

CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 202

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Problem Set #6

Issued: 12 Feb 09
Due: 19 Feb 09

Reading: Abraham, Marsden, and Ratiu (MTA), Sections 5.1 and 5.2

Problems:

1. MTA 5.1-1: properties of the adjoint map on $GL(n)$
2. MTA 5.1-2: tangent group
3. [Warner, page 135, #16; MTA 5.1-5]
 - (a) Let G be a Lie group. Show that the set of right invariant vector fields on G forms a Lie algebra under the Lie bracket operation and that it is naturally isomorphic to $T_e G$.
 - (b) Let $\phi : G \rightarrow G$ be the diffeomorphism defined by $\phi(g) = g^{-1}$. Prove that if $X \subset TG$ is a left invariant vector field on G then $\phi_*(X)$ is a right invariant vector field whose value at e is $-X(e)$. Further show that $X \mapsto \phi_*(X)$ gives a Lie algebra isomorphism of the Lie algebra of left invariant vector fields with the Lie algebra of right invariant vector fields on G .
(A Lie algebra isomorphism is a linear mapping $A : V \rightarrow V$ which preserves the Lie bracket: $A[\xi, \eta] = [A\xi, A\eta]$.)
4. MTA 5.2-1, parts (iii)–(v): calculations on $SO(3)$
5. MTA 5.2-5: the Euclidean group