CDS110 Recitation Section, Friday November 30

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1) Recap of performance specifications and loop shaping
2) Performance limitations: proof of the Bode Integral Formula
3) Hw 8, problem 3

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1) Performance specifications: steady state error, phase and gain margins and tracking error

Given a plant transfer function P(s), we want to design a controller C(s) such that the CLOSED LOOP transfer function (CLTF) will achieve:

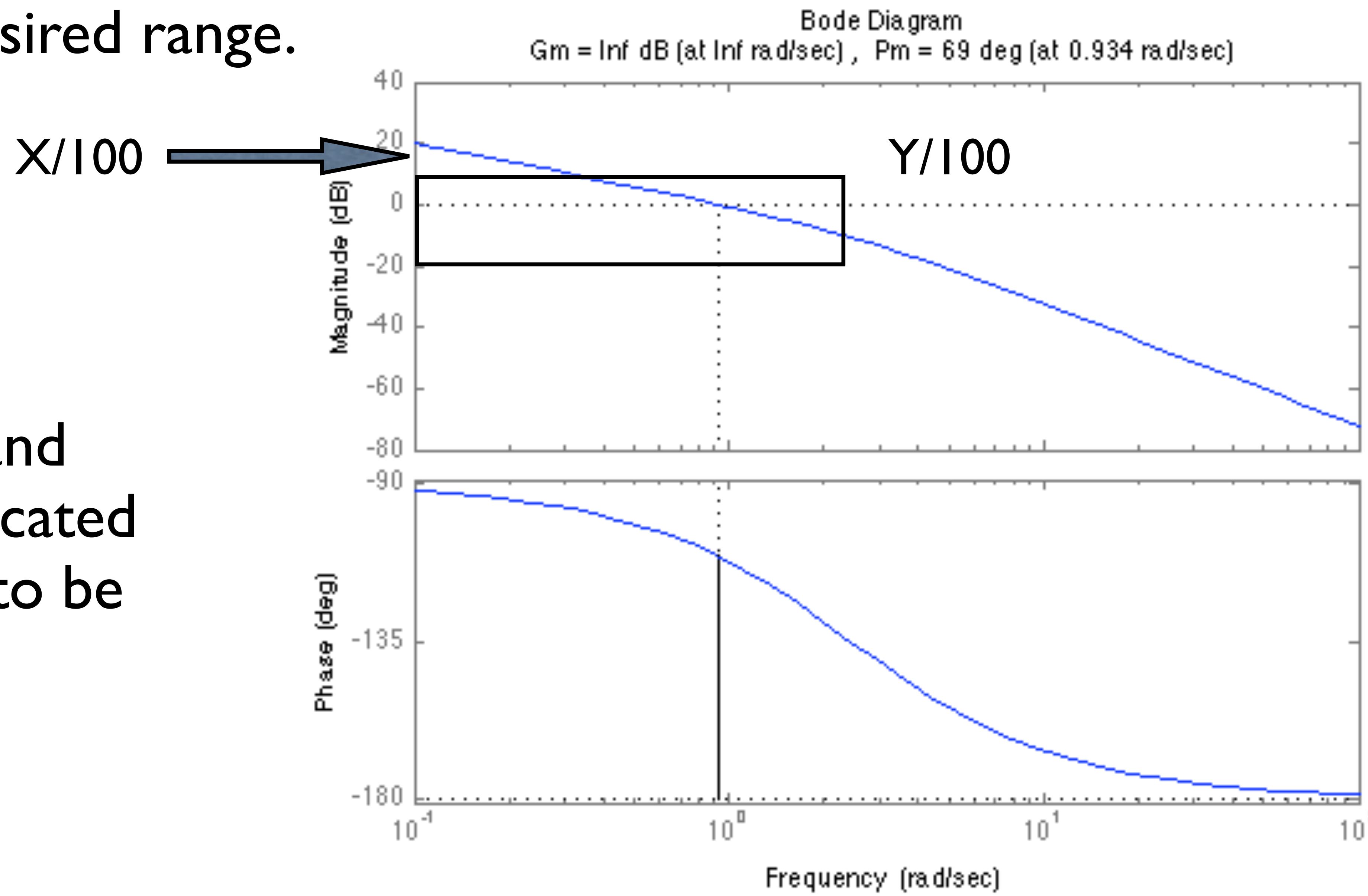
a) steady state error < X/100
b) tracking error < Y/100 over a certain frequency range
c) gain margin > Gm
d) phase margin > Pm

Remember: these properties are to be shaped on the OPEN LOOP transfer function (OLTF) L(s)=C(s)P(s).

