Dynamical Systems and Control in Micro/Nano-photonics
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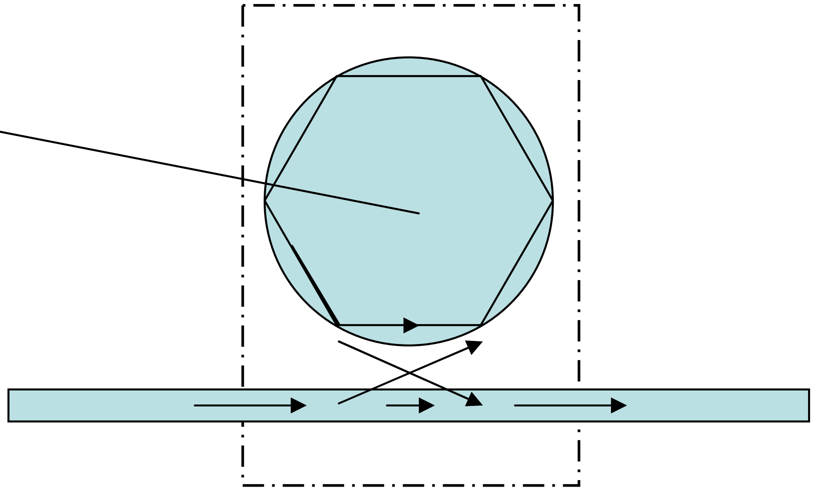
Motivation

- Microphotonic structures can support wavelength-scale optical fields and high quality factors
  - Very large per photon electric field strengths ($E \sim 1/V^{1/2}$)
  - Long photon lifetimes ($\tau_{ph} \sim 1/Q$)
  - Significant light-matter interaction for a small number of photons
- This enhanced light-matter interaction can be exploited to create novel devices
  - Low power switching
  - Low threshold lasers
- The enhanced light-matter interaction can also cause deleterious effects
  - Optical loss due to absorption by generated free carriers
- Nonlinear effects in a silicon microdisk system have recently been studied
  - System of coupled differential equations has been used as a model
  - Direct numerical integration yields good agreement with experiment
  - Can the tools of nonlinear dynamics be used to generate more physical intuition about the global behavior of the system?
  - Can the tools of control systems be used to force the system to behave in some desired way for specific device applications?

The Problem



- Consider an optical resonator
  - It can be considered a SISO system if we add input and output terminals (optical fiber)
  - EOM for that idealized system
    - $da/dt = (-\gamma_a/2 + i(\omega-\omega_o))a + kS$
- Add in “reality”
  - Semiconductor properties
    - Absorption processes
      - Linear $P_{abs,linear} \sim |a|^2$
      - Two photon (TPA) $P_{abs,TPA} \sim |a|^4$
      - Free-carrier (FCA) $P_{abs,FCA} \sim N|a|^2$
    - Charge carrier dynamics
      - TPA generates free carriers $\sim |a|^4$
      - Free carriers decay $\sim -\gamma_n N$
    - Temperature dependent!
      - Absorption increases temp $\sim P_{abs}$
      - Temp decays $\sim -\gamma_{th}\Delta T$
  - State of system characterized by
    - a, N, ΔT
    - Full E’sOM- to be restated later

