Packet-based Estimation in Lossy Networks

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Admin stuff

- Contact info:
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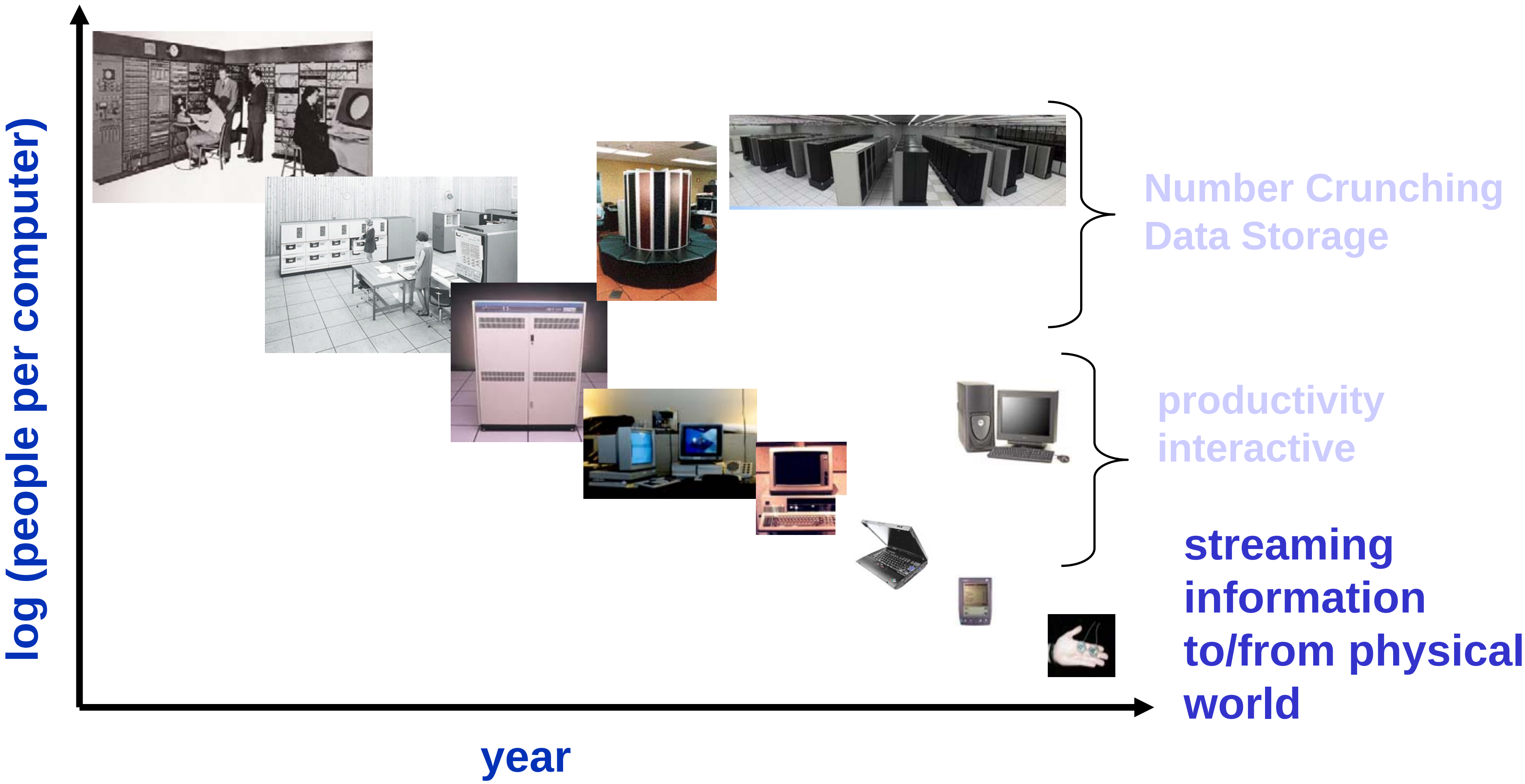
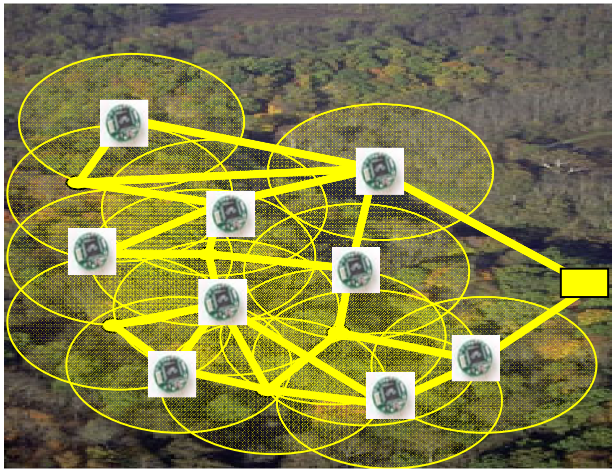
  - Phone: 510 367-1848
- Project Ideas, questions, research, coffee
- Plan:
  - Packet-based estimation: 1-1.5 lectures
  - Packet-based control: 1.5-2 lectures
- Resources (to be posted on the wiki):
  - Kalman Filtering with Intermitent Observations
    - IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 49, NO. 9, SEPTEMBER 2004
  - Chapter 3,4 of my dissertation.
- Additional readings to be posted.

Bell’s Law –
new computer class per 10 years

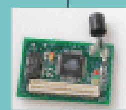
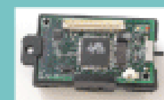
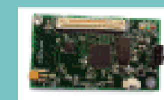


Vast Networks of Tiny Devices

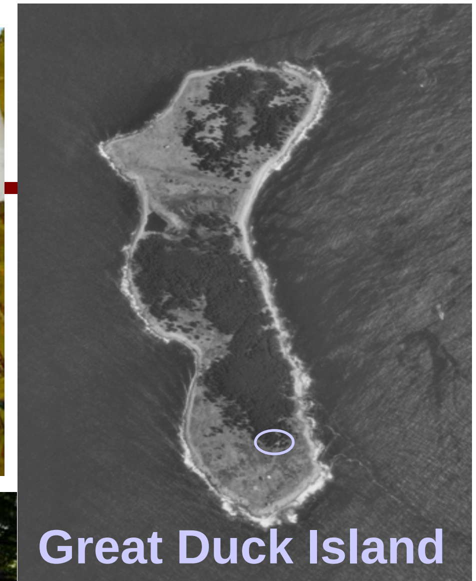
- Past 25 years of internet technology built up around powerful dedicated devices that are carefully configured and very stable
  - local high-power wireless subnets at the edges
  - 1-1 communication between named computers
- Here, ...
- every little node is potentially a router
- work together in application specific ways
- connectivity is highly variable
- must self-organize to manage topology, routing, etc
- and for power savings, radios may be off 99% of the time

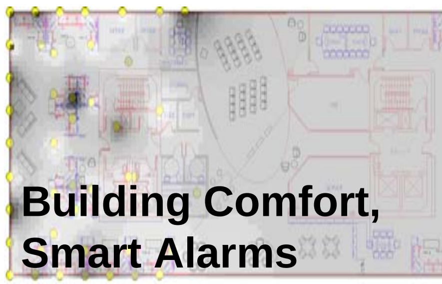


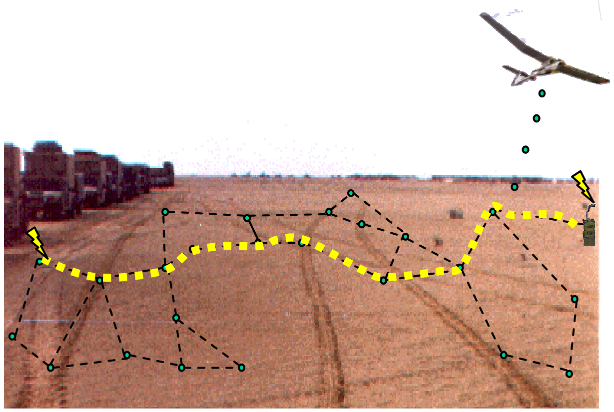
Mote Evolution

Mote Type Year	*WeC* 1998	*René* 1999	*René 2* 2000	*Dot* 2000	*Mica* 2001	*Mica2Dot* 2002	*Mica 2* 2002	*Telos* 2004
								
Microcontroller								
Type	AT90LS8535		ATmega163		ATmega128			TI MSP430
Program memory (KB)	8		16		128			48
RAM (KB)	0.5		1		4			10
Active Power (mW)	15		15		15		60	0.5
Sleep Power (μW)	45		45		75		75	2
Wakeup Time (μs)	1000		36		180		180	6
Nonvolatile storage								
Chip	24LC256				AT45DB041B			ST M24M01S
Connection type	I^2C				SPI			I^2C
Size (KB)	32				512			128
Communication								
Radio	TR1000				TR1000	CC1000		CC2420
Data rate (kbps)	10				40	38.4		250
Modulation type	OOK				ASK	FSK		O-QPSK
Receive Power (mW)	9				12	29		38
Transmit Power at 0dBm (mW)	36				36	42		35
Power Consumption								
Minimum Operation (V)	2.7		2.7		2.7			1.8
Total Active Power (mW)	24				27	44	89	38.5
Programming and Sensor Interface								
Expansion	none	51-pin	51-pin	none	51-pin	19-pin	51-pin	10-pin
Communication	IEEE 1284 (programming) and RS232 (requires additional hardware)							USB
Integrated Sensors	no	no	no	yes	no	no	no	yes

