trivial in the decentralized case.
  (Trade-off curve is different from classical RHC, increasing horizon length can lead to infeasibility.)

- Influencing quality of solutions and feasibility by changing weight on cost terms.



[Krogh et al., 2001]

Problem definition

Collection of dynamical systems

- decoupled
- independently actuated

$$x^i_{k+1} = f^i(x^i_k, u^i_k)$$

- constrained

$$x^i_k \in \mathcal{X}^i, \;\; u^i_k \in \mathcal{U}^i$$

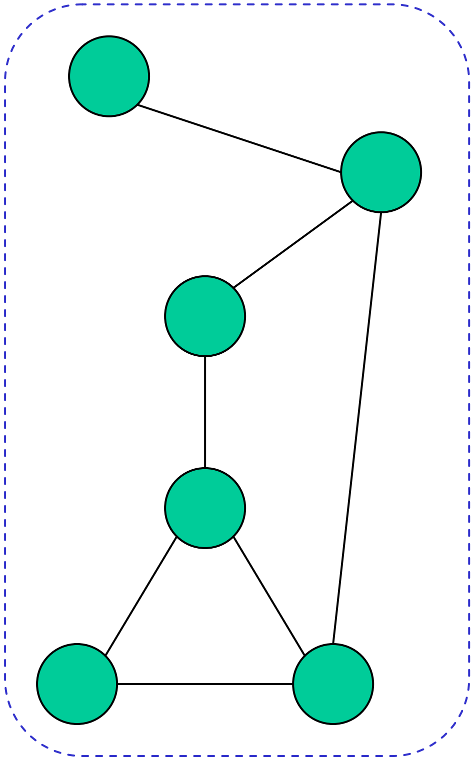
Optimal control problem

- coupling in performance objective
- coupling in constraints

Graph structure for describing:

- information exchange
- constraints between nodes
- coupling in performance objective

$$\mathcal{G}_k = \{\mathcal{V}_k, \mathcal{A}_k\}$$



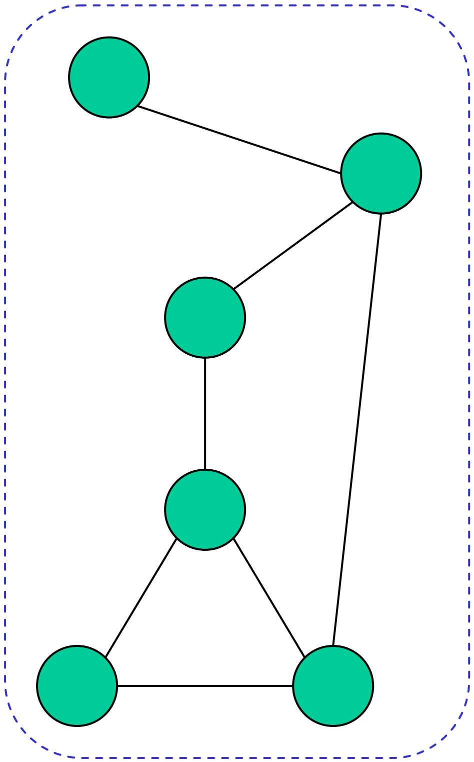
Problem definition

Optimal control problem

$$\min_{\{\tilde{u}_0, \tilde{u}_1, \ldots\}} \sum_{k=0}^{\infty} l(\tilde{x}_k, \tilde{u}_k)$$

$$\text{subj. to } \begin{cases} x^i_{k+1} = f^i(x^i_k, u^i_k), \\ g^{i,j}(x^i_k, u^i_k, x^j_k, u^j_k) \leq 0, \\ \quad i = 1, \ldots, N_v, \ (i,j) \in \mathcal{A}_k \ \ k \geq 0 \\ x^i_k \in \mathcal{X}^i, \ \ u^i_k \in \mathcal{U}^i, \ k \geq 0 \end{cases}$$