1) Performance specifications: steady state error, phase and gain margins and tracking error - RECAP

a) Steady state error: the transfer function Her is defined as 1/(1+L(s)). Approximate 1/(1+L(0))≈1/L(0). The specification 1/L(0)<X/100 => we want that L(0)>100/X.

b) Tracking error over a certain frequency range: now the transfer function Hyr is L/(1+L), but the error will be (1- L/(1+L)) therefore we will still use Her. Using the same approximation of a), we have to check that L(jw)>Y/100 in the desired range.

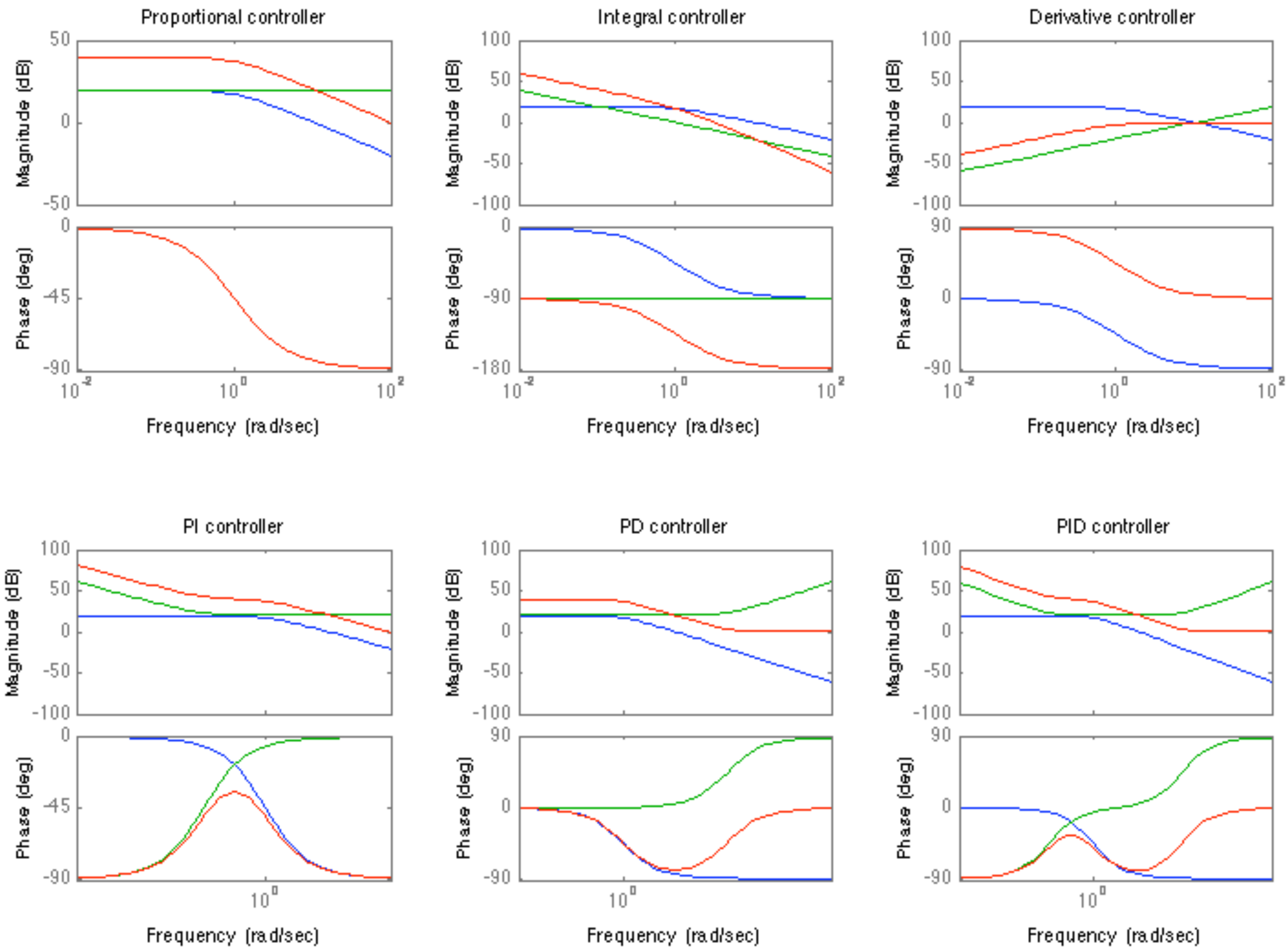


c) & d) Gain margin and phase margin are indicated in the figure and are to be shaped through the controller as well.

