CDS 270-2: Lecture 6-2
Impact of Communication Noise on Estimation over Wireless Links

Yasamin Mostofi

May 3, 2006

Goals:

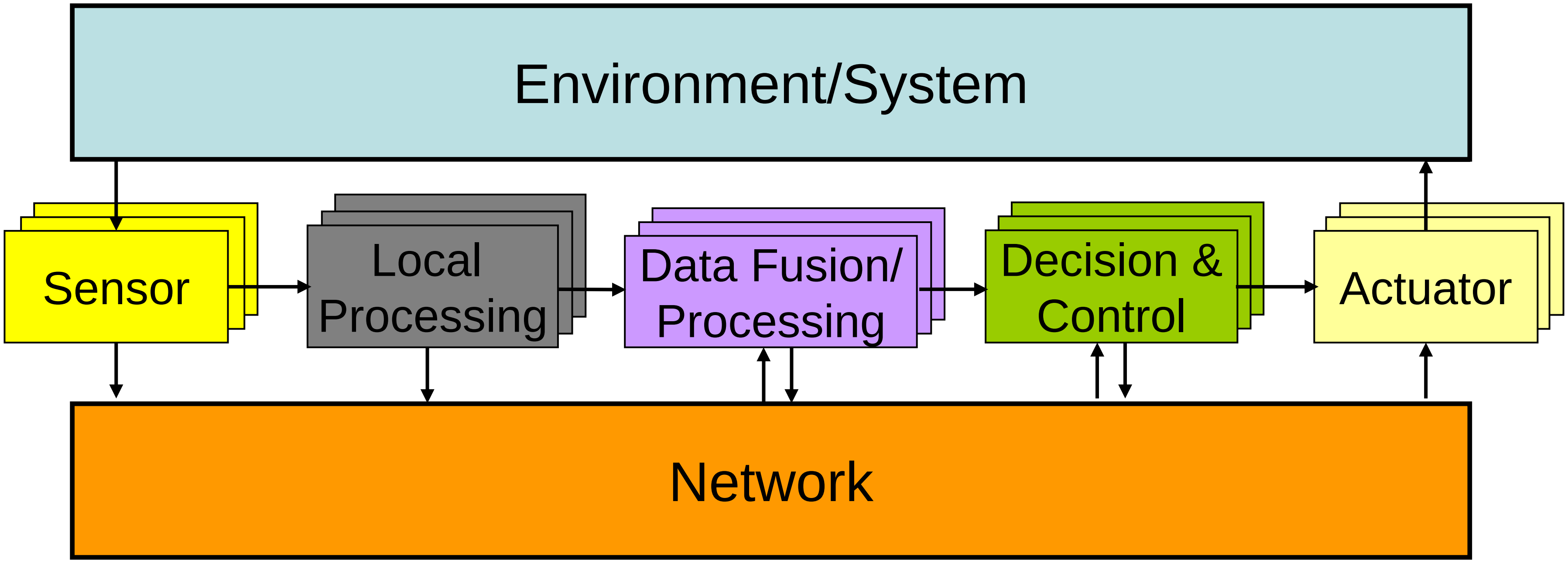
- To understand the impact of noisy wireless communication links on estimation over wireless
- To evaluate the performance of Kalman filtering over noisy mobile links

Reading for May 3th and 5th

Can also be found at http://www.cds.caltech.edu/~yasi/

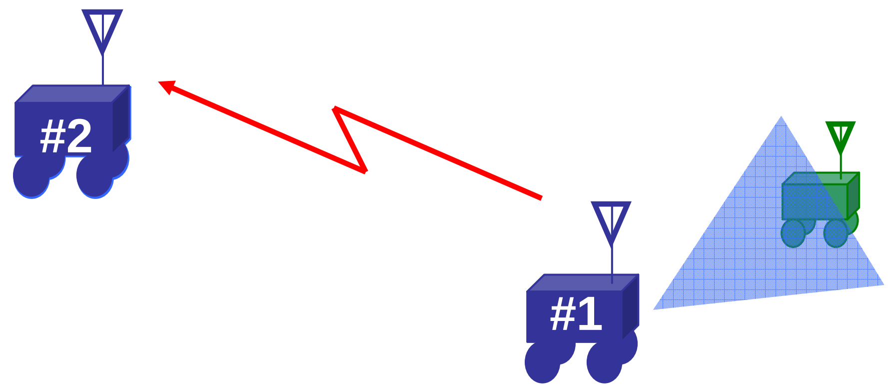
- Y. Mostofi and R. Murray, “Receiver Design Principles for Estimation over Fading Channels," Proceedings of Conference on Decision and Control (CDC), December 2005
- Y. Mostofi and R. Murray, “On Dropping Noisy Packets in Kalman Filtering over a Wireless Fading Channel”, Proceedings of American Control Conference (ACC), June 2005
- Y. Mostofi and R. Murray, "Effect of Time-Varying Fading Channels on the Control Performance of a Mobile Sensor Node," Proceedings of IEEE 1st International Conference on Sensor and Ad Hoc Communications and Networks (Secon), October 2004, Santa Clara, CA 05

