

CALIFORNIA INSTITUTE OF TECHNOLOGY  
Control and Dynamical Systems

CDS 202

Problem Set #2

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Issued: 15 Jan 09  
Due: 22 Jan 09

Reading: Abraham, Marsden, and Ratiu (MTA):

- Review Sections 2.1 and 2.2 (covered in CDS 201) as needed
- Read Sections 2.3–2.4
- Read Section 3.1–3.3

Problems:

1. MTA 2.3-1: Derivative of bilinear maps
2. MTA 2.3-4: Composition of a nonlinear and linear maps
3. MTA 3.1-4 (i) and (ii): Manifold structure of the Möbius band.

*Optional:* Try to use your intuition about Möbius band to answer the following questions (then try them to see if you are right):

- Consider a Möbius band of finite width, like the one shown in Figure 3.4.4. What happens is you cut it down the center with a pair of scissors? Is the resulting set a manifold? Is it connected?
- Repeat the experiment, but this time cutting the Möbius band one third of the distance from one of the edges.

4. MTA 3.1-5: Compactification of  $\mathbb{R}^n$ .

5. [Guillemin and Pollack, page 5, #3]

Let  $M$ ,  $N$ , and  $P$  be smooth manifolds and let  $f : M \rightarrow N$  and  $g : N \rightarrow P$  be smooth maps.

- (a) Show that the composite map  $g \circ f : M \rightarrow P$  is smooth.
- (b) Show that if  $f$  and  $g$  are diffeomorphisms, so is  $g \circ f$ .

(You may use the fact that the composition of smooth functions between open subsets of Euclidian spaces are smooth.)

6. MTA 3.3-1: Graphs of manifolds