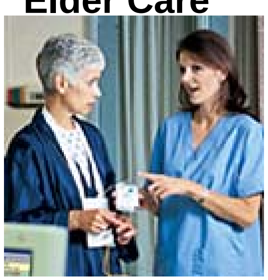
Some Applications





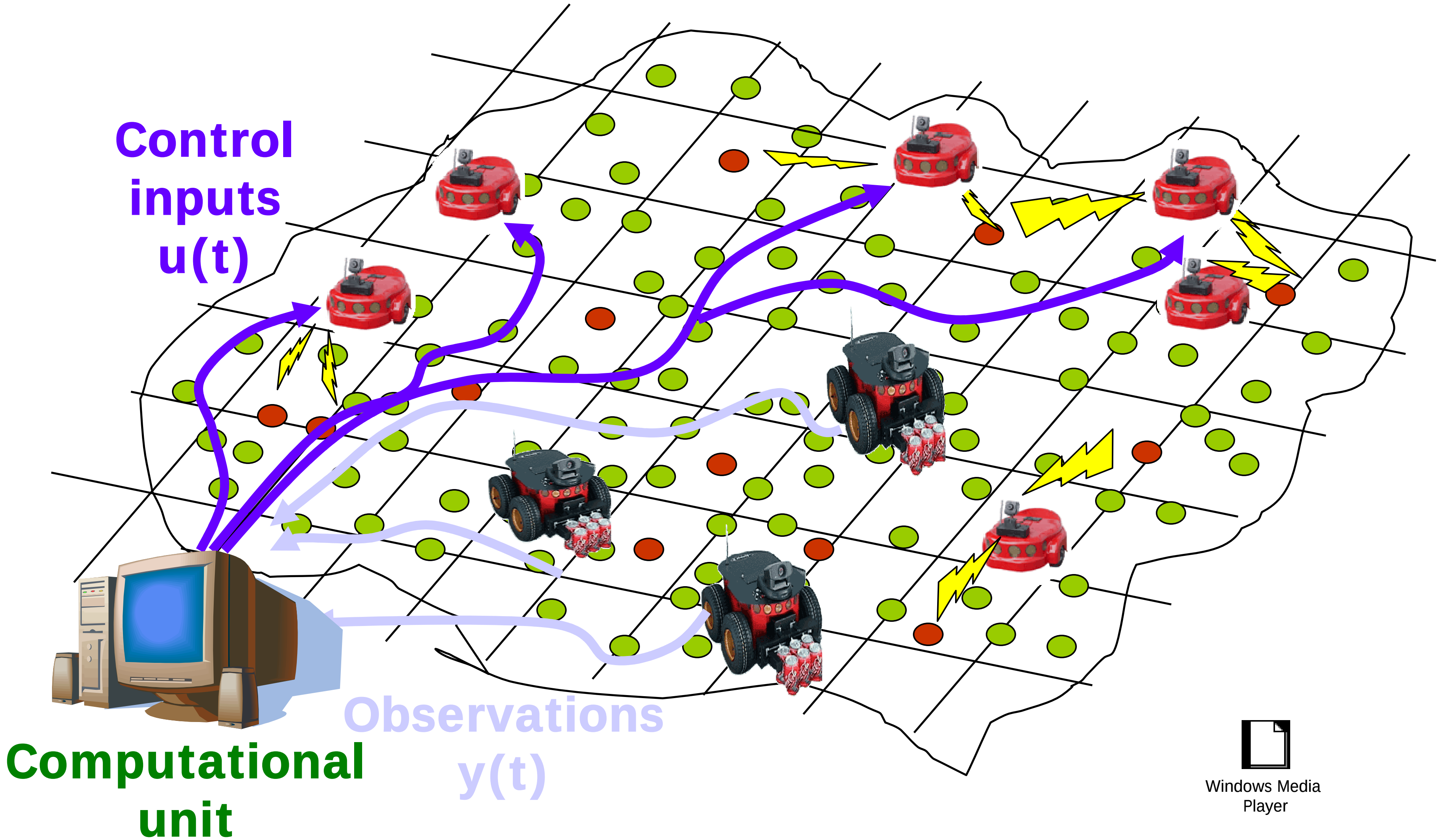


Elder Care



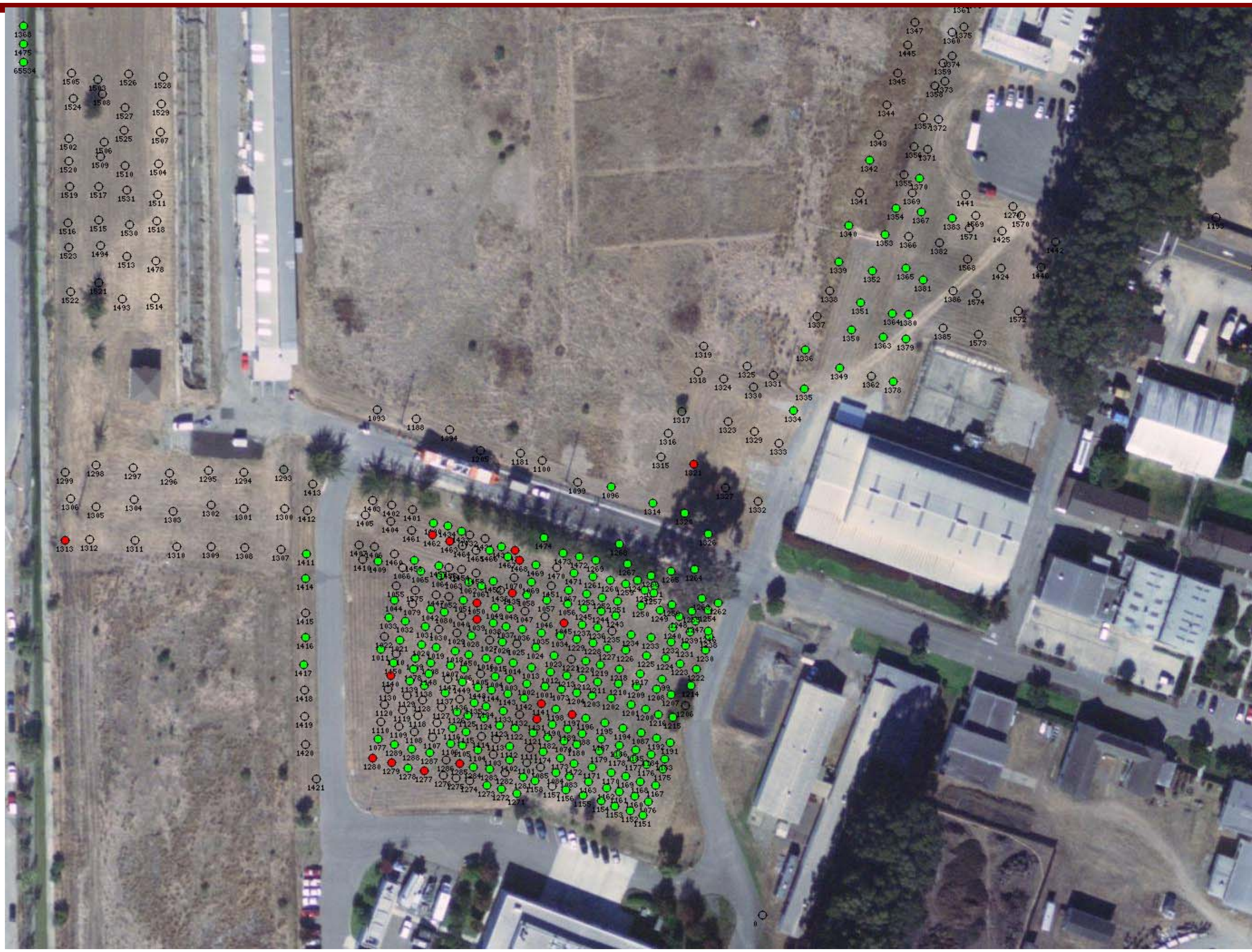
Factories