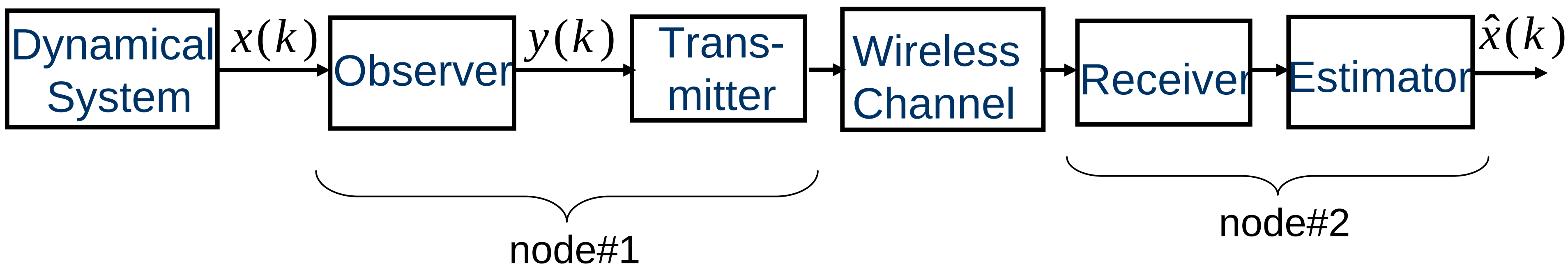
Networked Sensing, Estimation & Control



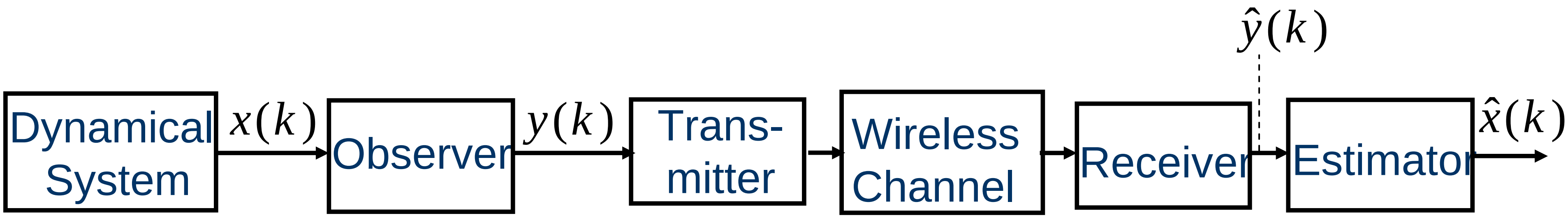
Today: Modeling and impact of wireless links