$$\mathcal{G}_k = \{\mathcal{V}_k, \mathcal{A}_k\}$$



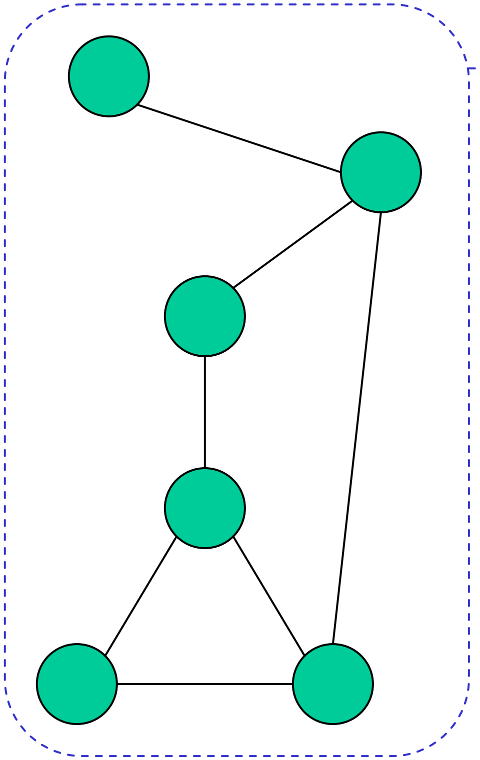
Coupling in performance objective

$$l(\tilde{x}, \tilde{u}) = \sum_{i=1}^{N_v} l^i(x^i, u^i, \tilde{x}^i, \tilde{u}^i)$$

Coupling in constraints

$$g^{i,j}(x^i, u^i, x^j, u^j) \leq 0, \quad (i,j) \in \mathcal{A}$$

Finite time, centralized RHC problem



$$\min_{\{U_t\}} \sum_{k=0}^{N-1}$$ **Global perf. index and terminal cost**

subj. to

- **Models of subsystems**
- **Physical constraints of subsystems**
- **Interaction constraints**
- **Terminal constraints**

Motivation

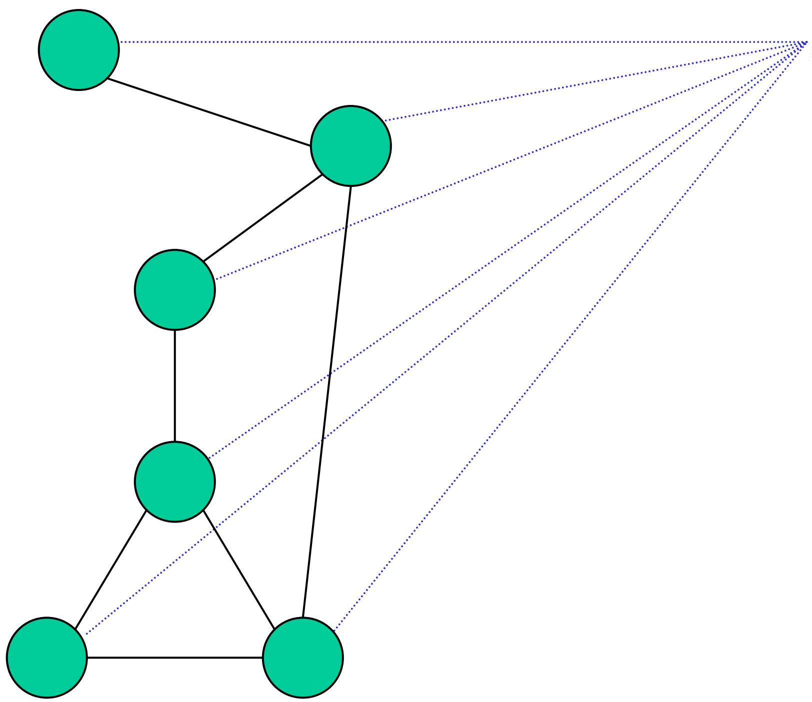
- Centralized optimization problem computationally prohibitive.

Simple idea

- Break centralized RHC controller into local problems of smaller size.
- Use information about neighbors, predict neighbors' trajectories. (Driving in traffic analogy.)
- Implement own control solution.