1) LOOP SHAPING: once you plot Bode for your OLTF, you can shape it by choosing the right controller.

RULES: log of magnitudes are summed, phases are summed.

TABLE: effects of elementary controllers on P(s)=10/(s+1)



Kp=10
Ki=1
Kd=.1

P ---
C ---
PC ---

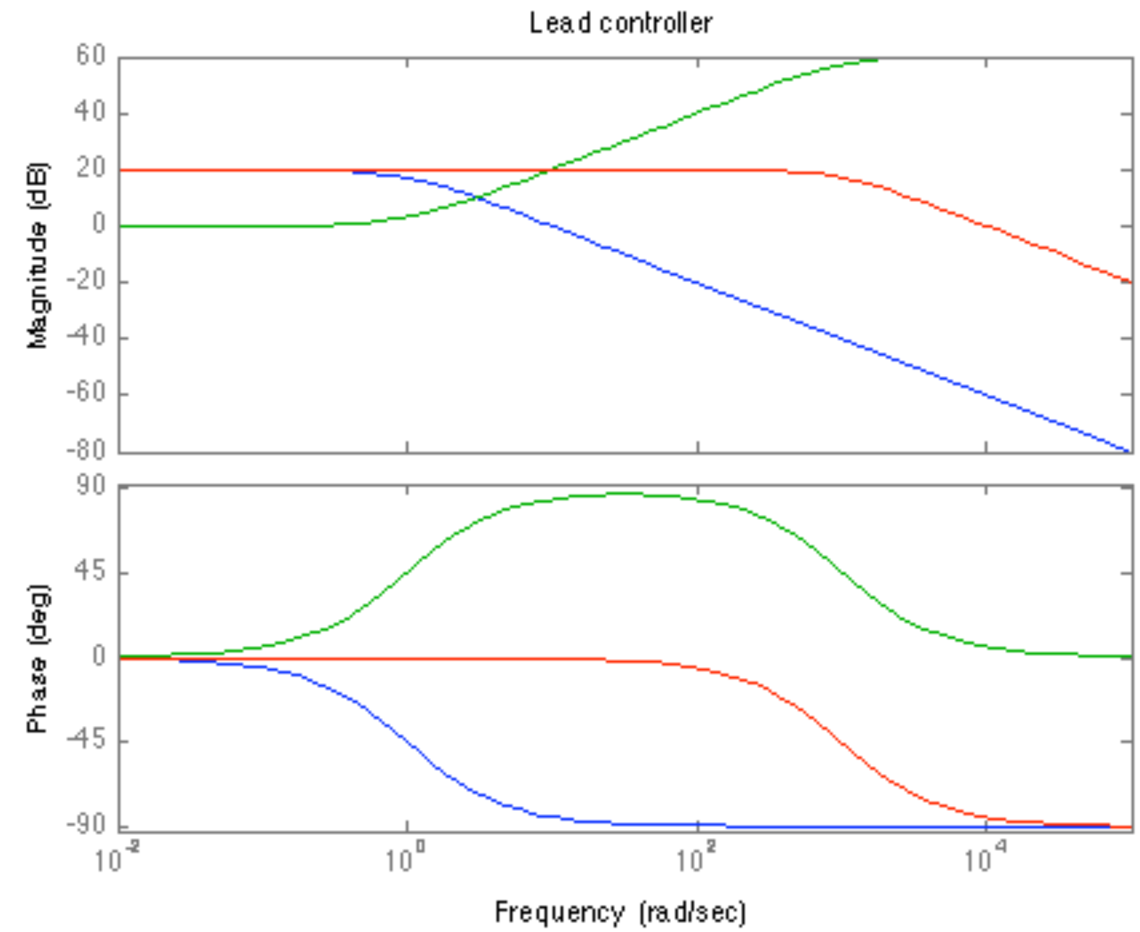
Lead Controller: zero first, then pole

$C(s)=1000\ (s+1)/(s+1000)$

P ---

C ---

PC ---



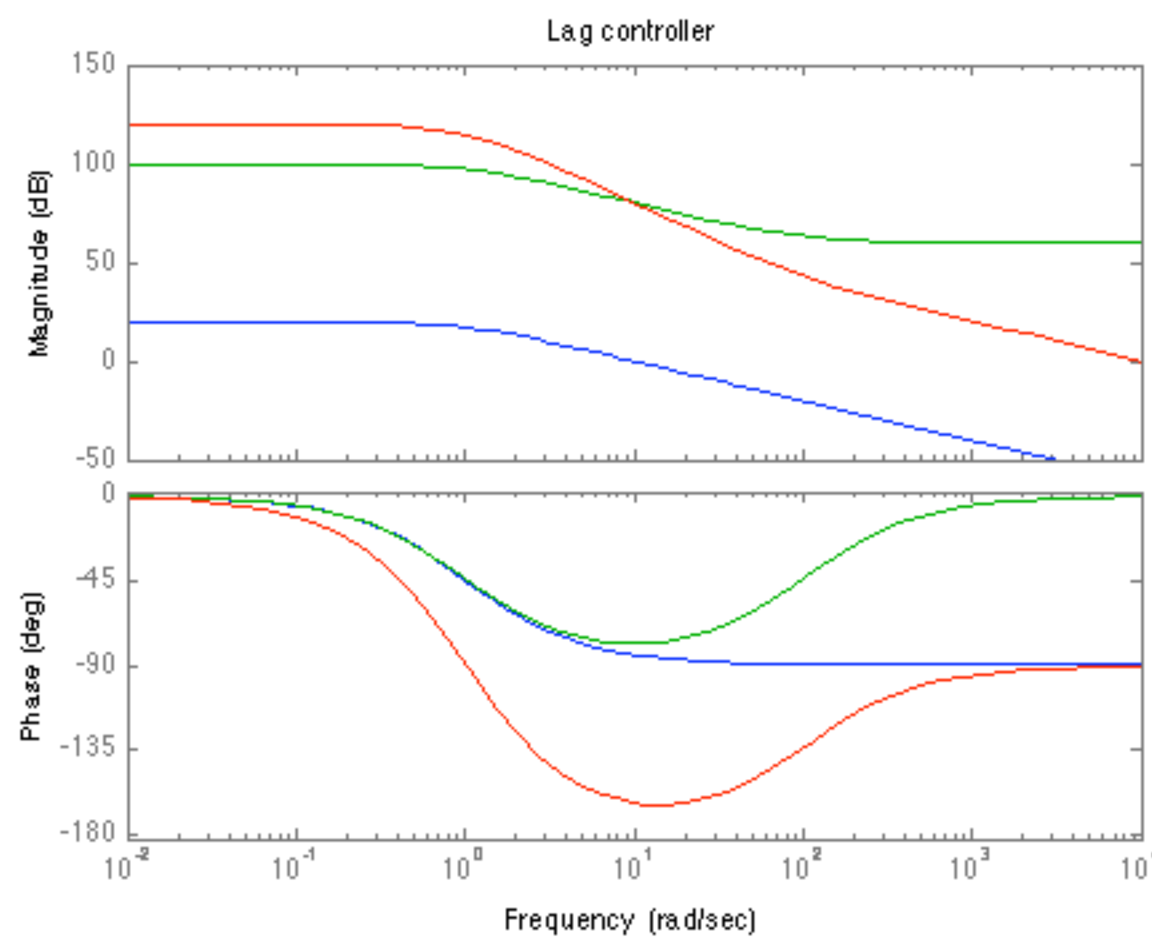
Lag Controller: pole first, then zero

$C(s)=1000\ (s+1000)/(s+1)$

P ---

C ---

PC ---



2) Performance limitations: proof of the Bode Integral Formula

Consider the OLTF L(s), and assume it has N poles in the RHP (unstable). Define the sensitivity function S(s)=1/(1+L(s)), and assume it goes to zero faster than 1/s for s going to infinity. Then:

$$\int_0^{+\infty} |\log(S(i\omega))| \, d\omega = \pi \sum_{k=1}^{N} Re(p_k)$$

Preliminary observations:

a) S(s) has zeros where L(s) has poles. So unstable poles of L(s) are mapped to RHP zeros of S(s).