System Model





System Model



To focus on communication noise, assume scalar quantities

Linear dynamical system: $x(k+1) = Ax(k) + w(k)$

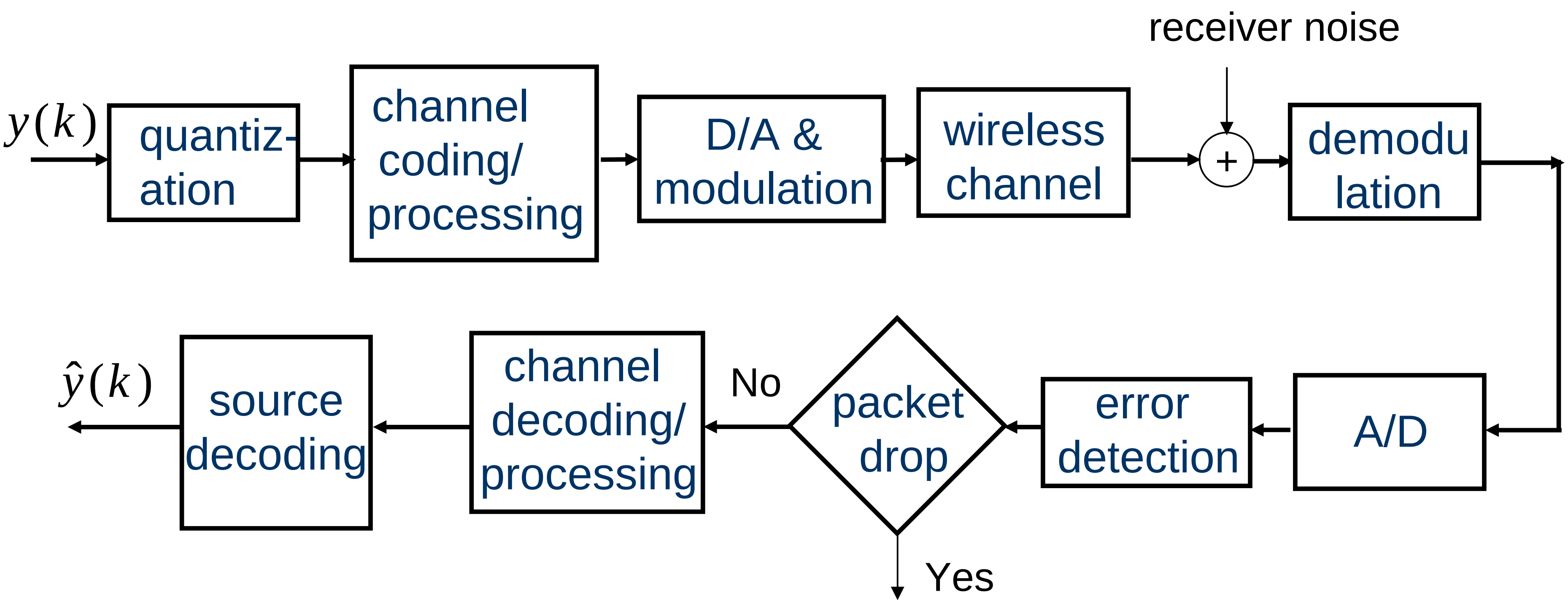
Observation: $y(k) = Cx(k) + v(k)$

$w(k)$: Zero mean noise with variance of Q

$v(k)$: Zero mean noise with variance of R

$\hat{x}(k)$: Kalman filter estimate of $x(k)$

Wireless Transmission



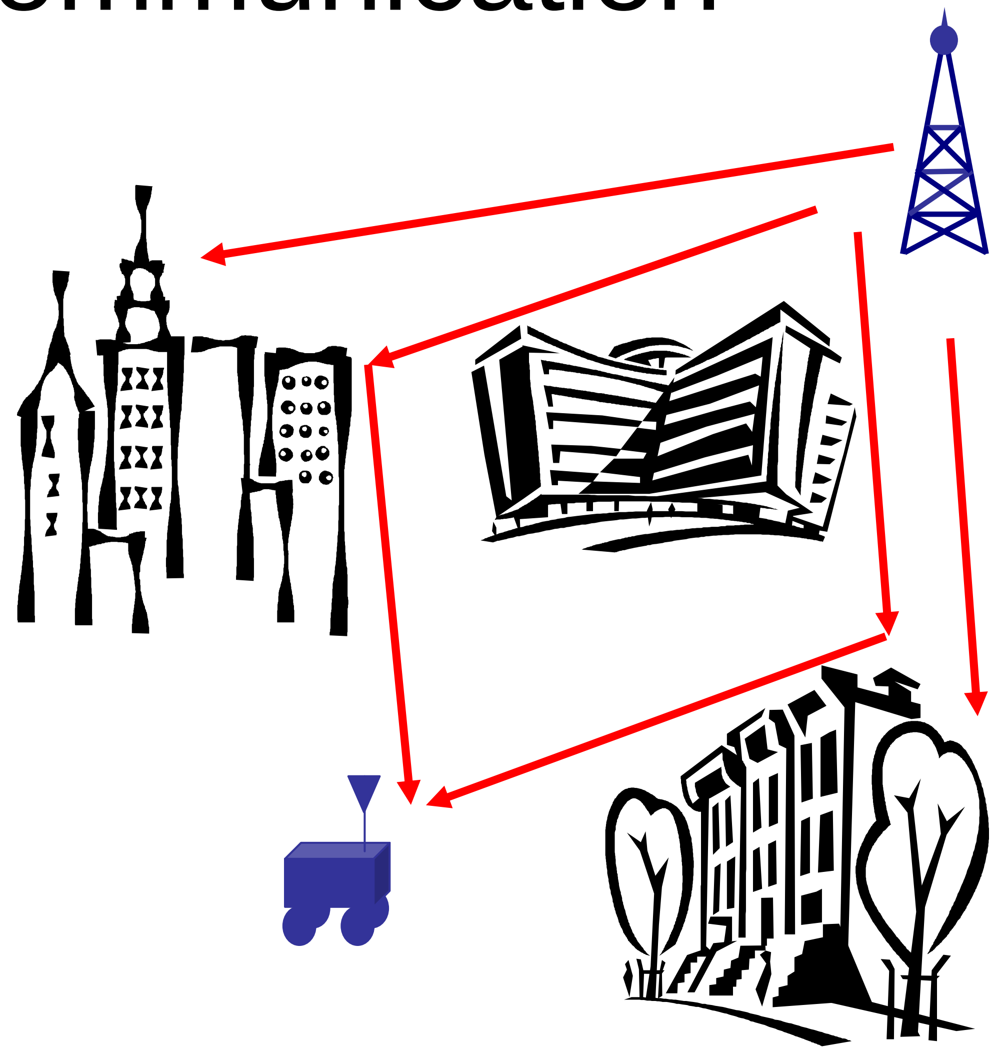
Past 4 classes: Studied cases where only noise-free samples came through

Error Detection & Packet Drop

- Drop criteria changes depending on the application
- Voice applications are delay-sensitive:
  - Calls are dropped only if crucial bits are corrupted
  - There is error detection for crucial bits
  - The rest of the error is either corrected or tolerated
- Data applications are not delay-sensitive:
  - Packets are only kept if no error is detected
- What is optimum for control over wireless? (we will get to this later)

Wireless Communication

- Impairments:
  - Signal attenuation
  - Multipath, fading & shadowing
  - Time-varying links
  - Limited bandwidth
  - Collision
- One measure of link quality:
  - **Received Signal to Noise Ratio** = Ratio of received signal power to receiver noise power



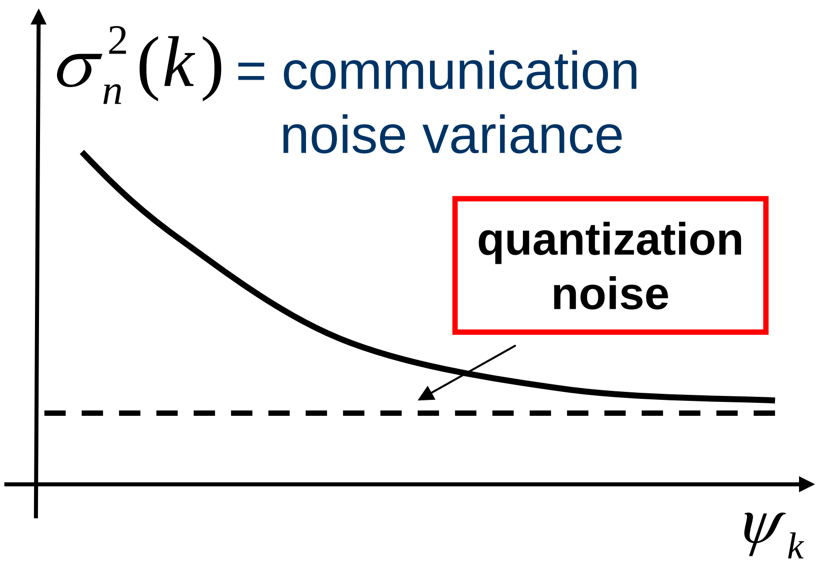
Communication Noise

- Impairments result in noisy reception:

$$\hat{y}(k) = y(k) + n(k) = Cx(k) + v(k) + n(k)$$

- $n(k)$ is communication noise with variance of $\sigma_n^2(k)$
- $\sigma_n^2(k)$ is a function of received Signal to Noise Ratio
- Communication noise appears as additional observation noise
- This model provides the right abstraction for estimation and control

Communication Noise Variance



$\sigma_n^2(k)$ **:**

- Function of TX/RX technologies like quantization, noise figure, modulation, channel coding, ..