Decentralized RHC Scheme

Decentralized RHC problem



Local performance index

$$\min_{\tilde{U}_t^i} \sum_{k=0}^{N-1} l^i(x^i_{k,t}, u^i_{k,t}, \tilde{x}^i_{k,t}, \tilde{u}^i_{k,t}) + l^i_N(x^i_{N,t}, \tilde{x}^i_{N,t})$$

$$\text{subj. to} \begin{cases} x^i_{k+1,t} = f^i(x^i_{k,t}, u^i_{k,t}), \; k \geq 0 \\ x^i_{k,t} \in \mathcal{X}^i, \quad u^i_{k,t} \in \mathcal{U}^i, \\ \qquad k = 1, \ldots, N-1 \\ x^j_{k+1,t} = f^j(x^j_{k,t}, u^j_{k,t}), \\ \qquad (j,i) \in \mathcal{A}, \; k \geq 0 \\ x^j_{k,t} \in \mathcal{X}^j, \quad u^j_{k,t} \in \mathcal{U}^j, \\ \qquad (j,i) \in \mathcal{A}, \\ \qquad k = 1, \ldots, N-1 \\ g^{i,j}(x^i_{k,t}, u^i_{k,t}, x^j_{k,t}, u^j_{k,t}) \leq 0, \\ \qquad (i,j) \in \mathcal{A}, \\ \qquad k = 1, \ldots, N-1 \\ g^{q,r}(x^q_{k,t}, u^q_{k,t}, x^r_{k,t}, u^r_{k,t}) \leq 0, \\ \qquad (q,i) \in \mathcal{A}, \; (r,i) \in \mathcal{A}, \\ \qquad k = 1, \ldots, N-1 \\ x^i_{N,t} \in \mathcal{X}^i_f, \quad x^j_{N,t} \in \mathcal{X}^j_f, (i,j) \in \mathcal{A} \\ x^i_{0,t} = x^i_t, \quad \tilde{x}^i_{0,t} = \tilde{x}^i_t, \end{cases}$$

Own model and constraints (first group); Neighbors' model and constraints (second group); Interaction constr. with neighbors, (third group); between neighbors (fourth group)

Stability analysis

Main issue

- Although the idea of predicting the neighbors' behaviors is intuitive and can be observed in practice (e.g. driving in traffic, birds, etc.), the problem formulation by itself does not guarantee stability and feasibility.

Approach

- Use value function of individual nodes (subsystems) as Lyapunov functions.
- This approach is used also in traditional decentralized control for dynamically coupled subsystems, together with certain bounds on the interactions.

Prediction mismatch drives the problem

- Multiple optima
  (non-strictly convex cost function, non-convex constraints)
- Graph structure
  (different set of neighbors)

prediction mismatch → $$\sum_{j \mid (i,j) \in \mathcal{A}} \varepsilon^{i,j} \leq J_0^{i*}$$ ← initial conditions