$$da/dt = (-\gamma_a/2 + i(\omega-\omega_o))a + kP_{in}^{1/2}$$

$$dN/dt = -\gamma_n N + \beta|a|^4$$

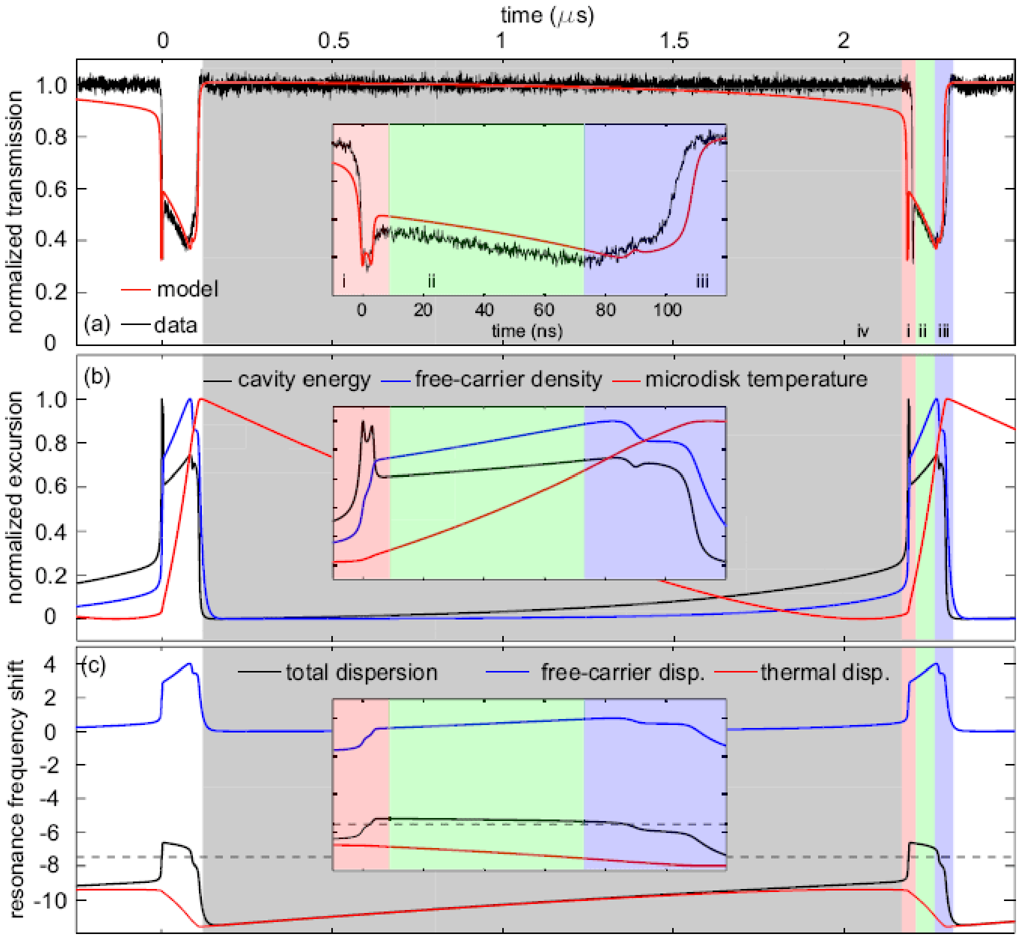
$$d\Delta T/dt = -\gamma_{Th}\Delta T + \beta_1|a|^4 + \Gamma N|a|^2$$

$$\omega_o = \omega_o(1-1/n_{si}\, dn_{si}/dT\, \Delta T - 1/n_{si}\, dn_{si}/dN\, N)$$

$$\gamma_a = \gamma_{a,o} + c_1\Gamma N|a|^2 + c_2\beta|a|^4$$

The Problem (cont'd)

- Numerically integrate model
  - Interesting dynamics
- Investigate further
  - Global behavior vs.
    - Q
    - V_{eff}
    - Input wavelength



Non-linear dynamic equations for a single mode

Wave amplitude: real and imaginary parts	$a = u + iv$
Detuning between input and the resonant frequency	$\delta\omega_0 = \omega - \omega_0$
Free carrier density	N
Temperature offset of the microdisk	ΔT
Effective volume of the cavity	V_{eff}

$$
\begin{aligned}
\frac{du}{dt} &= -\delta\omega_0 v - A_1\Delta T v - A_2 N v - \gamma_{a,o} u - \frac{A_3(u^2+v^2)u}{V_{eff}} - A_4 N u \\
\frac{dv}{dt} &= \delta\omega_0 u + A_1\Delta T u + A_2 N u - \gamma_{a,o} v - \frac{A_3(u^2+v^2)v}{V_{eff}} - A_4 N v - A_5\sqrt{\gamma_{a,0}} \\
\frac{dN}{dt} &= -A_6 N + \frac{A_7(u^2+v^2)^2}{V_{eff}^2} \\
\frac{d\Delta T}{dt} &= -A_8\Delta T + \frac{A_9\gamma_{a,o}(u^2+v^2)}{V_{eff}} + 2\frac{A_3A_9(u^2+v^2)^2}{V_{eff}^2} + 2\frac{A_4A_9N(u^2+v^2)}{V_{eff}^2}.
\end{aligned}
$$

Possible analytical strategies

- Fix all parameters except $\delta\omega_0$, A_7/V_{eff}^2 and $\gamma_{a,0}$ which are the more interesting parameters in applications.

- $\omega \in [1.25, 1.26]10^{15} rad/s$, $V_{eff} \in [1, 1000](\lambda_0/n_{Si})^3$, $\gamma_{a,0} \in [20MHz, 20GHz]$

- Equilibrium points and their stability.

- Relation between parameter values and equilibrium points (Bifurcation analysis).

Possible numerical strategies

- Existing numerical simulations and experiments suggest that periodic solutions exist (Johnson et. al. 2006).
- Approximate analytically the periodic solutions using Poincaré-Lindstedt method..
- Numerically compute a Poincaré map.
- LOCBIF for bifurcation analysis.
- Bifurcation control