Control and communication over Sensor Networks

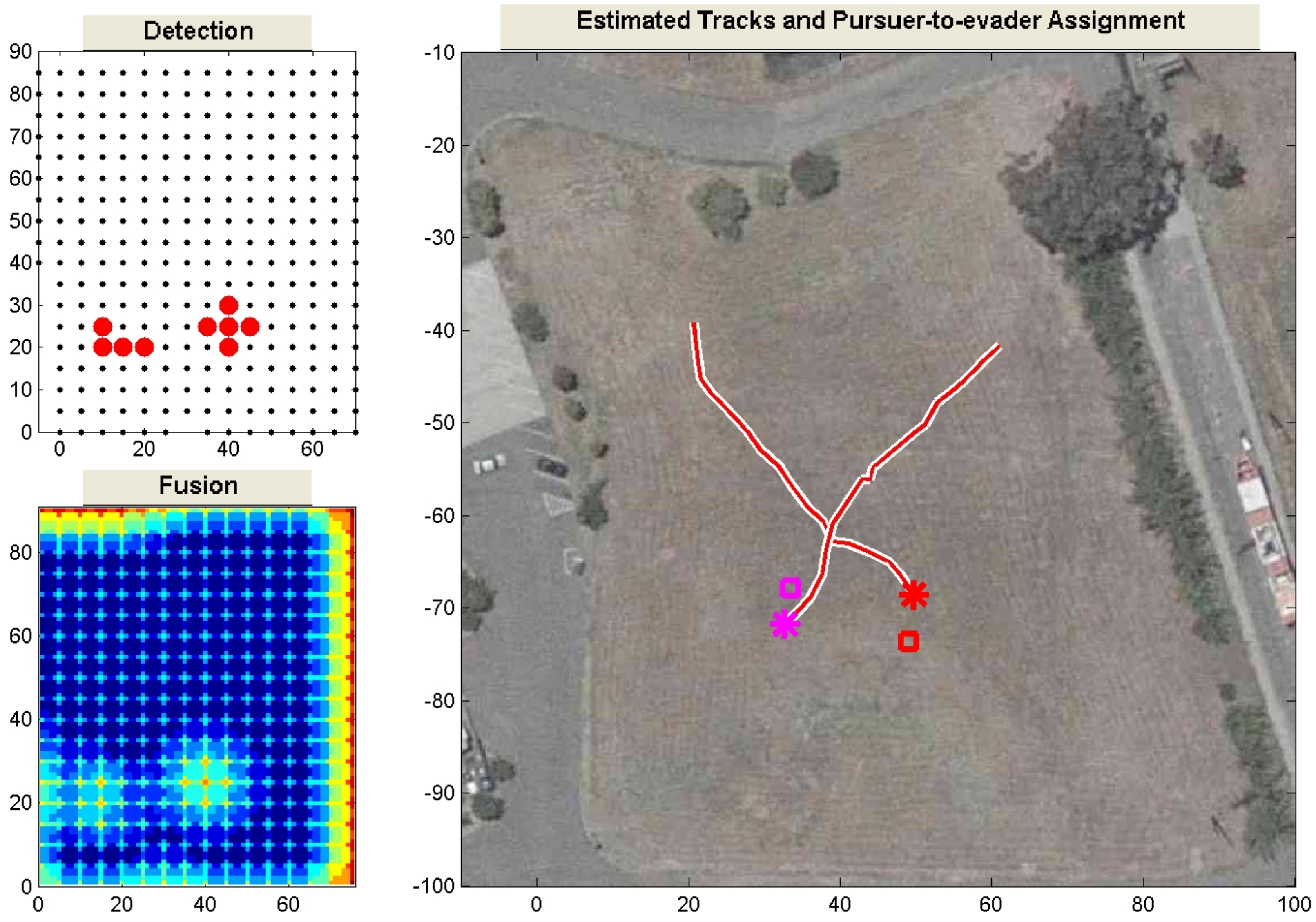


Windows Media Player

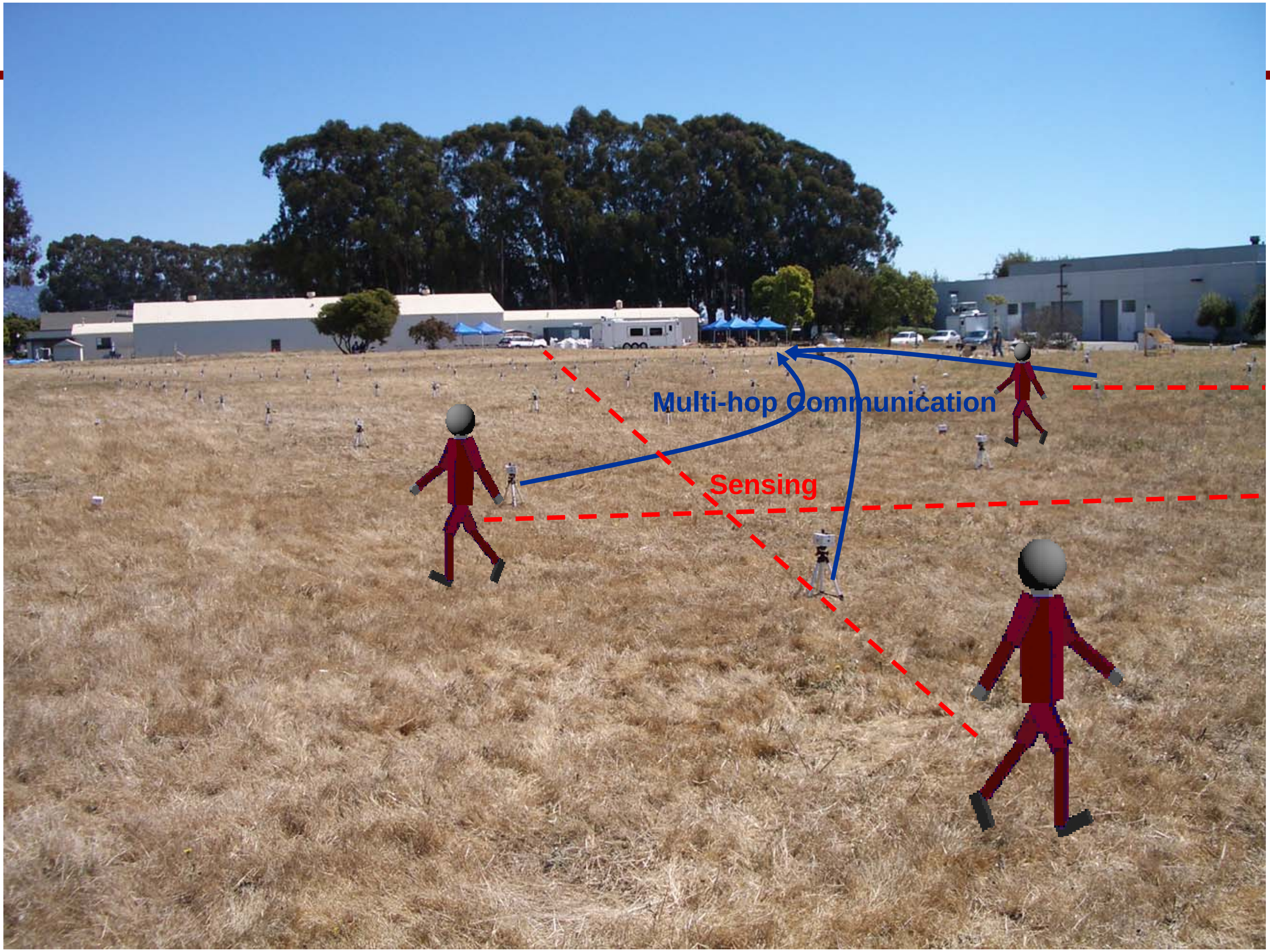
NEST final demo: 557 nodes network deployed