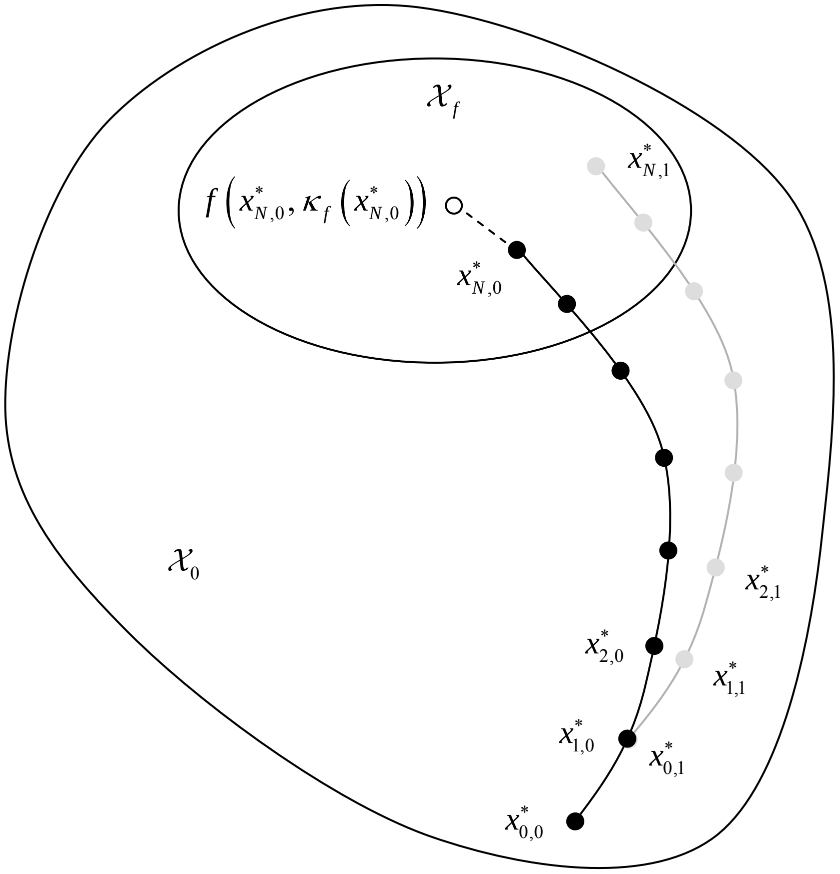
stability condition

Proof of sufficient conditions for DRHC stability

Main ideas of the proof

- Use shifted optimal sequence as a feasible sequence in the next step (use neighors' solution sequence)
- Use value function as a Lyapunov function

Illustrated in class for two subsystems…



Stability analysis

$$\sum_{j \mid (i,j) \in \mathcal{A}} \varepsilon^{i,j} \leq \underbrace{J_0^{i*}}$$

Initial conditions

$$J_0^{i*} = \|Qx_0^i\|_p + \|Ru_{0,0}^{i,i}\|_p + \sum_{j|(i,j)\in\mathcal{A}} \|Qx_0^j\|_p + \sum_{j|(i,j)\in\mathcal{A}} \|Ru_{0,0}^{j,i}\|_p$$
$$+ \sum_{j|(i,j)\in\mathcal{A}} \|Q(x_0^i - x_0^j)\|_p$$

Stability analysis

$$\sum_{j \mid (i,j) \in \mathcal{A}} \varepsilon^{i,j} \leq J_0^{i*}$$

Prediction mismatch

$$\varepsilon^{i,j} = \sum_{k=1}^{N-1} \left(2\|Q(x_{k,0}^{j,j} - x_{k,0}^{j,i})\|_p + \|R(u_{k,0}^{j,j} - u_{k,0}^{j,i})\|_p \right)$$

Stability analysis

$$\sum_{j \mid (i,\, j) \in \mathcal{A}} \varepsilon^{i,\, j} \leq J_0^{i*}$$

+ terminal point constraint

Asymptotic stability of each node

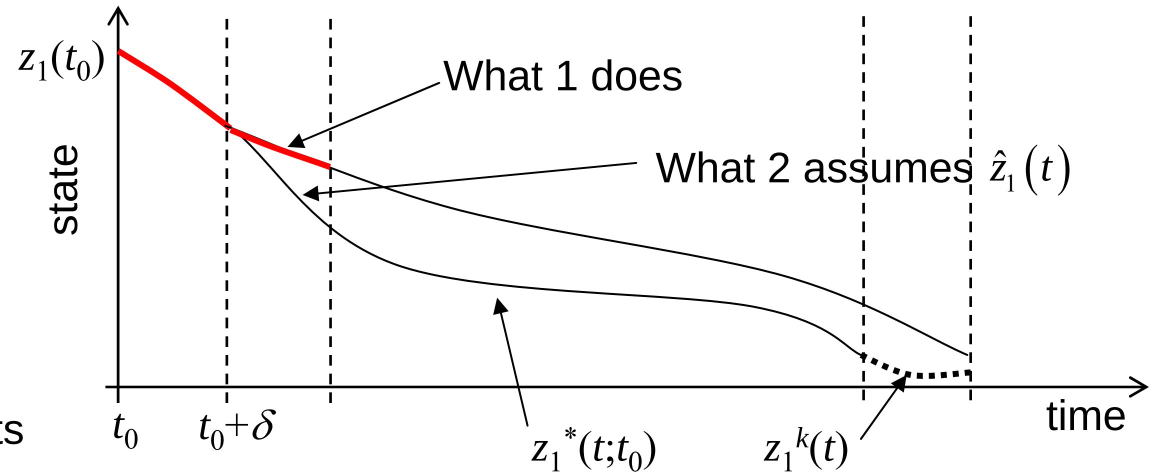
Testing the condition

- Unconstrained LTI: checking $M_i \geq 0$ (M_i is of limited size)
- Constrained LTI: solve systems of LMIs (limited size)