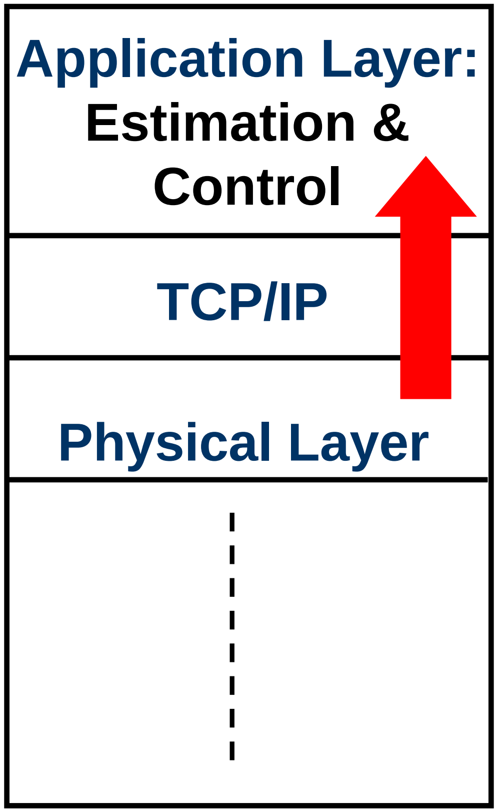
Distribution of ψ_k:

- Function of environment
- Common outdoor model:
  – exponential distribution

ψ_k: Received Signal to Noise Ratio at k^{th} transmission

Can the Estimator Know the Variance of Communication Noise?

- KF relies on using covariance of the observation noise
- In order for the estimator to know the quality of the communication link, a cross-layer information path is needed
- In general such paths can improve the performance considerably and have gotten considerable interest recently
- However, one has to be cautious since careless use of such paths can ruin the robustness of the system



Design Choices

- Estimation & control over wireless links are new applications
  - Need new design paradigms
- Possible design choices:
  - Receiver can keep all the samples
  - Receiver can optimize the packet drop
  - Cross-layer: estimator can use link quality information
- Consider *unstable processes*. We are interested in:
  - Analytical expressions to evaluate the performance of Kalman filtering over wireless noisy links
  - Optimizing packet drop (Friday lecture)
  - Stability condition (Friday lecture)

Performance Evaluation Example

- Consider the following example:
  - Receiver that keeps all the packets
  - KF that uses knowledge of channel quality (trust coefficient)
  - One class of channels:

    $$\sigma_n^2(k) = \frac{\beta}{\psi_k} \text{ for } \beta > 0$$

  - Exponential distributed ψ_k
  - Channel gets uncorrelated from one sample to next
  - In order to focus on communication impact, assume the following for this example: $R = 0, Q = 0 \;\&\; C = 1$

Performance Evaluation

$$P_k = \overline{\left[x(k) - \hat{x}(k)\right]^2}_{|\psi_{k-1},\ldots,\psi_0}$$

We are interested in finding $\overline{P_k}$

$$\Gamma = \frac{1}{\overline{\psi_k}}$$

$$P_{k+1} = \frac{A^2\beta \times P_k}{\beta + \psi_k \times P_k}$$

$\overline{P_{k+1,i}}$: average of P_{k+1} over $\psi_k, \psi_{k-1}, \ldots\ldots, \psi_{k-i}$

$$\overline{P_{k+1,0}} = E\left(\frac{A^2\beta P_k}{\beta + \psi_k P_k} \mid P_k\right) = A^2\Gamma\beta P_k \int_0^\infty \frac{e^{-\Gamma\psi_k}}{\beta + \psi_k P_k} d\psi_k = A^2\Gamma\beta \times \Pi\left(\frac{\Gamma\beta}{P_k}\right)$$

with $\Pi(z) = e^z \text{Expint}(z)$, where $\text{Expint}(z) = \int_z^\infty \frac{e^{-t}}{t} dt$

Performance Evaluation (cont.)

$$\overline{P_{k+1,0}} = A^2\Gamma\beta \times \Pi(\frac{\Gamma\beta}{P_k}) = A^2\Gamma\beta \times \Pi(\frac{\Gamma(\beta+\psi_{k-1}P_{k-1})}{A^2P_{k-1}})$$

Lemma 1: Consider an exponentially dist. ψ with $\Gamma = \frac{1}{\overline{\psi}}$. We will have the following for an arbitrary $q>0$ and $A>1$:

$$\overline{\Pi(\frac{\Gamma(\beta+q\psi)}{q\times A^{2i}})} = \frac{\Pi(\frac{\Gamma\beta}{q\times A^{2i}})}{1-A^{-2i}} - \frac{\Pi(\frac{\Gamma\beta}{q})}{1-A^{-2i}} \quad i\geq 1$$

Performance Evaluation (cont.)