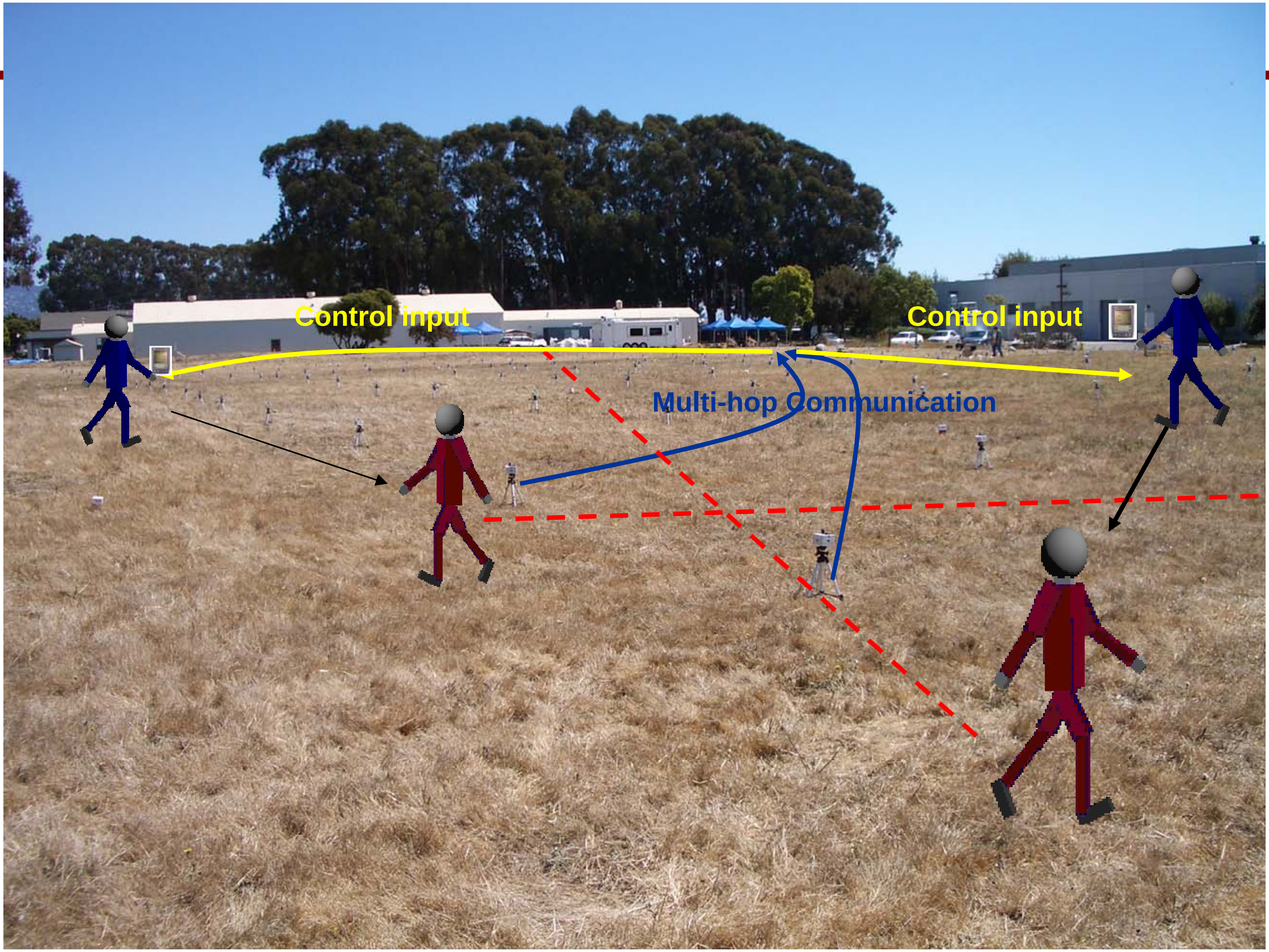
Multi-person tracking demo GUI



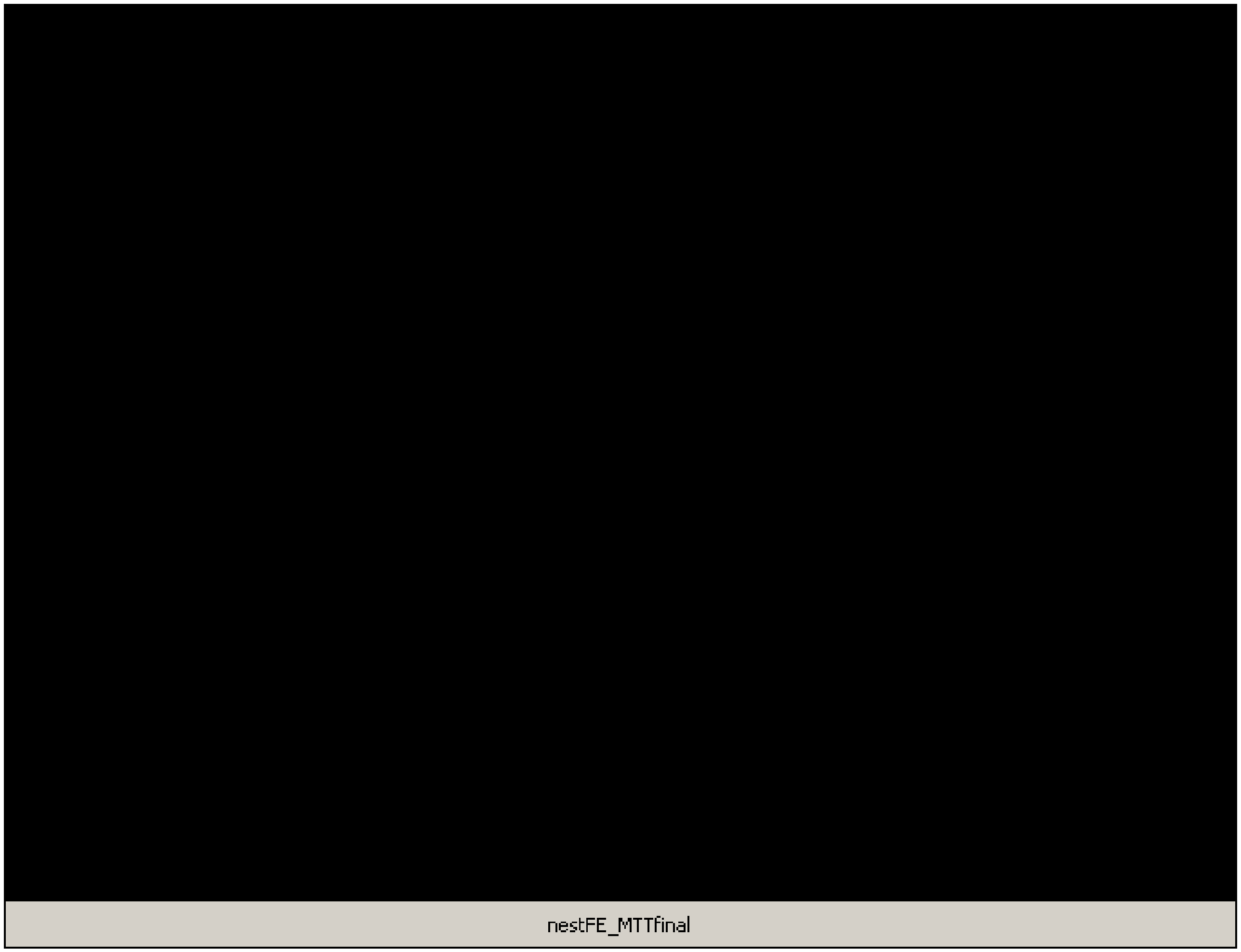
Multiple Person tracking



Pursuit evasion games

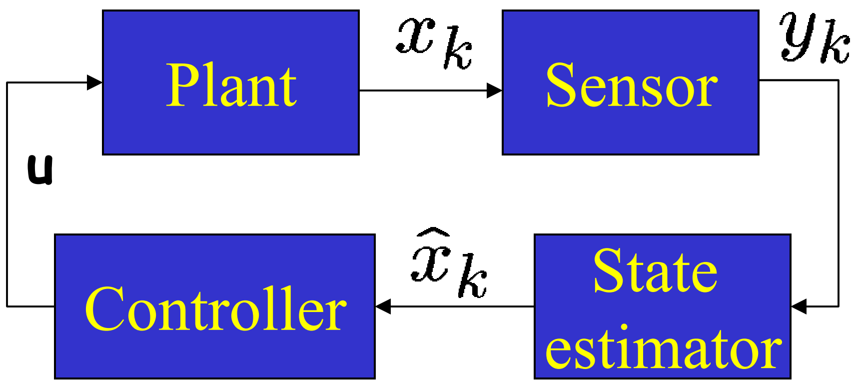


Experimental results:
Pursuit evasion games



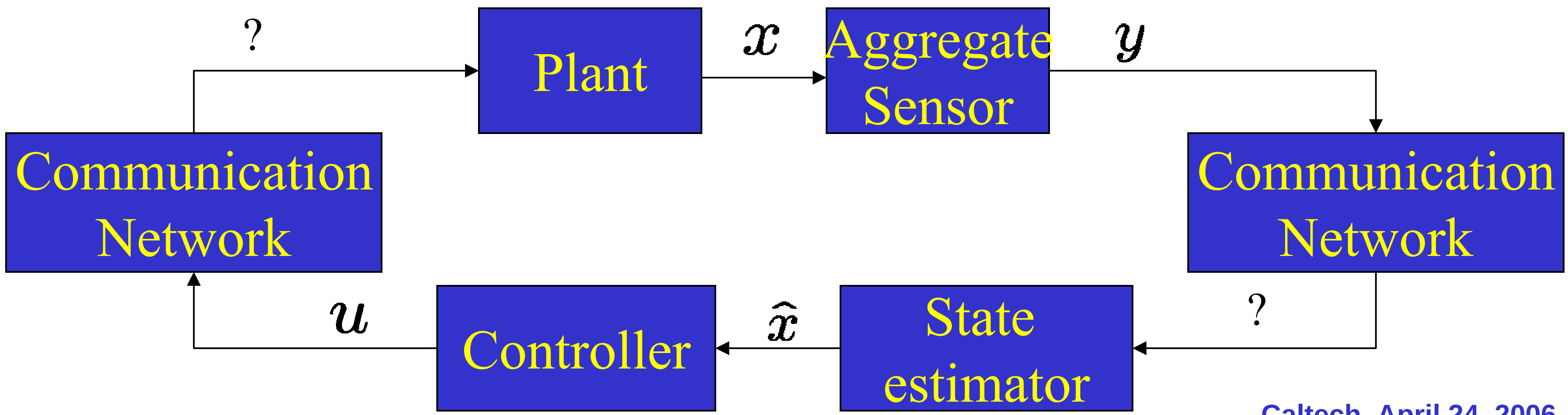
Classical control theory vs networked embedded control systems

Classical control design



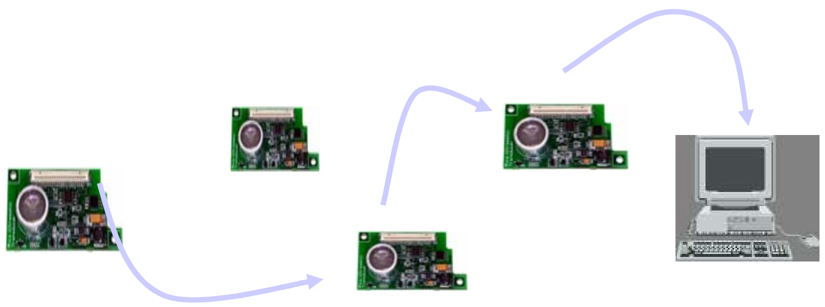
- Availability of data when needed
- Instantaneous communication
- Fixed delays