b) Logarithm of a complex number $z \in \mathbb{C}$

$$log(z) = log(|z|) + i\, arg(z)$$

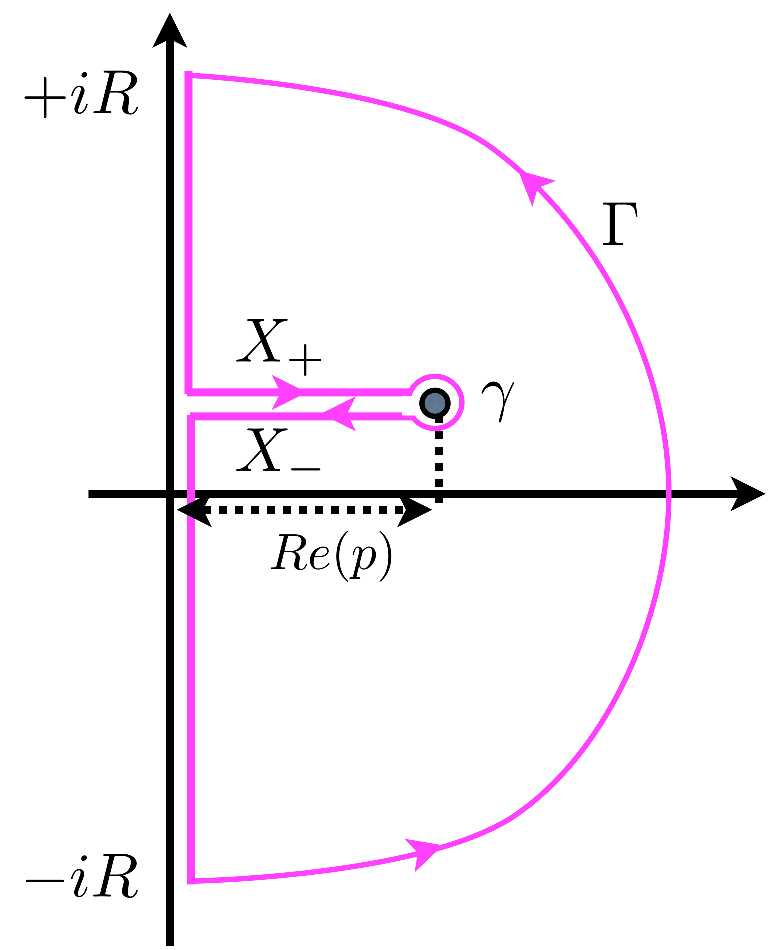
Example: $log(-2) = log(2) - i\pi$

The argument is the *principal argument* $\theta \in [-\pi,\, pi)$

Consider the case where we have one pole in the RHP, and pick the D contour integral in the picture. $\bar{\gamma} = X_+ + \gamma + X_-$

$$\int_D log(S(s))ds = \int_{iR}^{-iR} log(S(s))ds + \int_\Gamma log(S(s))ds + \int_{\bar{\gamma}} log(S(s))ds = 0$$

$$\begin{aligned} \int_{iR}^{-iR} log(S(s))ds &= -i\int_{-iR}^{iR} log(S(s))ds \\ &= 2i\int_0^{iR} |log(S(s))|ds \end{aligned}$$



$$lim_{R\to\infty}\int_\Gamma log(S(s))ds = 0$$

$$lim_{\gamma\to 0}\int_\gamma log(S(s))ds = 0$$

$$\int_{X_+} log(S(s))ds + \int_{X_-} log(S(s))ds =$$

$$\int_{X_+} log(S(s)) - log(S(s-2i\pi))ds = 2i\pi Re(p)$$

Because:

$$log(S(s)) - log(S(s-2i\pi)) =$$

$$|S(\cancel{s)}| - |S(s \cancel{-2}i\pi)| + i\,arg(S(s) - S(s-2\pi)) = 2i\pi$$

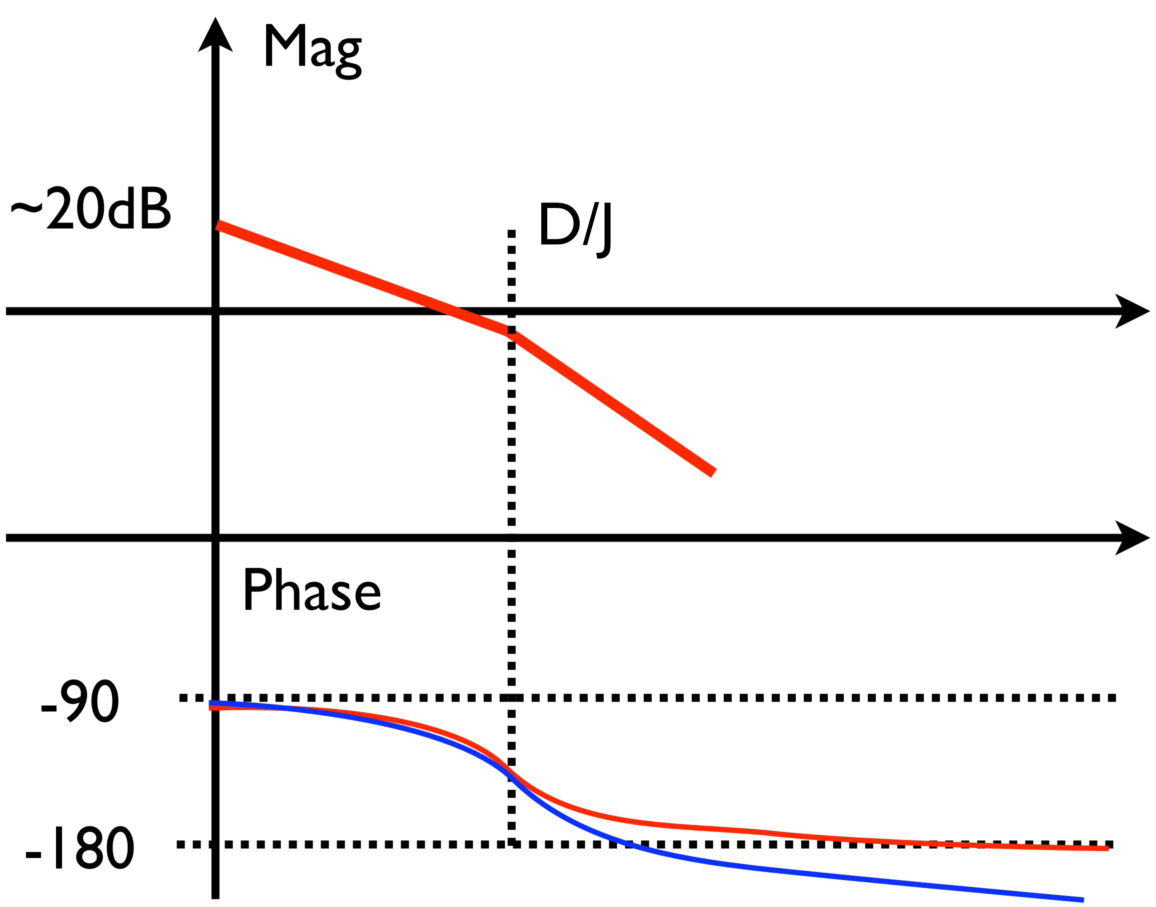
and the the length of X is Re(p)

QED

3) Hw 8, problem 3 - let's analyze this system

$$P(s) = \frac{1}{s(sJ+D)} e^{-s\tau} \qquad e^{-s\tau} \approx \frac{1 - s\,\tau/2 + s^2\,\tau/12}{1 + s\,\tau/2 + s2\,\tau/1}$$

Asymptotic Bode plot w/o delay:



$$\left| \frac{1}{-\omega^2 J + i\omega D} \right| = 1 \qquad \omega \approx .9339$$

$$M_\phi \approx 69^\circ$$

The delay term will not change the gain, but adds a phase which is a function of the frequency:

$$\phi_D(\omega) \approx 0.6\,|\omega|$$

Converted in degrees!

Performance specifications required:
a) zero steady state error
b) tracking error less than 10% between 0 and .5Hz
c) overshoot not more than 10%

a) We already have a pole at the origin, so the steady state error is zero anyways. Check:

$$lim_{s\to 0} \frac{1}{1+L(s)} = 0$$

b) We can draw the usual box: we need to increase the gain here!

c) Need to relate the overshoot with the phase margin. Based on what found at Problem 1, one should figure that we need a phase margin of at least 70 deg.
Idea: use a for loop - pick a proportional controller, increase the gain. Compute the step response of the CLTF at each gain and compare phase margin of OLTF and corresponding overshoot.
Or use second order system formulas.

Summarizing: we need to increase the gain above 20 dBs between 0 and .5 Hz, but also increase the phase in that range.

A lead compensator (as required) will do.

$$C(s) = k\,\frac{s+a}{s+b}$$

Place the zero first, and then the pole. Roughly, in between you will have a 90 deg increase in phase. Increasing k will help with the tracking specification.