Using Lemma 1:

$$\overline{P_{k+1,1}} = A^2\Gamma\beta\left[\frac{\Pi(\frac{\Gamma\beta}{A^2P_{k-1}})}{1-A^{-2}} - \frac{\Pi(\frac{\Gamma\beta}{P_{k-1}})}{1-A^{-2}}\right]$$

Similarly : $\overline{P_{k+1,m}} = \sum_{z=0}^{m} B_{z,m}\Pi(\frac{\Gamma\beta}{A^{2z}P_{k-m}}),$

where $B_{0,0} = A^2\Gamma\beta.$ The goal is to find $B_{z,m=k}$.

Let $T_k(z,m) = \Pi(\frac{\Gamma\beta}{A^{2z}P_{k-m}})$. Then $\overline{P_{k+1,m}} = \sum_{z=0}^{m} B_{z,m}T_k(z,m).$

Performance Evaluation (cont.)

$\overline{P_{k+1,m}} = \sum_{z=0}^{m} B_{z,m} T_k(z,m)$. Substituting P_{k-m} as a function of P_{k-m-1} and averaging over ψ_{k-m-1} will result in the following for $-1 \le m \le k-1$ (using Lemma 1):

$$\overline{P_{k+1,m+1}} = \sum_{z=0}^{m} \frac{B_{z,m}}{\xi_{z+1}} T_k(z+1, m+1) - \sum_{z=0}^{m} \frac{B_{z,m}}{\xi_{z+1}} T_k(0, m+1)$$

$$= \sum_{i=1}^{m+1} \frac{B_{i-1,m}}{\xi_i} T_k(i, m+1) - \left[\sum_{z=0}^{m} \frac{B_{z,m}}{\xi_{z+1}} \right] T_k(0, m+1)$$

$$= \sum_{i=0}^{m+1} B_{i,m+1} T_k(i, m+1) \quad \text{where } \xi_i = 1 - \frac{1}{A^{2i}}$$

Performance Evaluation

- Finally

$$\overline{P_{k+1}} = \sum_{i=0}^{k} B_{i,k} e^{\frac{\Gamma\beta}{A^{2i}P_0}} \text{Expint}(\frac{\Gamma\beta}{A^{2i}P_0})$$

$$B_{i,k} = \begin{cases} -\sum_{z=0}^{k-1} \frac{B_{z,k-1}}{\xi_{z+1}} & i = 0 \\ \frac{B_{i-1,k-1}}{\xi_i} & i \neq 0 \end{cases}$$

$$0 \leq i \leq k$$

$$\Gamma = \frac{1}{\overline{\psi}}$$

$$B_{0,0} = A^2\Gamma$$

$$\xi_i = 1 - \frac{1}{A^{2i}}$$

Stability Condition

- $\overline{P_k}$ will be bounded as long as $\overline{\psi} \neq 0$

Proof : $P_{k+1} = \dfrac{A^2 \beta \times P_k}{\beta + \psi_k P_k}$, P_{k+1} is a concave function of P_k.

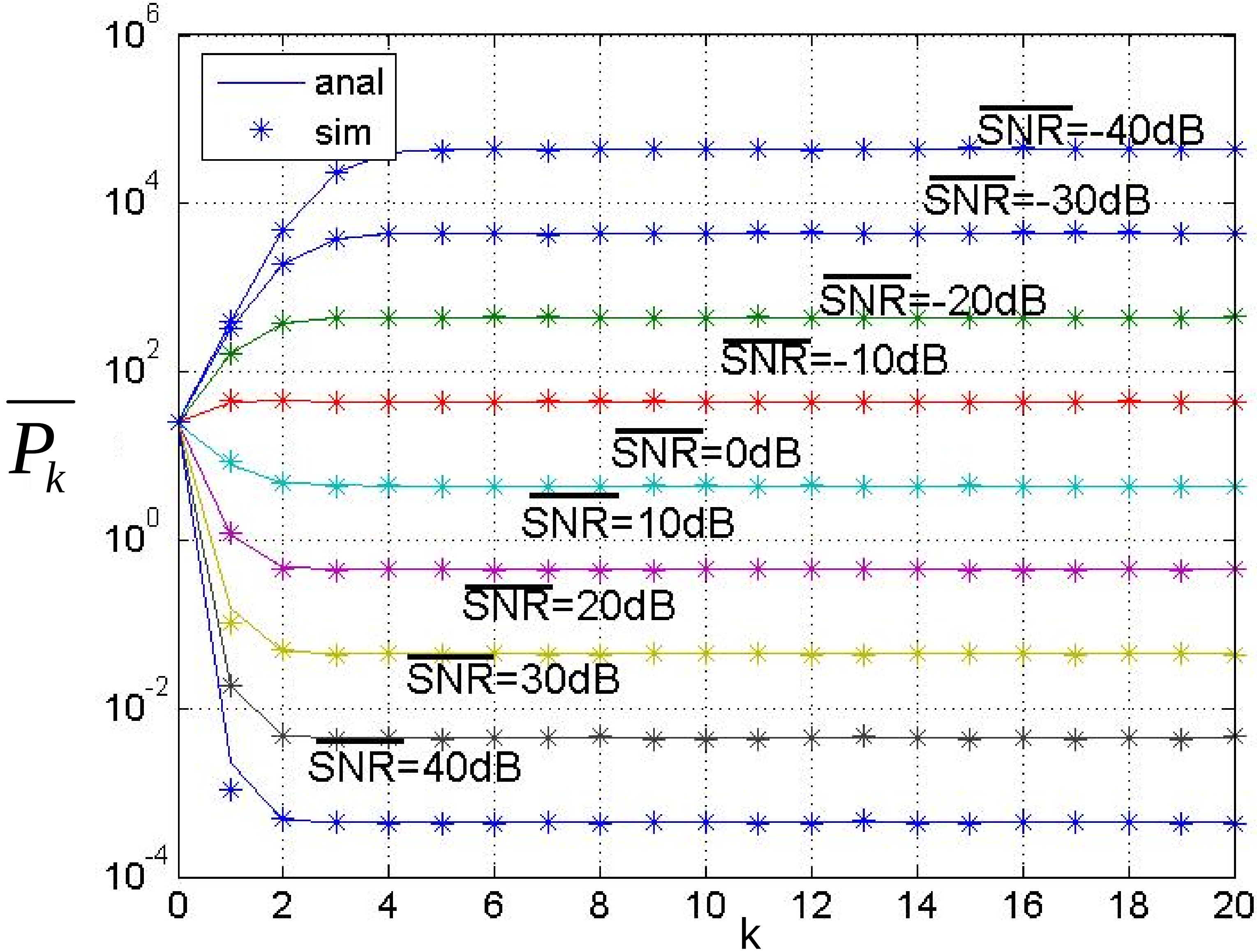
Using Jensen's inequality :