Networked control in Sensor Networks

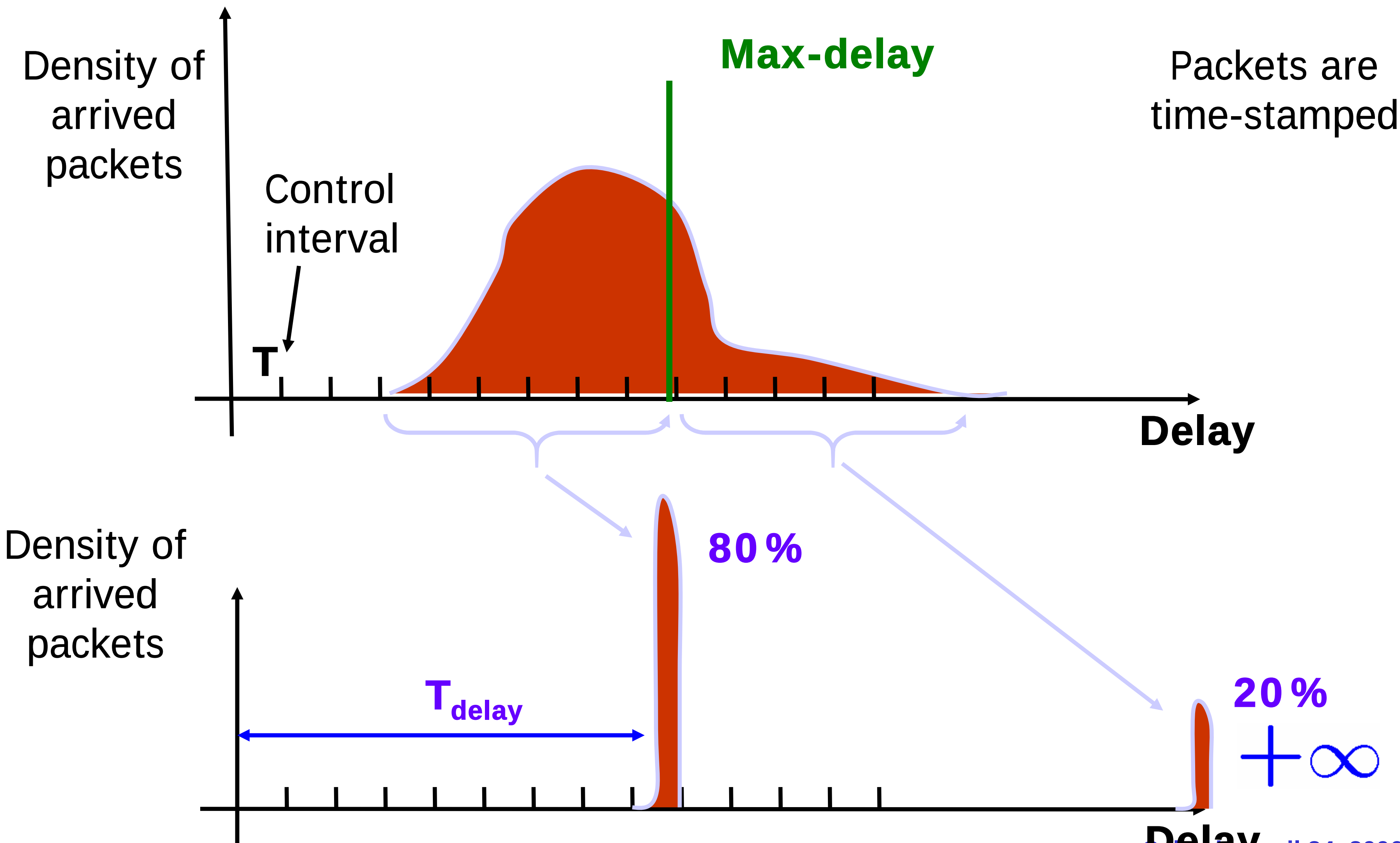


Issues: closing the loop around Wireless Sensor Networks

- **Issues w/ Sensor Networks and Data Networks ?**
  - Random time delay
  - Random arrival sequence
  - Packet loss
  - Limited Bandwidth



Modeling


Density of arrived packets
Max-delay
Packets are time-stamped
Control interval
T
Delay
80%
Density of arrived packets
T_{delay}
20%
$+\infty$
Delay

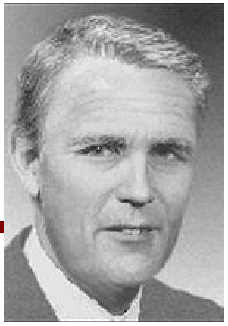
Estimation: Problem formulation

- MIMO Discrete time LTI system

$$\begin{aligned} x_{t+1} &= Ax_t + w_t \\ y_t &= Cx_t + v_t, \end{aligned}$$

- w_t and v_t are Gaussian random variables with zero mean and covariance matrices *Q* and *R* positive definite.
- where $x_t \in \mathbb{R}^n$ is the state vector,
- $y_t \in \mathbb{R}^m$ is the output vector,

Optimal State Estimator: Kalman Filter



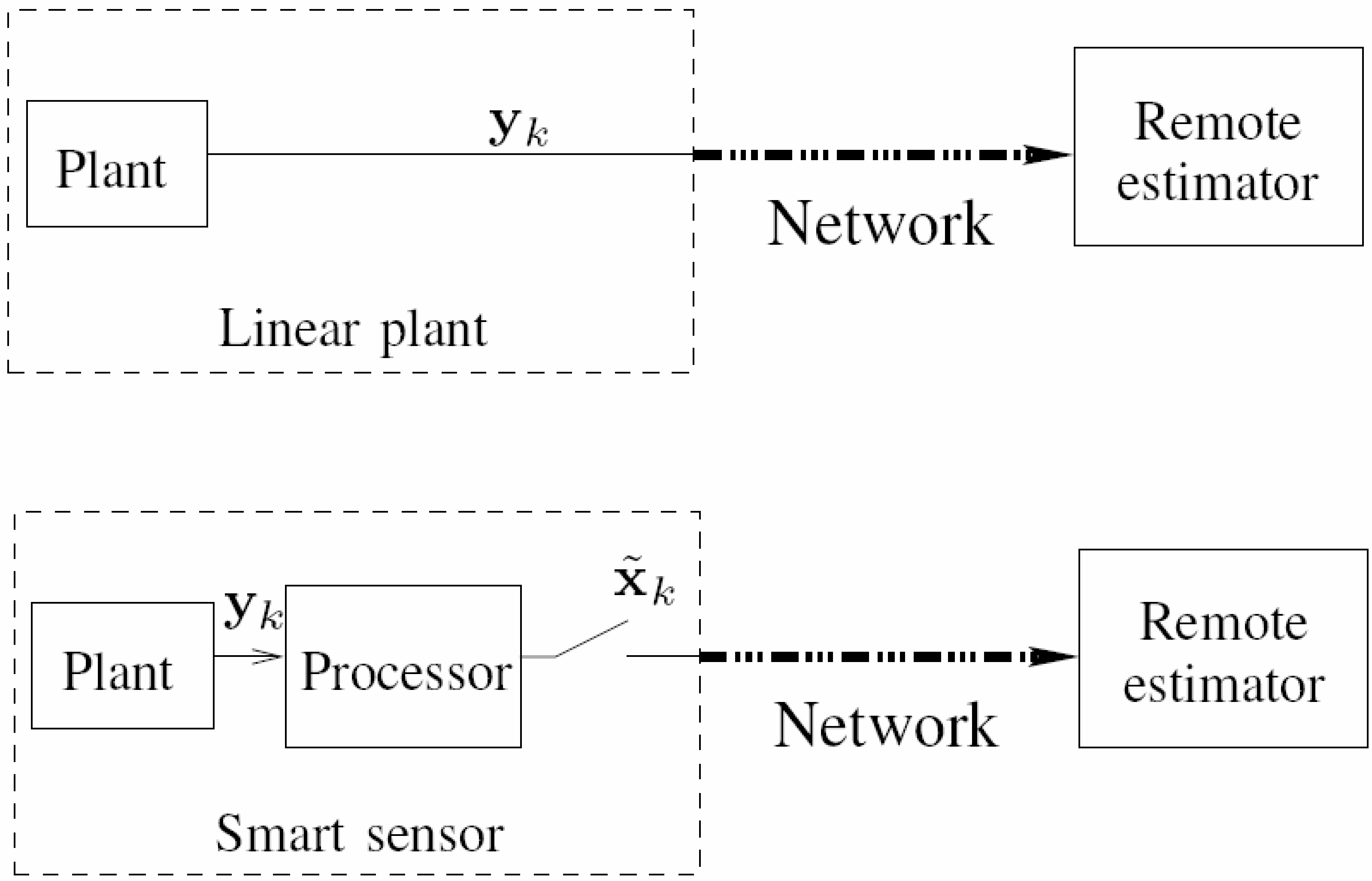
- Background:
  - A recursive linear minimum variance estimator.
  - Assuming linear system and Gaussian noise, Kalman filter is the optimal estimator.
  - It gives an estimate of the state x_t with bounded covariance error, which converges to a steady state value
  - Under the hypothesis of stabilizability of the pair (A,Q) and detectability of the pair (A, C), the estimation error covariance of the Kalman filter converges to a unique value from any initial condition

Problem formulation

- *Goal:* *given observations* y_t *find the best estimate (minimum variance) for* x_t
- *But* y_t *may not arrive at each time step when traveling over a sensor network*