Exchange of optimizers for stability (W. Dunbar)

Subsystems

- Independently actuated, dynamically decoupled
- Independently constrained



Coupling

- Performance objective
- NO interconnection constraints

Stability can be obtained using

- A given cost structure
- Sufficiently fast, synchronous updates
- Exchange of most recent optimal control trajectory between coupled subsystems
- Compatibility constraint: stay within bounded path of what was transmitted

$$\min_{u_1(\cdot)} \int_{t_k}^{t_k+T} L_1\left(z_1(\tau), \hat{z}_2(\tau), u_1(\tau)\right) d\tau + G_1\left(z_1(t_k+T)\right)$$

$$\text{s.t.} \quad \dot{z}_1(t) = f_1\left(z_1(t), u_1(t)\right)$$

$$u_1(t) \in U_1, \quad z_1(t_k+T) \in Z_{f_1}$$

$$\left\| z_1(t) - \hat{z}_1(t) \right\| \leq \delta^2 \kappa$$

Approaches to address feasibility of coupling constraints

Communication
- Exchange of optimizers between neighbors

[Dunbar – Murray]
[Richards – How]

Robust constraint fulfillment
- Decoupled terminal regions don't work
  (maybe stabilizing decentralized LTI terminal controllers)
- Worst-case approaches (too conservative)
- Time-varying, increasing uncertainty about neighbors

[Jia, Krogh]

Implicit safety guarantees
- Feasible basis states as hard terminal constraints in receding horizon path planning

[Schouwenaars]

Hierarchy in the interconnection graph $\mathcal{G}$
- Feasible set projection

[Gokbayrak – Cassandras]
[Stipanovic – Tomlin]
[Keviczky – Borrelli – Balas, CDC'04]

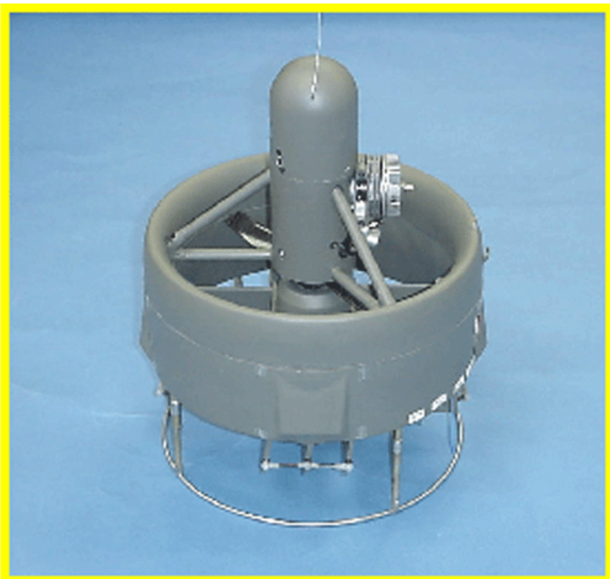
Hybrid receding horizon control
- Include "right-of-way" coordination rules
  in problem formulation using binary decision variables

[Keviczky – Vanek – Borrelli – Balas, HSCC'05, ACC'06]

Recovering from infeasibility
- Emergency maneuvers using invariant sets

[Borrelli – Keviczky – Balas, CDC'04]

Organic Air Vehicle (OAV) formation flight



Honeywell Laboratories

Objective

Autonomous arrangement of a set of OAVs **large** and **tight** formations

Model

Large scale, piecewise linear models, dynamically decoupled

Constraints

1.) Speed and acceleration constraints on OAV
2.) Collision avoidance constraints (non-convex)

More realistic simulation setup

Required theory can quickly grow to encompass many areas.

Simulation techniques can become complex, relying on

- computational geometry
- mathematical programming
- constrained optimal control
- invariant set computation
- hybrid switching systems

Many open problems

- guarantee constraint fulfillment in DRHC without excessive conservatism
- systematic design procedures to achieve guaranteed stability and performance
- avoid using synchronous clocks, centralized time-keeping

6-vehicle formation