$$\overline{P_{k+1}} = E_{\psi_k}\left(E_{P_k}\left(P_{k+1}\right)\right) \leq E_{\psi_k}\left(\frac{A^2 \beta \times \overline{P_k}}{\beta + \psi_k \overline{P_k}}\right)$$

$$= A^2 \beta \Gamma \times e^{\Gamma \beta / \overline{P_k}} \text{Expint}\left(\frac{\Gamma \beta}{\overline{P_k}}\right)$$

if $\overline{P_k} > \dfrac{\Gamma \beta}{\mu_0} \Rightarrow \overline{P_{k+1}} < \overline{P_k}$ where $A^2 \mu_0 e^{\mu_0} \text{Expint}(\mu_0) = 1$

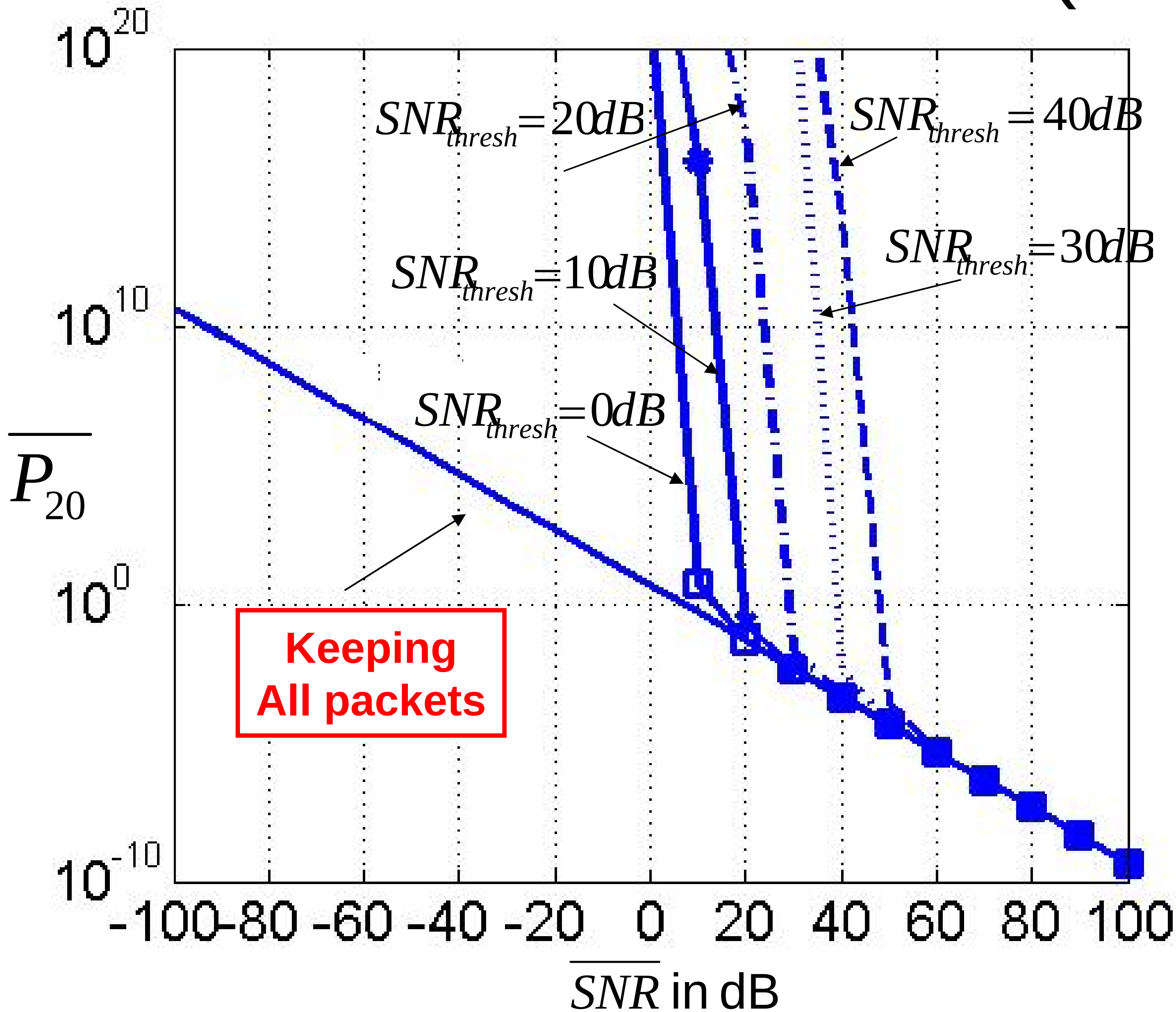
Performance Evaluation



$$A = 4$$

$$\beta = .1$$

Performance Evaluation (cont.)