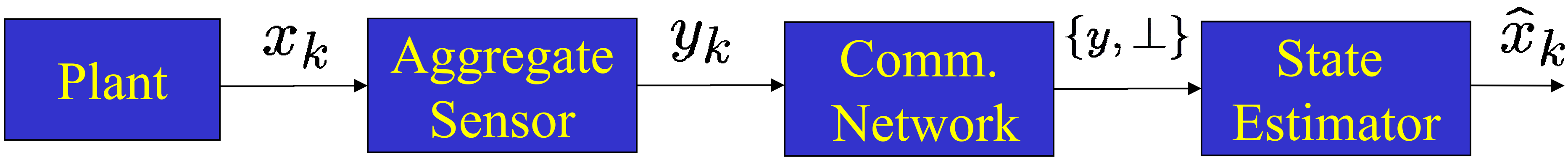
Intermittent observations

Two configurations



Pictures courtesy of Joao Hespanha

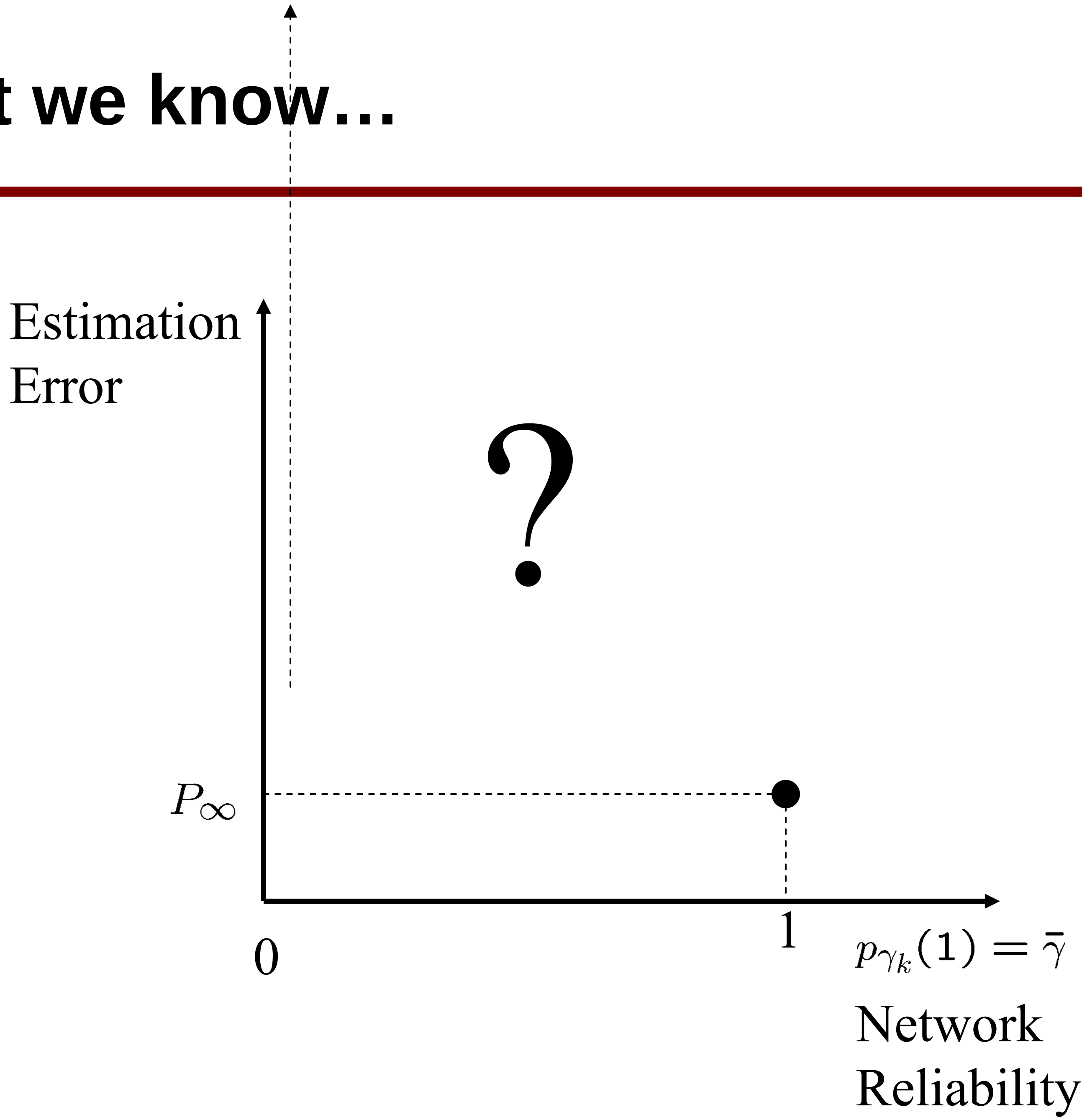
Optimal estimation



$$
\begin{aligned}
x_{k+1} &= Ax_k + w_k \\
y_k &= Cx_k + v_k
\end{aligned}
$$

$$p_{\gamma_k}(1) = \bar{\gamma}$$

What we know…


Estimation
Error
?
P_∞
0
1
$p_{\gamma_k}(1) = \bar{\gamma}$
Network
Reliability

Measurement noise modeling

- The arrival of the observation at time t is modeled as a binary random variable γ_t, with probability distribution $p_{\gamma_t}(1) = \bar{\gamma}$ and with γ_t independent of γ_s if *t* = *s*.
- The output noise v_t is defined in the following way:

$$p(v_t|\gamma_t) = \begin{cases} \mathcal{N}(0, R) & : \quad \gamma_t = 1 \\ \mathcal{N}(0, \sigma^2 I) & : \quad \gamma_t = 0, \end{cases}$$

for some σ^2

Some definition:

$$\hat{x}_{t|t} \triangleq \mathbb{E}[x_t|\mathbf{y_t}, \gamma_\mathbf{t}]$$

$$P_{t|t} \triangleq \mathbb{E}[(x_t - \hat{x}_t)(x_t - \hat{x}_t)'|\mathbf{y_t}, \gamma_\mathbf{t}]$$

$$\hat{x}_{t+1|t} \triangleq \mathbb{E}[x_{t+1}|\mathbf{y_t}, \gamma_\mathbf{t}]$$

$$P_{t+1|t} \triangleq \mathbb{E}[(x_{t+1} - \hat{x}_{t+1})(x_{t+1} - \hat{x}_{t+1})'|\mathbf{y_t}, \gamma_\mathbf{t}]$$

$$\hat{y}_{t+1|t} \triangleq \mathbb{E}[y_{t+1}|\mathbf{y_t}, \gamma_\mathbf{t}]$$

Optimal Filter Equations

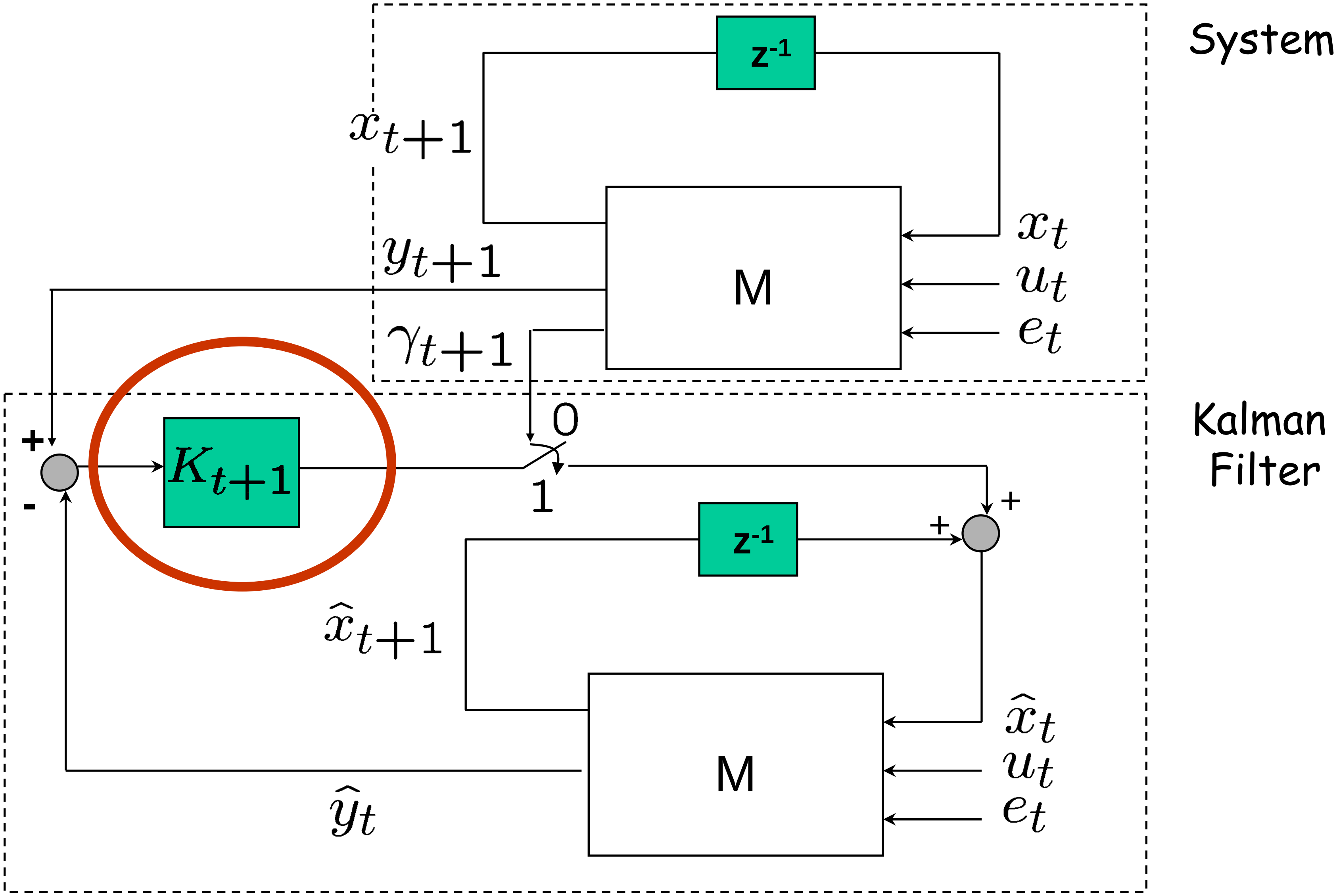
$$
\begin{aligned}
\hat{x}_{t+1|t} &= A\hat{x}_{t|t} \\
P_{t+1|t} &= AP_{t|t}A' + Q \\
\hat{x}_{t+1|t+1} &= \hat{x}_{t+1|t} + P_{t+1|t}C'(CP_{t+1|t}C' + \gamma_{t+1}R + (1-\gamma_{t+1})\sigma^2 I)^{-1}(y_{t+1} - C\hat{x}_{t+1|t}) \\
P_{t+1|t+1} &= P_{t+1|t} - P_{t+1|t}C'(CP_{t+1|t}C' + \gamma_{t+1}R + (1-\gamma_{t+1})\sigma^2 I)^{-1}CP_{t+1|t}.
\end{aligned}
$$