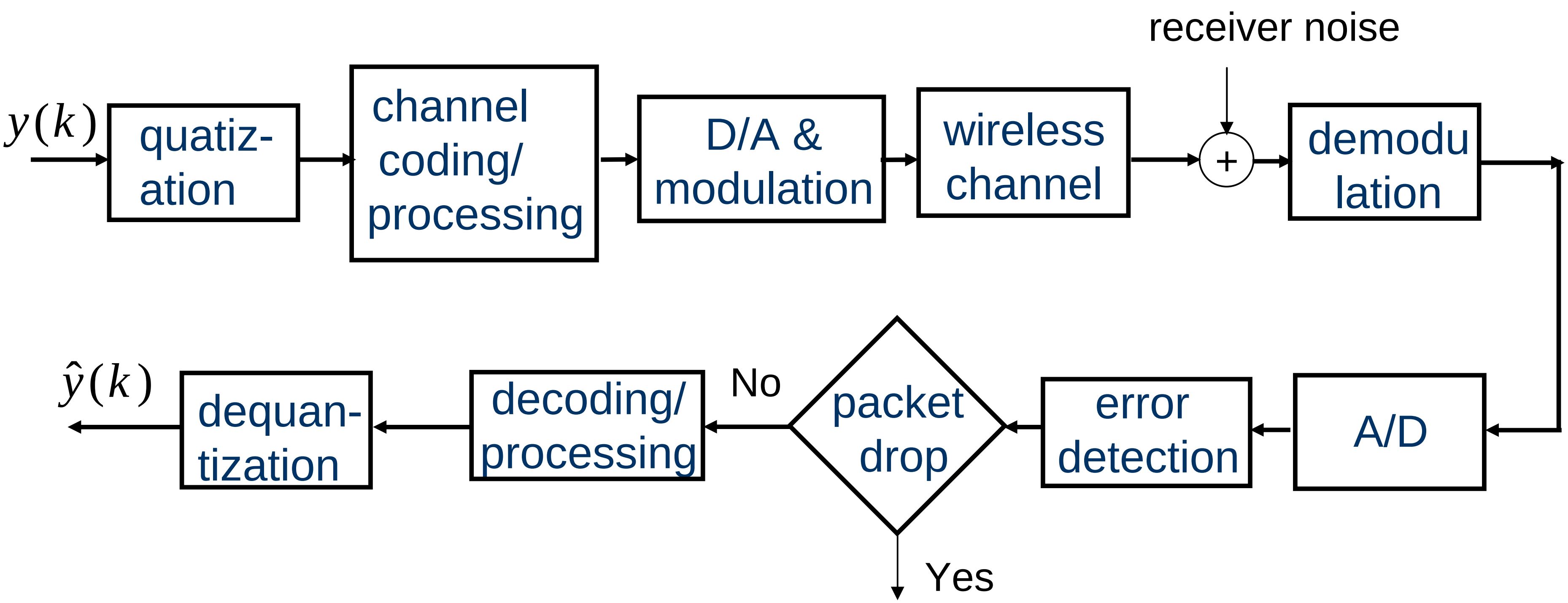
Summary (so far)

- We looked at a receiver that keeps all the packets and uses a cross-layer information path
- We derived analytical expression to evaluate performance
- We showed that the design is always stable

Possible Extensions

- Derive average estimation error variance for:
  - Other communication noise variances
  - General noise variance
  - General Signal to Noise Ratio distribution
  - Vector case
- Derive expressions for other moments of estimation error variance