- Taking the limit as $\sigma \longrightarrow \infty$

$$
\begin{aligned}
\hat{x}_{t+1|t+1} &= \hat{x}_{t+1|t} + \gamma_{t+1}P_{t+1|t}C^T(CP_{t+1|t}C^T + R)^{-1}(y_{t+1} - C\hat{x}_{t+1|t}) \\
P_{t+1|t+1} &= P_{t+1|t} - \gamma_{t+1}\underbrace{P_{t+1|t}C^T(CP_{t+1|t}C^T + R)^{-1}}_{K_{t+1}}CP_{t+1|t}
\end{aligned}
$$

- Note:
  - $\hat{x}_{t+1|t+1}$ and $P_{t+1|t+1}$ are random variables, since they depend on γ_{t+1}
  - We need to give a statistical description of $P_{t+1|t+1}$

Block diagram



First Approach

- Let's try to solve the difference equation for

$$E[P_{t+1|t}]$$

$$P_{t+1|t} = AP_{t|t-1}A^T + Q - \gamma_t\, AP_{t|t-1}C^T(CP_{t|t-1}C^T + R)^{-1}CP_{t|t-1}A$$

$$\mathbb{E}[P_{t+1|t}] = A\mathbb{E}[P_{t|t-1}]A^T + Q - \bar{\gamma}\mathbb{E}[AP_{t|t-1}C^T(CP_{t|t-1}C^T + R)^{-1}CP_{t|t-1}A]$$

We don't get a trivial recursion

Approach to the solution

- Let's try to find deterministic difference equations to bound $E[P_{t|t-1}]$

$$\mathbb{E}[P_{t+1|t}] = A\mathbb{E}[P_{t|t-1}]A^T + Q - \bar{\gamma}\,\underbrace{\mathbb{E}[AP_{t|t-1}C^T(CP_{t|t-1}C^T+R)^{-1}CP_{t|t-1}A]}_{*}$$

Using Jensen's inequality and monotonicity arguments:

$$A\mathbb{E}[P_{t|t-1}]A^T \geq * \geq A\mathbb{E}[P_{t|t-1}]C^T(C\mathbb{E}[P_{t|t-1}]C^T+R)^{-1}C\mathbb{E}[P_{t|t-1}]A$$

$$S_{t+1} = (1-\bar{\gamma})AS_tA' + Q \qquad V_{t+1} = AV_tA' + Q - \bar{\gamma}AV_tC^T(CV_tC^T+R)^{-1}CV_tA$$

Lower bound Upper bound

$$S_t \leq E[P_{t|t-1}] \leq V_t$$

Lower Bound

- The solution to the difference equation:

$$S_{t+1} = (1 - \bar{\gamma}) A S_t A' + Q$$

is a lower bound for $E[P_{t|t}]$.

- It diverges for:

$$\bar{\gamma} \leq \gamma_{min} = 1 - 1/\rho^2 \qquad \rho = max\{|eig(A)|\}$$

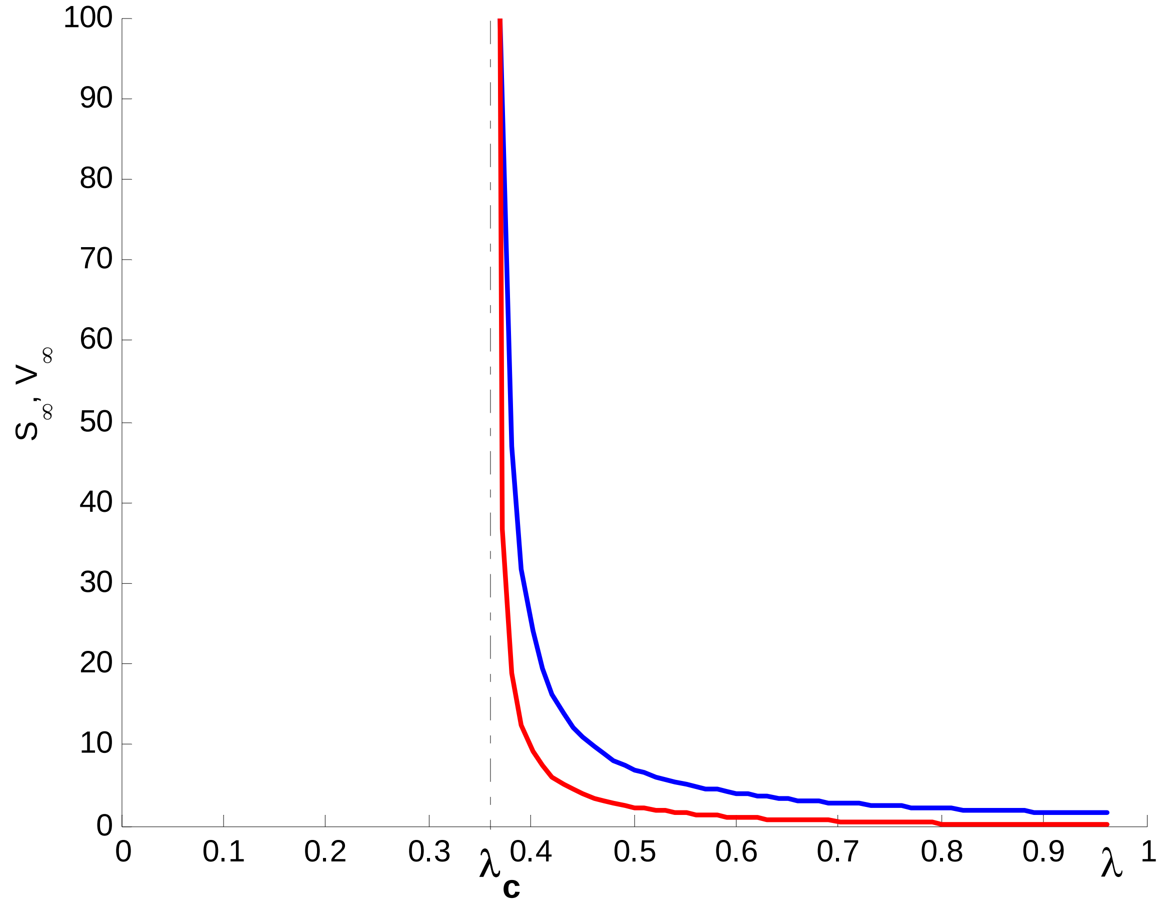
Upper Bound

- Modified Algebraic Riccati Equation (MARE):

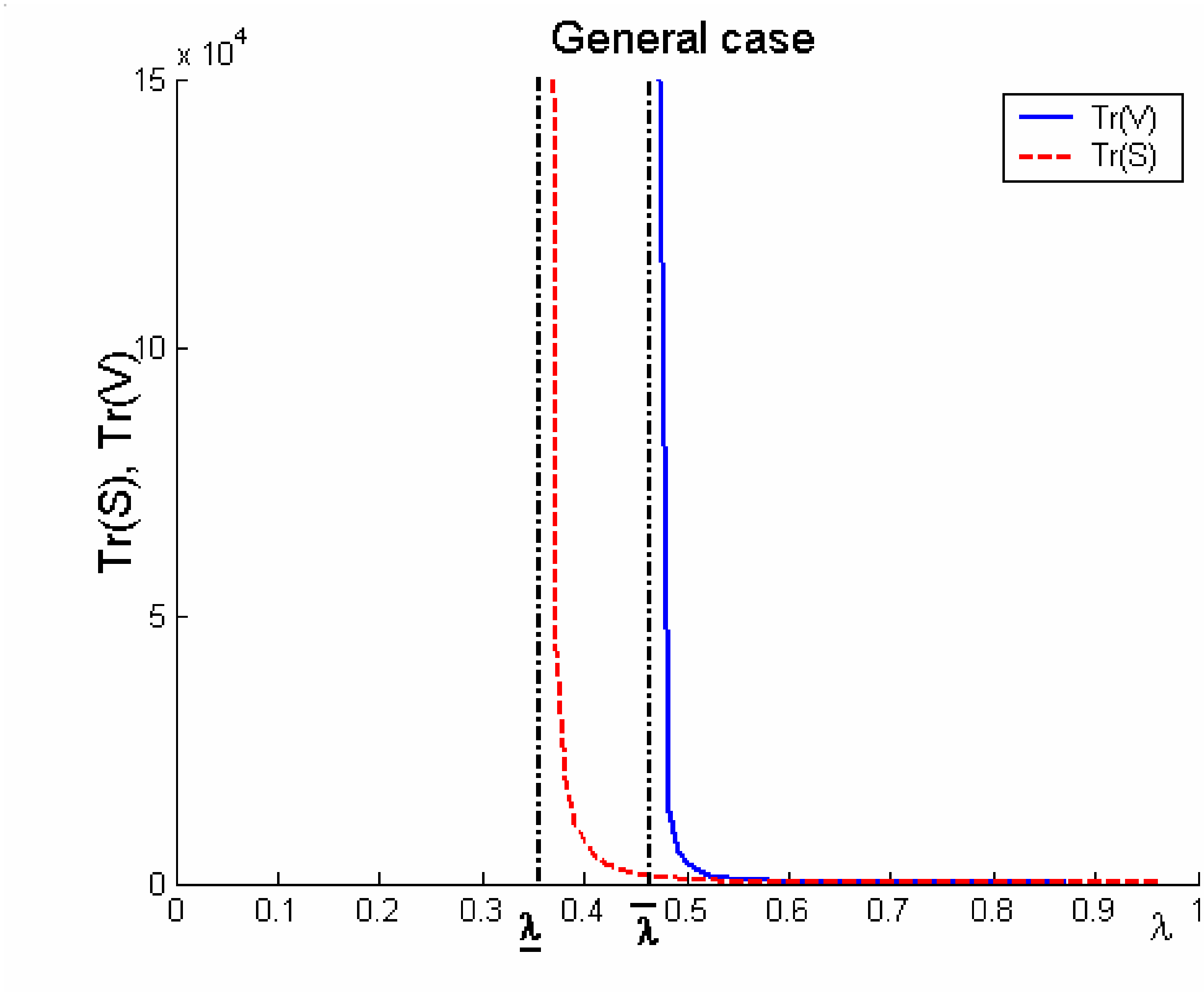
$$V_{t+1} = AV_tA' + Q - \bar{\gamma}AV_tC^T(CV_tC^T + R)^{-1}CV_tA$$

- Converges for $\bar{\gamma}$=1 and diverges for $\bar{\gamma}=\gamma_c$ ➔ $\exists$ γ_{max}, such that $\bar{\gamma} > \gamma_{max}$ MARE converges (continuity argument)
- Questions:
  - How to find min γ_{max} ? ➔ feasibility LMI with bijection on $\bar{\gamma}$
  - How to find $V=g_\gamma(V)$ when it exists ? ➔ just iterate $V_{t+1}=g_\gamma(V_t)$, $V_0 \geq 0$
  - $\gamma_{max}=\gamma_c$? ➔ only if C is invertible

Lower & Upper Bound (Scalar Case)



General case



Theorem

$\lim_{t\to\infty} E[P_t] = \infty$ for $0 \le \bar{\gamma} \le \gamma_c$ and some initial condition $P_0 \ge 0$

$E[P_t] \le M_{P_0}$ $\forall t$ for $\gamma_c < \bar{\gamma} \le 1$ and any initial condition $P_0 \ge 0$

$$1 - \frac{1}{\max |\sigma_i|^2} = \gamma_{min} \le \gamma_c \le \gamma_{max}$$

If C is invertible then $\gamma_{min} = \gamma_c = \gamma_{max}$

Contribution

- Optimal dynamic filter among all possible filters

- We can prove the existence of a unique critical value γ_c such that $E[P_t]$ converges for all $\bar{\gamma} > \gamma_c$ and diverges otherwise

- Analytical solution for lower and numerical solution for upper bound for the critical probability

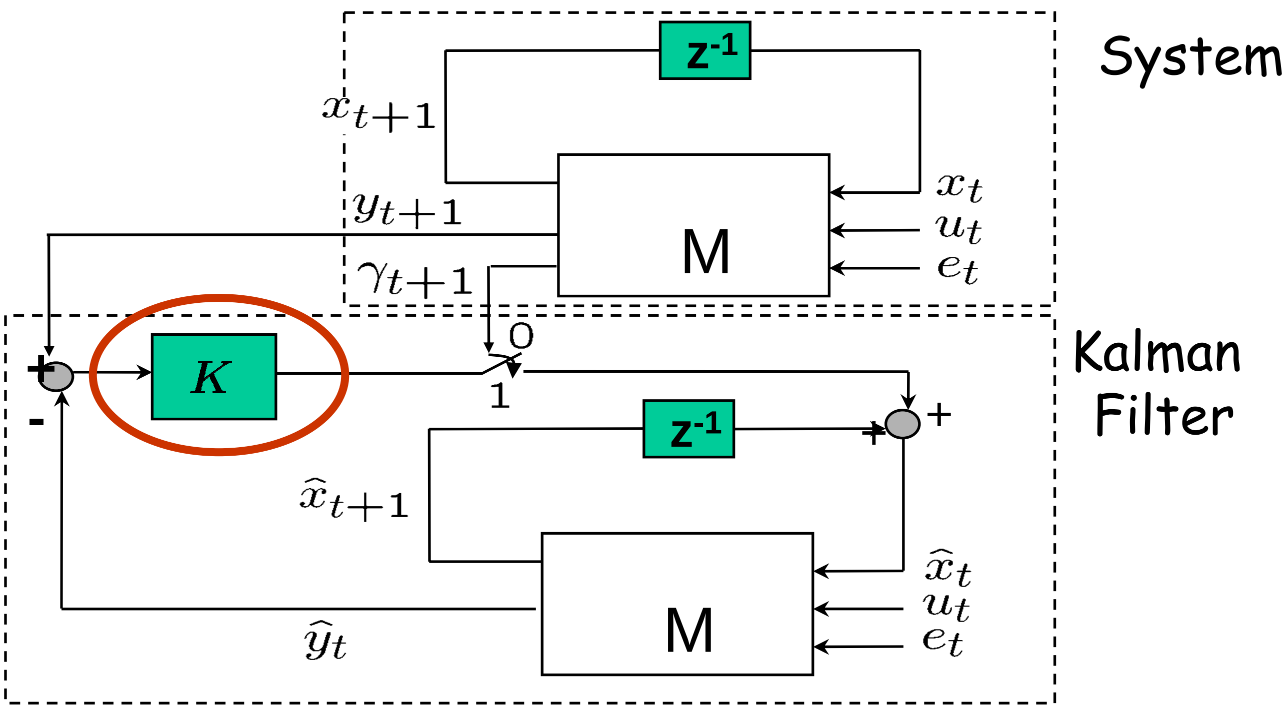
$$\gamma_{min} \leq \gamma_c \leq \gamma_{max}$$

- Numerical solution for lower and upper bounds for the estimation error covariance $E[P_{t|t-1}]$

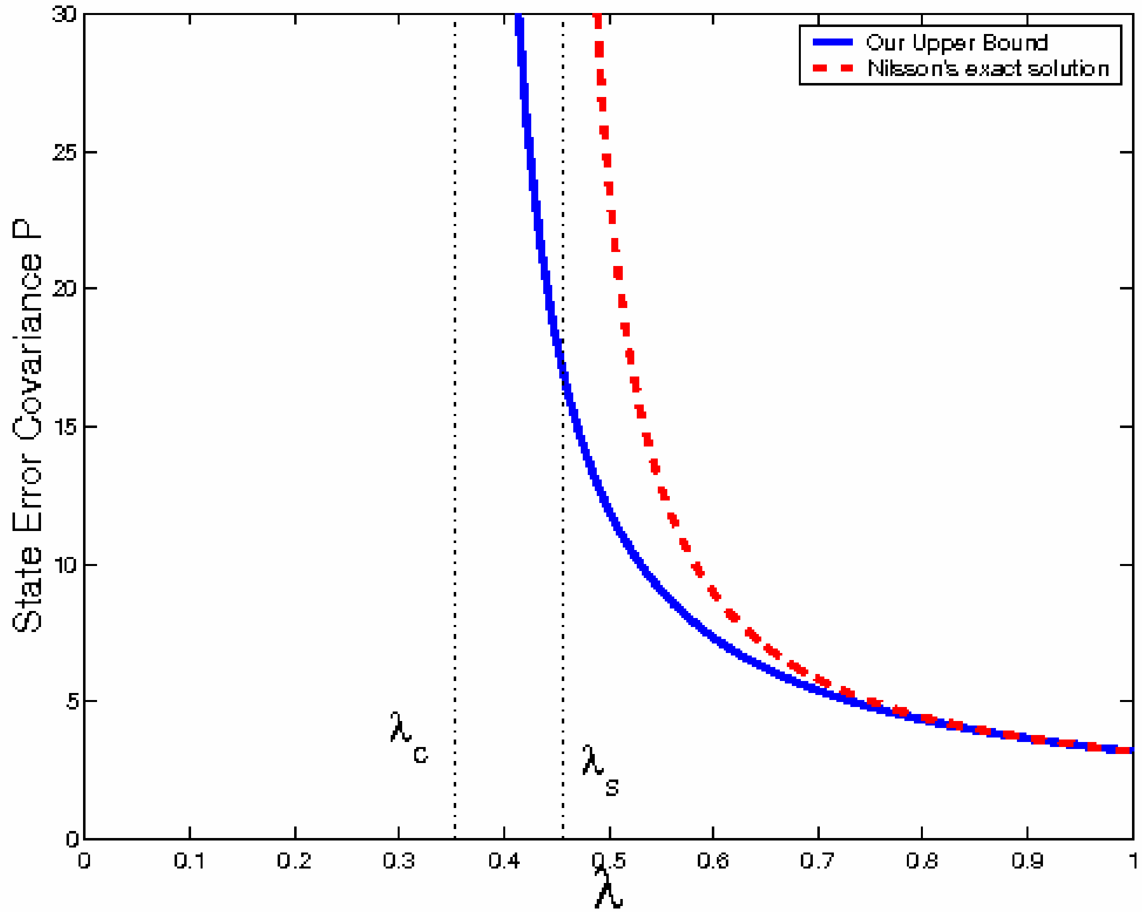
$$\underline{P} \leq E[P_{t|t-1}] \leq \overline{P}$$

Relation to Jump Linear Systems

- Nilsson et al. have solved the same problem using a jump linear system with two states, open loop filter and a closed loop one with constant gain
- They derive exact value of λ_c for the steady state Kalman filter, which is suboptimal



Performance comparison



Steady State filter shows lower performance as $\lambda \rightarrow 0$

Design Guidelines

- Estimation problem:
  - Characterize the reliability of your channel
    - Find your λ
  - Model the dynamical phenomenon you want to observe (linearize if necessary)
  - Observe the eigenvalues of the system
    - If $\lambda > \lambda_c$ your estimate will have bounded covariance on average
    - Else
      - slow down dynamics (change eigenvalues) if you have control over them
      - or increase the reliability of the channel