Wireless Transmission



$$\hat{y}(k) = y(k) + n(k)$$

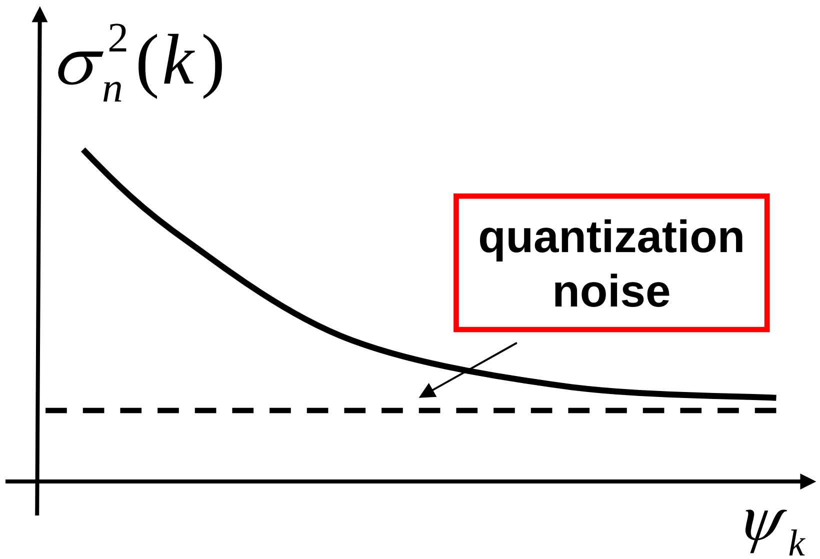
$n(k)$ is communicat ion noise with

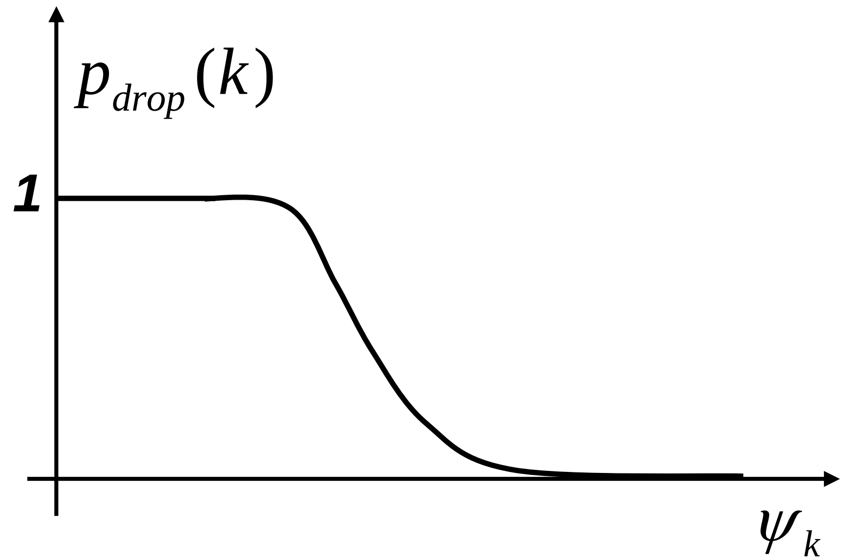
variance of $\sigma_n^2(k)$

Optimum Design

- What is the optimum packet drop for estimation and control over wireless links?
- What are the benefits of using channel knowledge in the estimator?
  - Stability & performance
- Consider general cases: general ψ and σ_n^2 and system parameters
- Ideal noise profile: keeping only noise-free samples
  - Suitable for non delay-sensitive applications

Abstraction in the Higher Layer





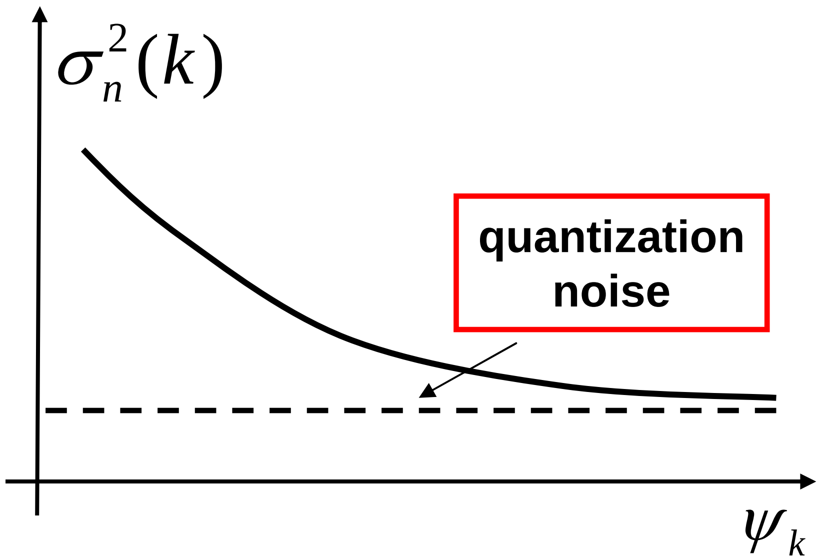
$\sigma_n^2(k)$ & $p_{drop}(k)$:

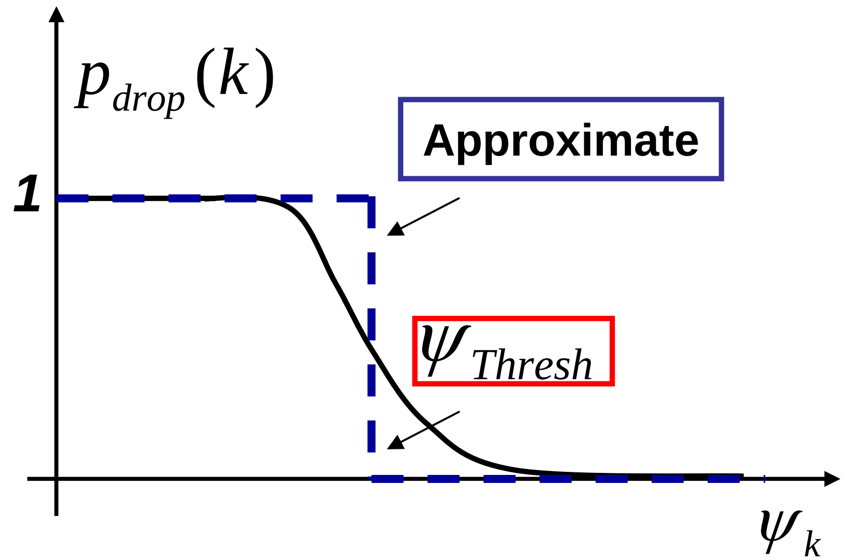
- Function of TX/RX technologies like quantization, noise figure, modulation, channel coding, ..

Distribution of ψ_k:

- Function of environment

Abstraction in the Higher Layer





$\sigma_n^2(k)$ & $p_{drop}(k)$:

- Function of TX/RX technologies like quantization, noise figure, modulation, channel coding, ..

Distribution of ψ_k:

- Function of environment

New Design Paradigms

Take impact of noisy samples into account

Only keep noise-free samples

	Non-ideal noise profile: *cross-layer*	Non-ideal noise profile: *no cross-layer*	Ideal noise-profile
drop	**Scenario#3** ?	**Scenario#2** ?	**Scenario#1 Sinopoli et al. & Liu et al.**
keep all	?	?	X

Scenario#1: Sinopoli et al. (TAC 04), to maintain stability:

$$\text{for non-mobile nodes}: p_{drop,scenario\#1} < A^{-2}$$

Next Class

- We will complete the table next time for general communication noise variance and Signal to Noise Ratio distribution